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Square roots from GSp_4 or $GL(2)xGL(2)$

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"If the formal square root of an abelian surface over Q looks exceedingly like an elliptic curve, it has to be an elliptic curve."
We discuss what such a proposition might mean, and prove the most straightforward version where the precise condition is simply that the L-function of the abelian surface possesses an entire holomorphic square root. The approach follows the Diophantine principle that algebraic numbers or zeros of L-functions repel each other, and is in some sense similar in spirit to the Gelfond–Linnik–Baker solution of the class number one problem.
We discuss furthermore this latter connection: the problems that it raises under a hypothetical presence of Siegel zeros, and a proven analog over finite fields. The basic remark that underlies and motivates these researches is the well-known principle (which is a consequence of the Deuring–Heilbronn phenomenon, to be taken with suitable automorphic forms f and g): an exceptional character χ would cause the formal $\sqrt{L(s,f)L(s,f\bigotimes\chi)L(s,g)L(s,g\bigotimes\chi)}$ to have a holomorphic branch on an abnormally big part of the complex plane, all the while enjoying a Dirichlet series formal expansion with almost-integer coefficients. This leads to the kind of situation oftentimes amenable to arithmetic algebraization methods. The most basic (qualitative) form of our main tool is what we are calling the "integral converse theorem for $GL(2)$," and it is a refinement of the unbounded denominators conjecture on noncongruence modular forms.
This talk will be partly based on a joint work with Frank Calegari and Yunqing Tang.

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