BSD and Estimates for Class Groups of Number Fields

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(Joint with A.Shankar)

We consider question of bounding (Fixed) torsion in class groups of number fields: Given two integers m, n, what is the smallest constant $e_{m,n}$ such that $\#Cl(K)[m]$ is bounded by $|Disc(K)|^{e_{m,n}+o(1)}$ for all number fields K of degree m? Conjecturally $e_{m,n} = 0$. Unconditionally, the bound of 1/2 follows directly from the Brauer-Siegel Theorem and this is the best we can do in most cases, though there are improvements in existing cases. We discuss a method to bound these quantities by re-interpreting the torsion in the class group as a finite Selmer group, and embedding it into an appropriate motive. We prove that if one assumes GRH + Refined BSD, one can prove the bound $e_{5,n} = 1/4$ independently of what n is. We also show that this can be made unconditional in the function field case, where even though BSD is open, in a sense, one still has the refined BSD formula.

The method requires us to find appropriate motives with suitably trivial modular Galois representations, and one of our hopes in giving this talk is to encourage the audience to aid us in finding such motives!

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