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Kummer Extensions, with Applications to Generalized Cross-Ratios, Functions on Hilbert Schemes, and the Gross-Zagier Conjecture

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Kummer extensions are extensions of Hodge structure of the form 0 -> Z(1) -> K -> Z(0) -> 0. The group of such extensions is isomorphic to C^x. I will show how to construct such extensions in two situations, one geometric and the other arithmetic. The arithmetic work is joint with Jeanine Van Order.
The geometric construction begins with a smooth projective variety P over the complex numbers. We are given algebraic cycles A and B on P. We assume A and B are homologous to 0 (Betti cohomology) and have disjoint supports. We assume further dim B=codim_P(A)-1 and H^{2codim A-1}(P)=(0). (Ex. P=P^1, A=a-a', B=b-b' disjoint 0-cycles). With these assumptions, the assumed cohomological vanishing means that the height biextension associated to the height <A,B> degenerates, yielding a Kummer extension. The extension class in C^x can be thought of as a generalized cross-ratio of A and B. In particular, the construction yields functions on the appropriate Hilbert schemes.
The arithmetic construction grows from work of A. Mellit who proved some special cases of Gross-Zagier in his thesis in 2008. One starts with a smooth, projective variety X of dimension n. One is given a motivic cohomology class in CH^p(X,1) = H^{2p-1}_M(X,Z(p)) and an algebraic cycle class in H^{2n-2p+2}(X,Z(n-p+1)). The (higher) Abel Jacobi class associated to the cycle in CH^p(X,1) corresponds to a Hodge extension 0 -> H^{2p-2}(X,Z(p)) -> V -> Z(0) -> 0. Then multiplication by the cycle class pushes out the extension to yield a Kummer extension 0 -> Z(1) -> K -> Z(0) -> 0.

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