

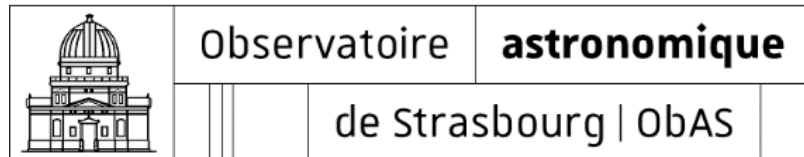
# Coverage of astronomical datasets



---

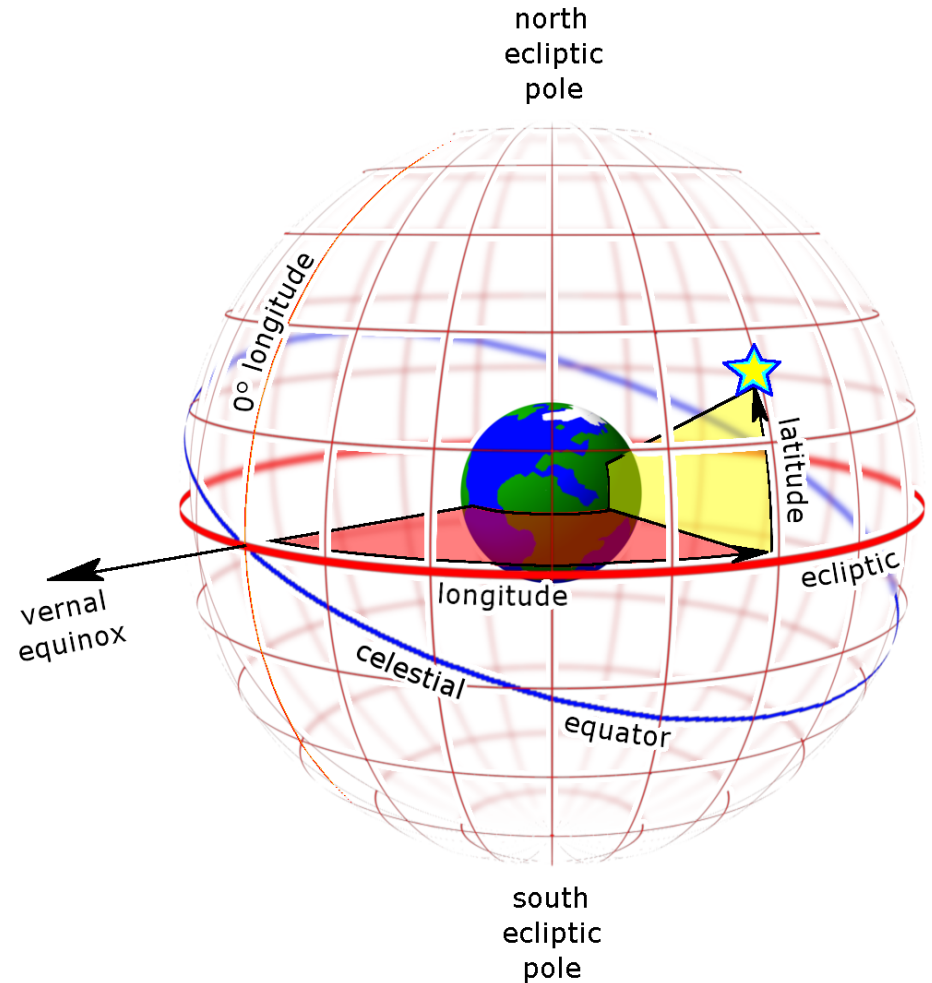
Summer School “Geometry and data”

Sébastien Derriere  
sebastien.derriere@astro.unistra.fr



# □ Astronomical observations

- Spherical coordinates
- Often, distances are not known



# □ JWST image

- Neptune & satellites
- Stars
- Galaxies

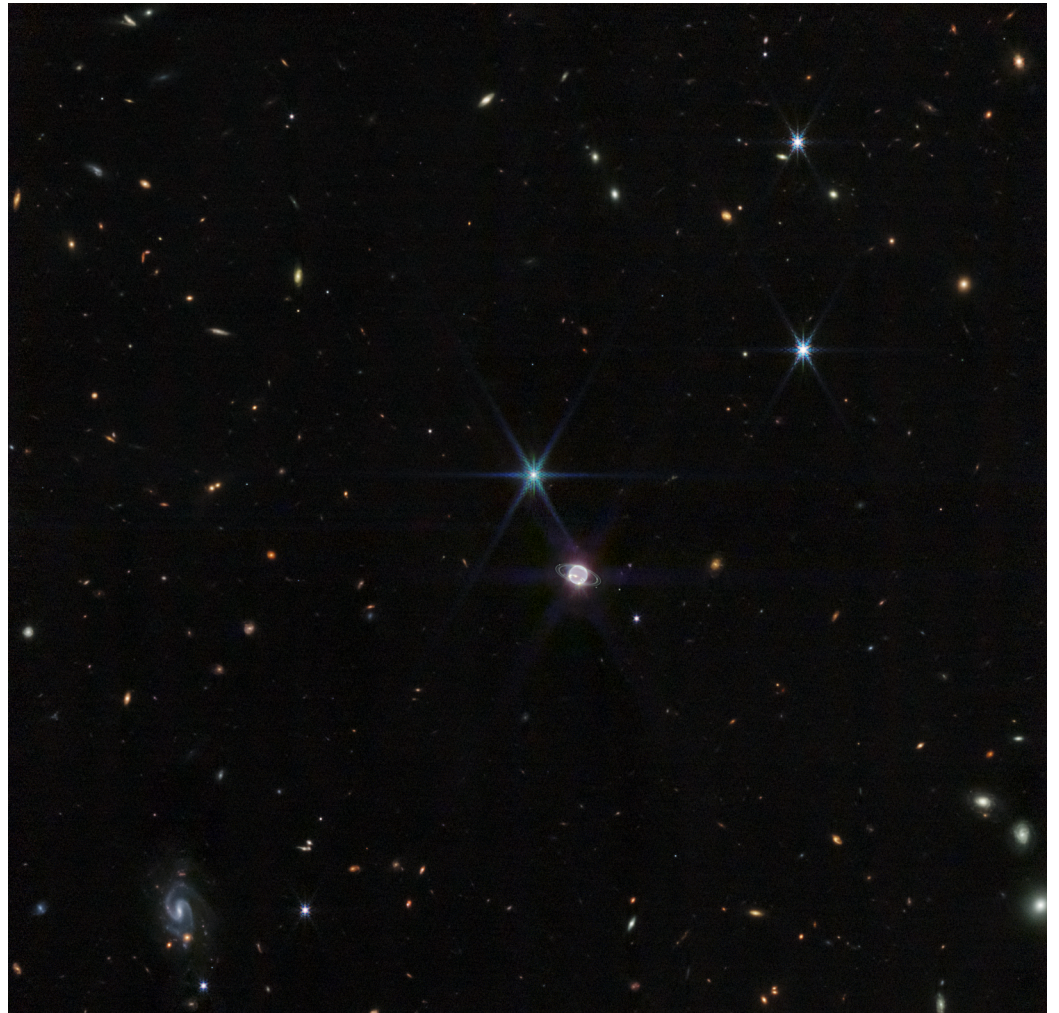


# □ Astronomical images

- 2D spatial sampling  
(field of view)

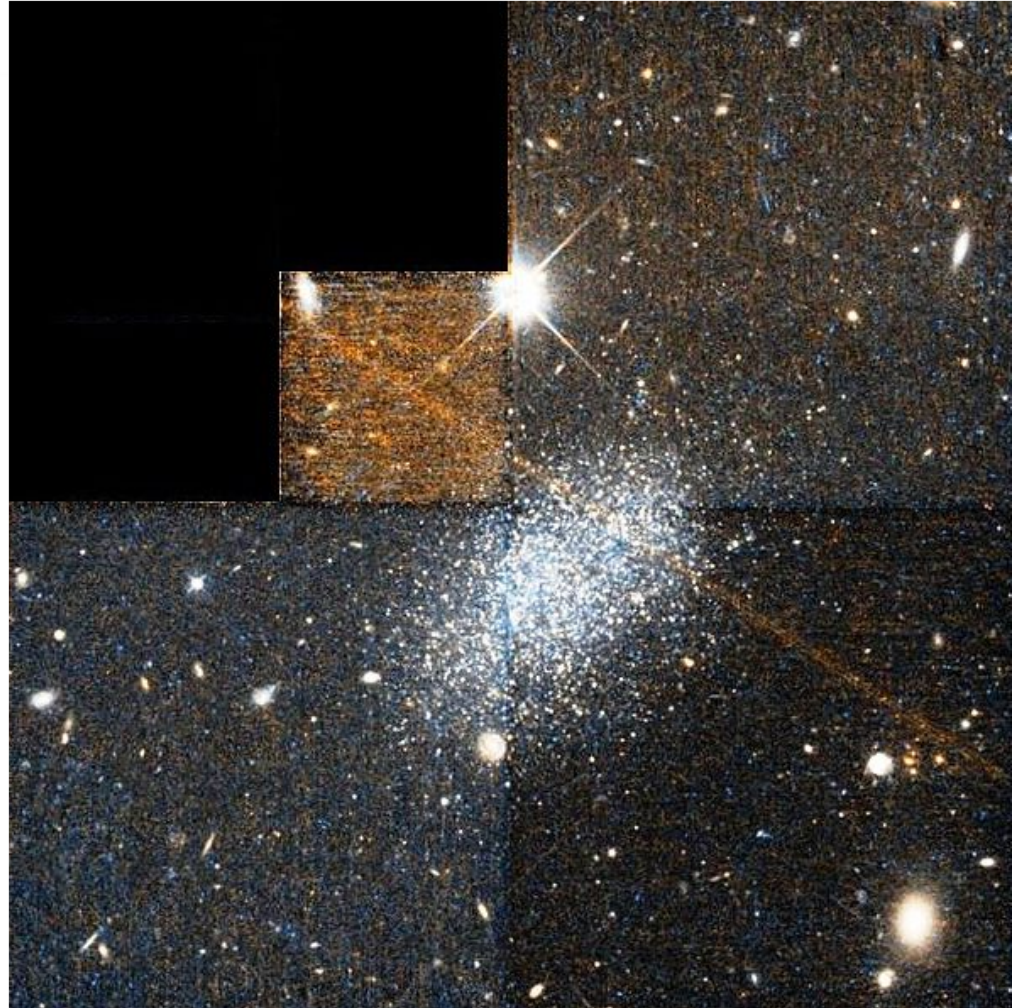
... but also

- Time sampling  
(exposure time)
- Frequency sampling  
(wavelength/energy)
- Limits in sensitivity
- ...



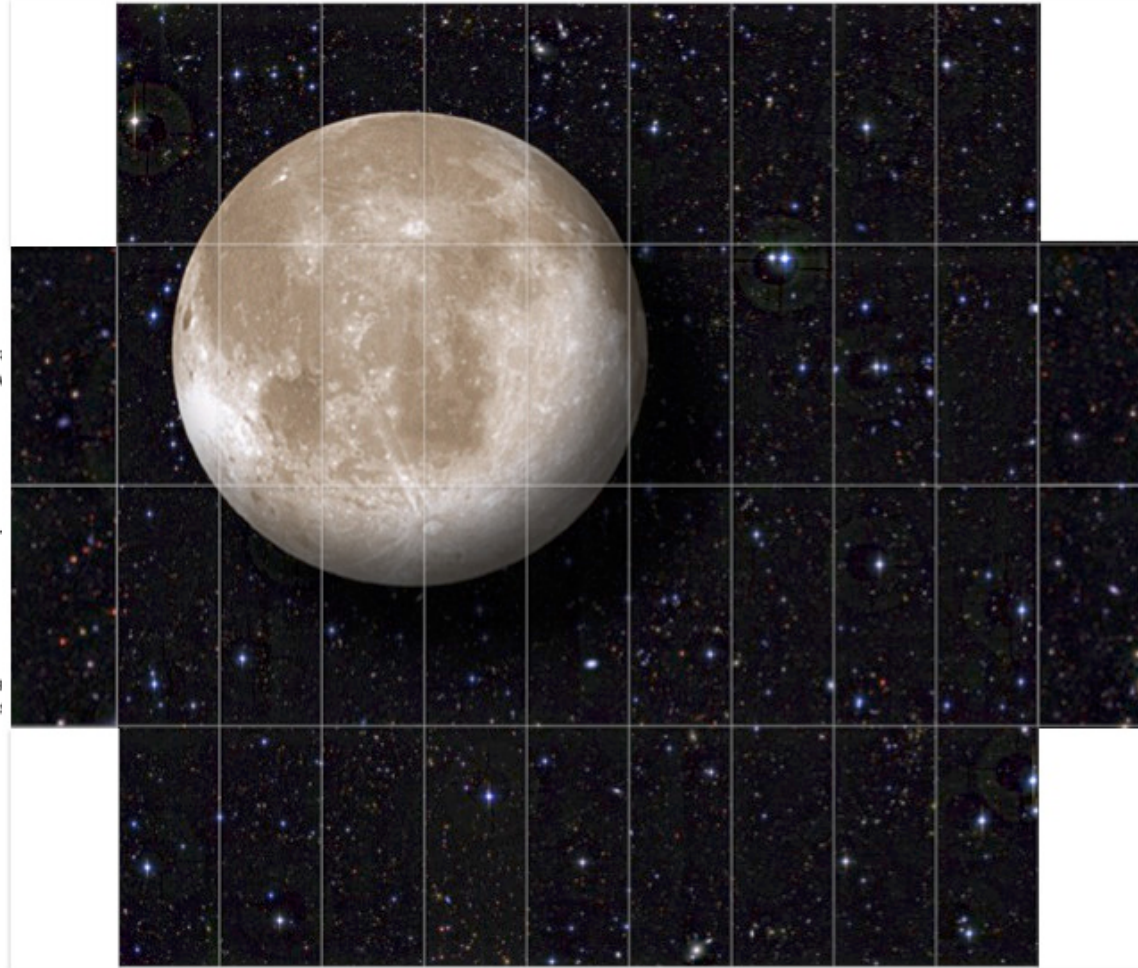
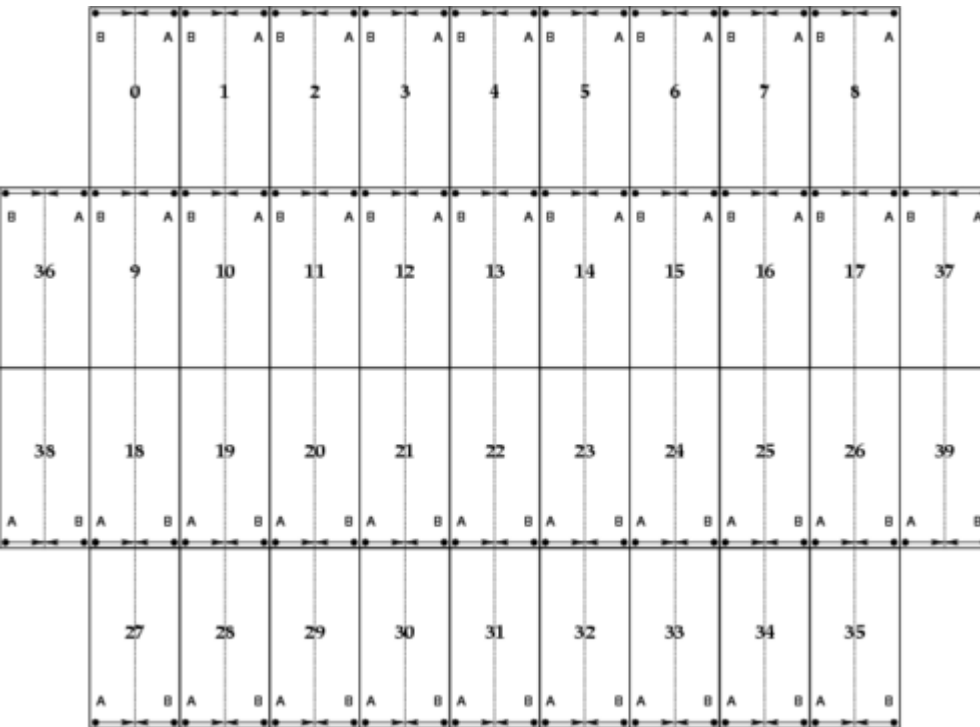
# □ Astronomical images

- Shape can be complex !  
(HST WFPC2)



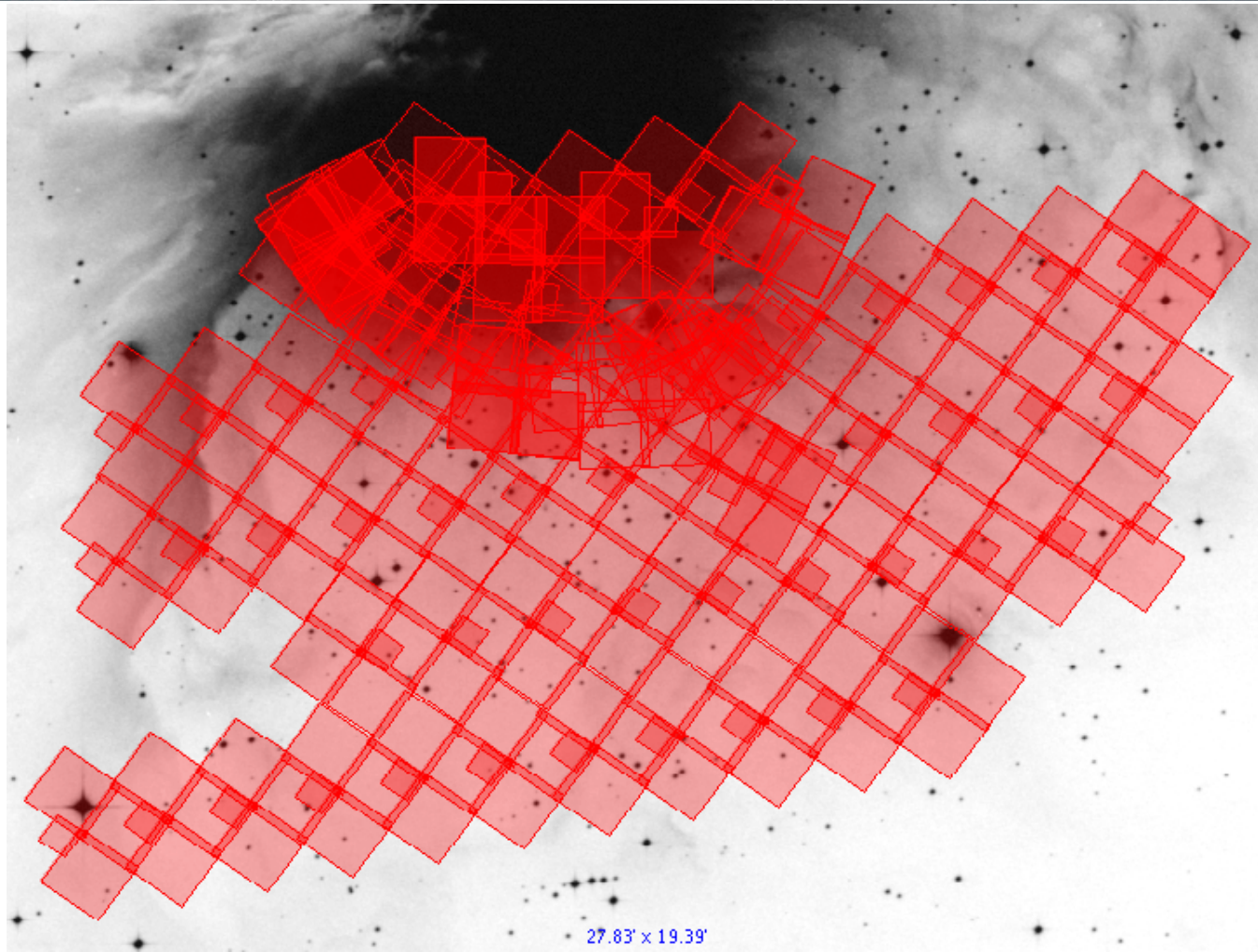
# □ Astronomical images

- Mosaics of detectors (MEGACAM, CFHT)



# □ Sky surveys

- Collections of images, covering a fraction of the sky
- How to provide seamless access to image sky surveys ?
- How do you describe the spatial coverage ?
  - Shape ?
  - Sky fraction ?



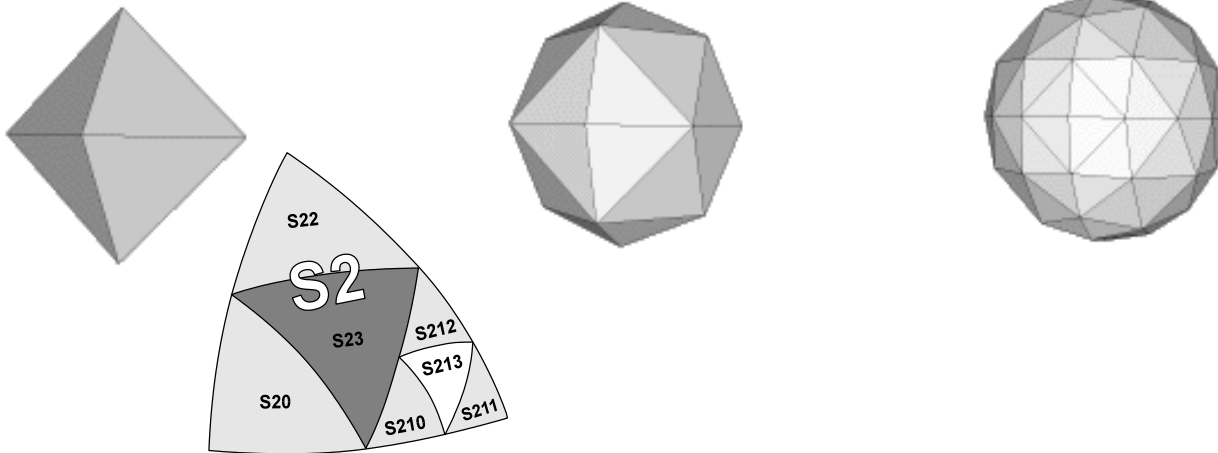
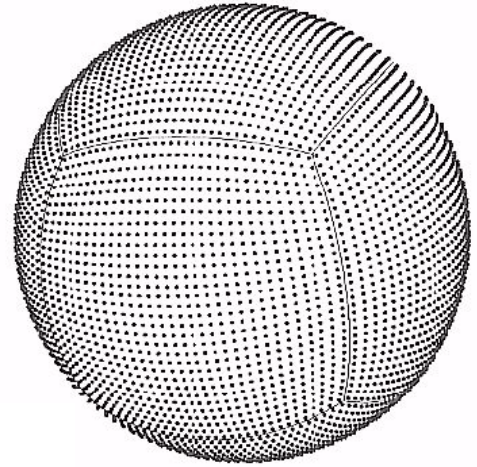
# □ Sky surveys : all-sky view and coverage ?

- Describe complex regions on the celestial sphere
- Compare, combine coverage of several datasets (intersect, ...)
- Global properties
  - Sky fraction (full-sky, ...)
  - Shape (multiple regions, gaps, holes, ...)



# □ Sky partitions : several schemes

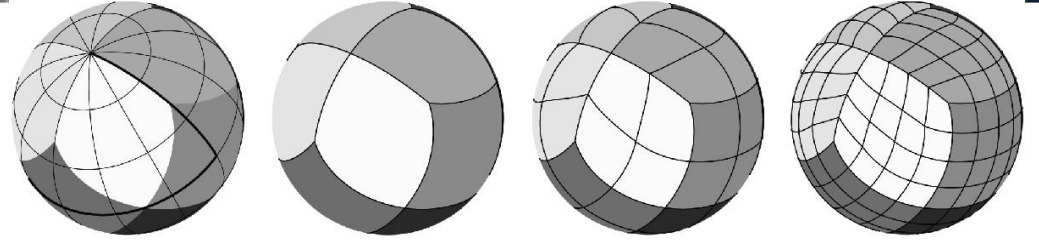
- Spherical-cubic projection, quad-cubes
  - 6 faces of a cube, subdivided recursively into 4 squares
- HTM : Hierarchical Triangular Mesh
  - 8 triangles, subdivided recursively into 4 triangles



# □ Sky partition : HEALPIX

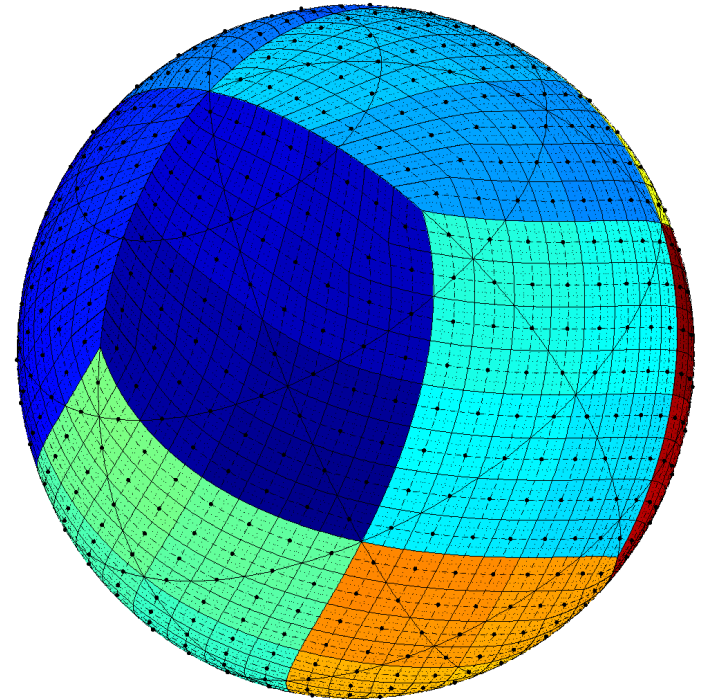
- HEALPix (Gorski et al. 2005)

- 12 quadrilateral pixels
- 2x2 division at each level
- Equal area, Iso-latitude



- All these partitions rely on quad-trees

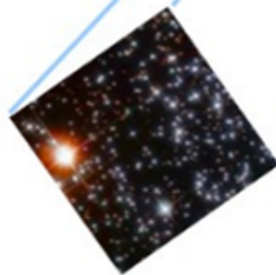
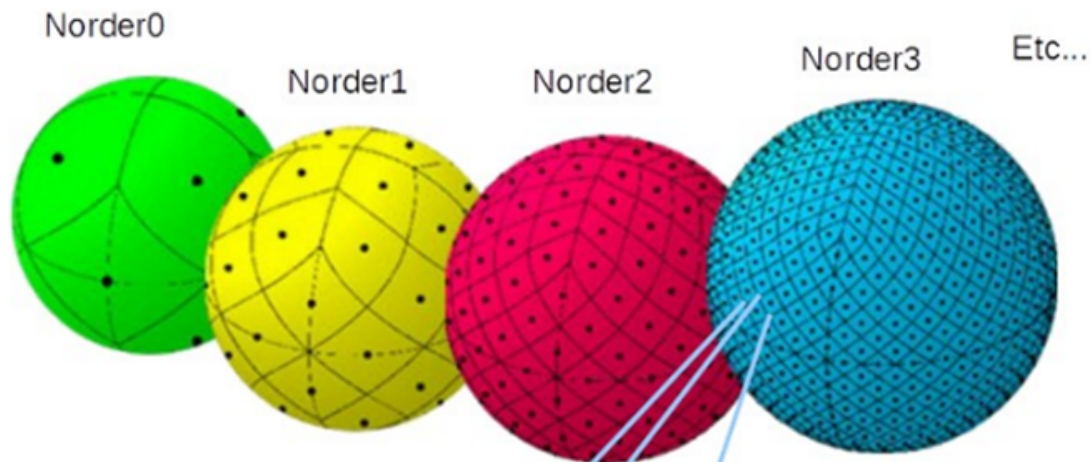
- Each cell is subdivided into 4 children, recursively



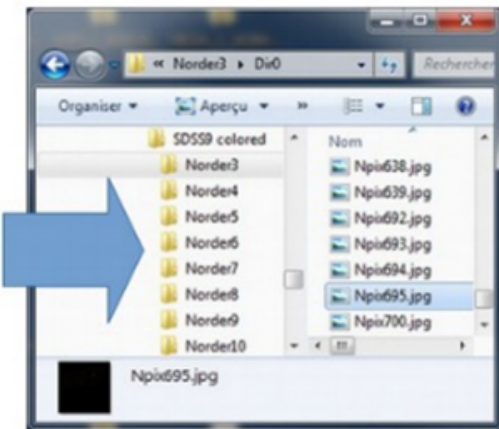
# HiPS = Hierarchical Progressive Surveys

- Mosaic of HEALPix tiles (HEALPix pixel geometry)

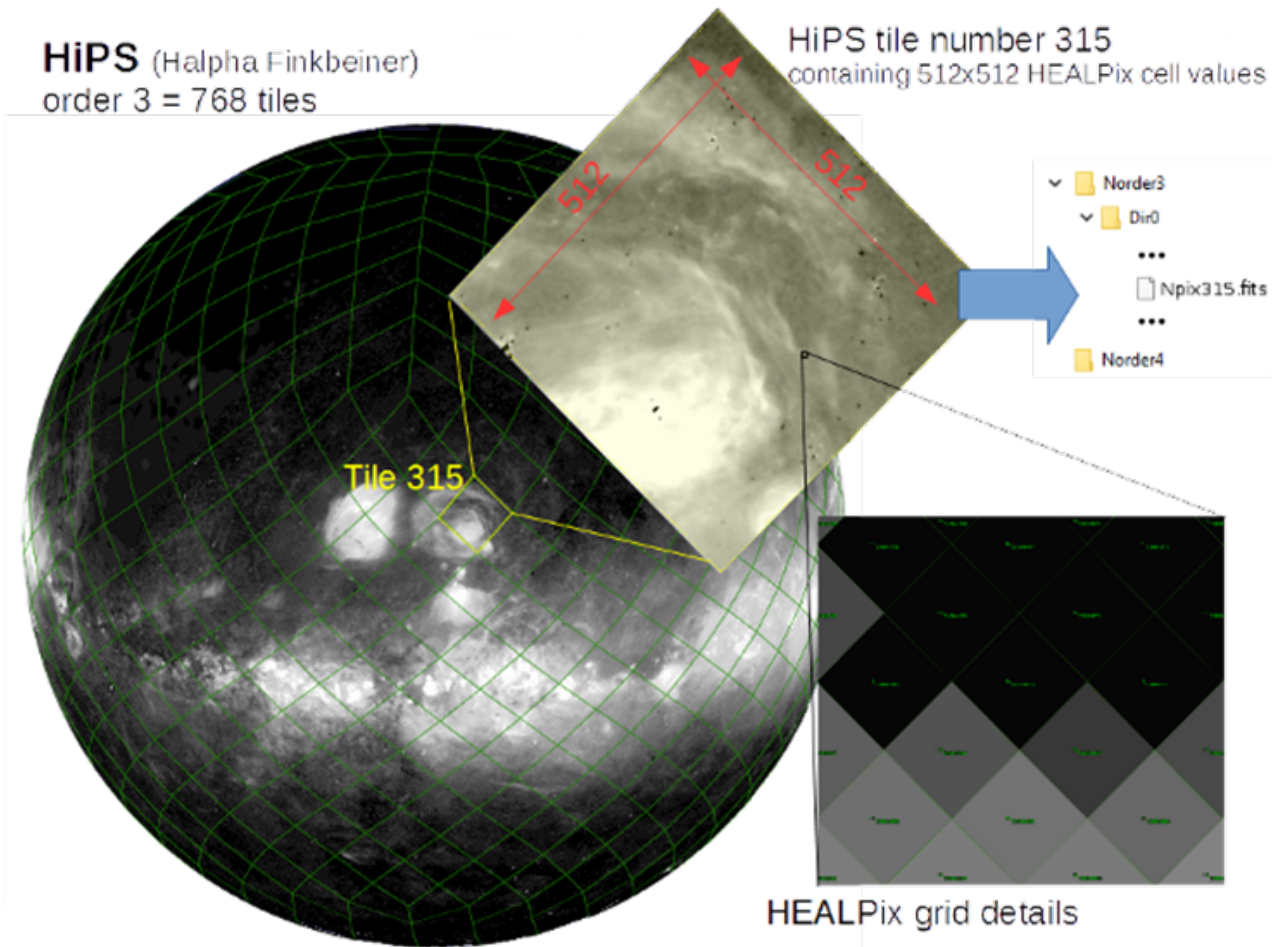
(Fernique et al. 2015)



Npix695.jpg



# HiPS = Hierarchical Progressive Surveys



# HiPS as a standard

- International Virtual Observatory alliance standard – [ivoa.net](http://ivoa.net)



## HiPS – Hierarchical Progressive Survey

Version 1.0  
IVOA Recommendation  
19<sup>th</sup> May 2017

This version:  
1.0: Recommendation 2017-05-19

Previous version(s):  
1.0: Proposed Recommendation 2017-04-06  
1.0: Proposed Recommendation 2017-04-03  
1.0: Proposed Recommendation 2017-02-07  
1.0: Working Draft 2016-06-23

Interest/Working Group:  
Applications: <http://www.ivoa.net/work/bin/view/VOA/voaApplications>

Editor:  
Pierre Ferrique

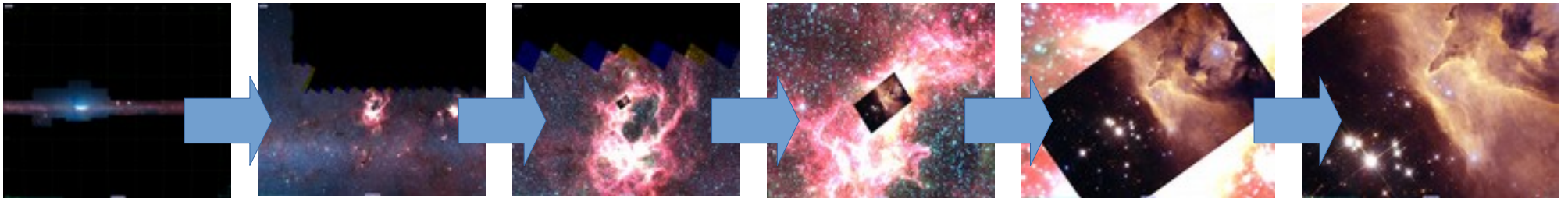
Authors:  
Pierre Ferrique, Mark Allen, Thomas Boch, Tom Donaldson, Daniel Durand,  
Ken Ebisawa, Laurent Michel, Jesus Salgado, Felix Stoehr

### Abstract

This document presents HiPS, a hierarchical scheme for the description, storage and access of sky survey data. The system is based on hierarchical tiling of sky regions at finer and finer spatial resolution which facilitates a progressive view of a survey, and supports multi-resolution zooming and

# □ HiPS : huge success

- **Hierarchical Progressive Survey**
  - *“The more you zoom in on a particular area, the more details show up”*
- Multi-resolution **HEALPix** data structure for **Images**, **Catalogues**, 3-dimensional data **cubes**, ...
- Conserves scientific data properties alongside visualisation considerations
- No databases or servers, just HTTP



# □ A few examples in Aladin

Aladin = sky atlas developed by CDS in Strasbourg.

Acces to 1200+ surveys !

<http://aladin.cds.unistra.fr/hips/>

Aladin v11.0 \*\*\* BETA VERSION (based on v11.057) \*\*\*

Fichier Edition Image Catalogue Graphique Couverture Outil Vue Interop Aide

Données disponibles → 27169  
in view out view

Commande 00:42:53.18 +41:31:16.5

DS5 PanSTARRS SDSS 2MASS WISE GALEX RXMM Gaia Simbad ICRS FoV HST Coronell

PanSTARRS DR1 color-z-zg-g

Des données sont en cours de chargement... voir la "pile"

V	RA ICRS	a RA ICRS...	DE ICRS	a DE ICRS...	Source	Plx	a Plx	PM	pmRA	a pmRA...
VizieR	11.44172569285	0.1901	41.26755131461	0.1333	369268987326157952	0.9265	0.2141	6.056	-3.474	0.212
VizieR	11.4382720459	2.7223	41.26547839034	1.108	369268987331277184	-0.7865	2.2003	10.614	9.629	2.728
VizieR	11.45387919932	0.0294	41.26459044935	0.0204	369268989182454380	0.795	0.0323	11.065	6.486	0.031
VizieR	11.45957249321	0.3166	41.26999334764	0.2769	3692689921885897008	1.3501	0.2482	11.769	11.184	
VizieR	11.46923128308	0.1285	41.26823614024	0.0621	3692689858310623808	0.2564	0.1484	1.7821	1.720	

# □ MOCs

- **Multi-Order Coverage map**  
*“Combine sky regions in few milliseconds”*
- A simple and efficient method to specify any kind of sky regions
- Based on **HEALPix** tessellation
- Existing **libraries**: Java, C, python
- Used in VO tools (Aladin, TOPcat, ...)



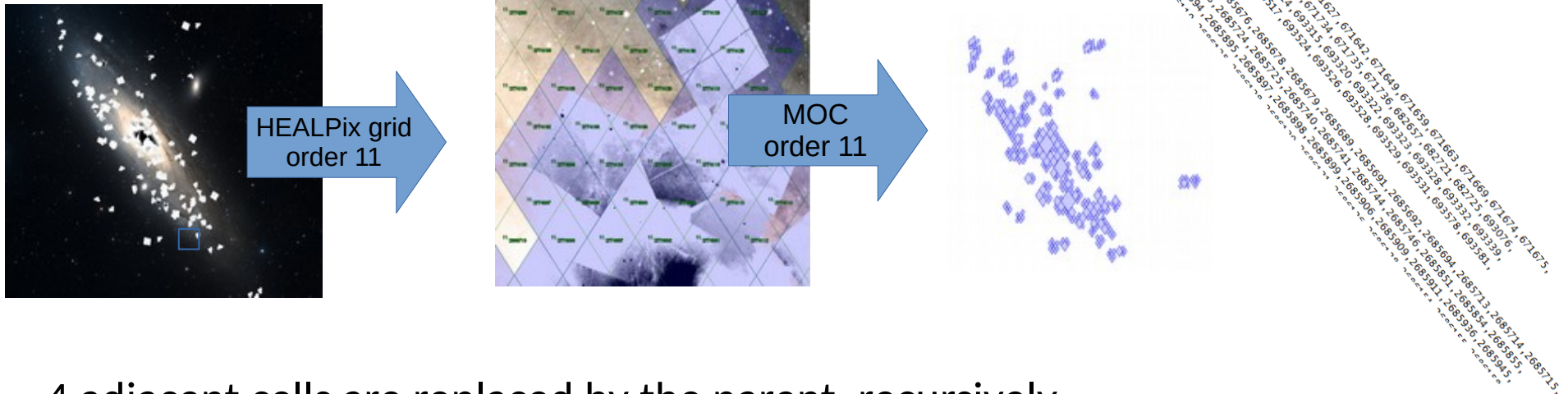
# □ What is a MOC ?

- How to describe the spatial coverage of HST observations around the Andromeda galaxy ?



# □ What is a MOC ?

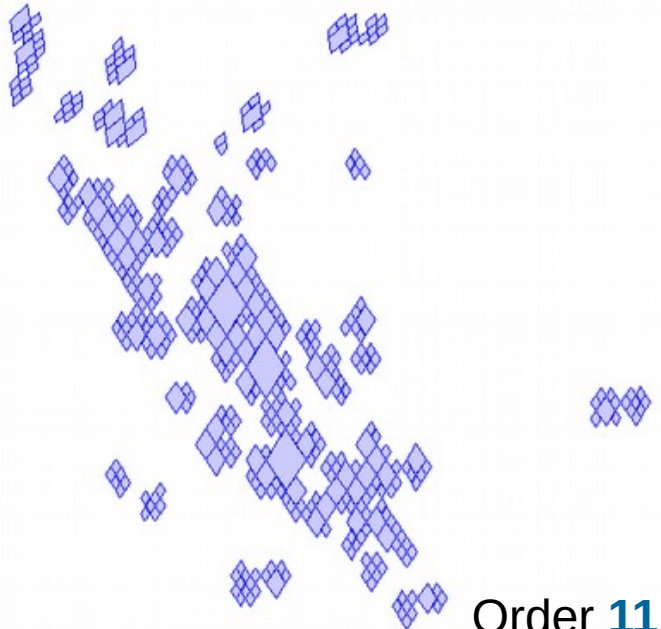
- “Just” the list of HEALPix cell numbers covering a region



- 4 adjacent cells are replaced by the parent, recursively  
→ intrinsic compression
- Store as a FITS table (or JSON)

# □ What is MOC ?

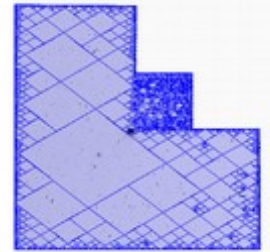
- MOC accuracy depends of the MOC **order**  
= the smallest HEALPix cell level used in the MOC



Order **11**  
=>1.718 arcsec



Order **20**  
=>201.3mas



Order **29**  
=>391  $\mu$ s

• • •

# □ MOC standard



## MOC: Multi-Order Coverage map Version 2.0 IVOA Recommendation 2022-07-27

Working group  
Applications

This version  
<http://www.ivoa.net/documents/moc/20220727>  
Latest version  
<http://www.ivoa.net/documents/moc/20220727>  
Previous versions  
Version 1.1  
Version 1.0

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Editor(s)

Pierre Fernique, Ada Nebot, Daniel Durand



# A few examples in Aladin

Very easy to perform arithmetic operations between MOCs : intersections, union, ...

The screenshot displays the Aladin v11.0 interface. The main window shows a multi-wavelength astronomical image with various overlays. A large orange/red region is labeled 'PanSTARRS DR1 color-z-zg-g'. A bright yellow star is labeled 'M31'. A green box highlights a specific region. The interface includes a menu bar (Fichier, Edition, Image, Catalogue, Graphique, Couverture, Outil, Vue, Interop, Aide), a command line (Commande: 00:42:53.18 +41:31:16.5), and a toolbar on the right. The bottom panel shows a table of data for the selected region.

V	RA ICRS	a RA ICRS...	DE ICRS	a DE ICRS...	Source	Plx	a Plx	PM	pmRA	a pmRA...
Wizier	11.44172569285	0.1901	41.26755131461	0.1333	369268987326157952	0.9265	0.2141	6.056	-3.474	0.212
Wizier	11.4382720459	2.7223	41.26547839034	1.108	369268987331277184	-0.7865	2.2003	10.614	9.629	2.728
Wizier	11.45387978932	0.0284	41.26459044935	0.0204	369268989182454380	0.7965	0.0323	11.065	6.486	0.031
Wizier	11.45957249321	0.3166	41.26909334244	0.2769	3692689921885897008	1.3501	0.2482	11.769	11.184	
Wizier	11.46923128308	0.1285	41.26802614024	0.0621	3692689258310623808	0.3564	0.1484	1.7821	1.720	

# □ MOC 2.0

- Extension to describe not only space coverage, but also time coverage : ST-MOC.
- I'll come back to this at the end with an example (it time allows) !

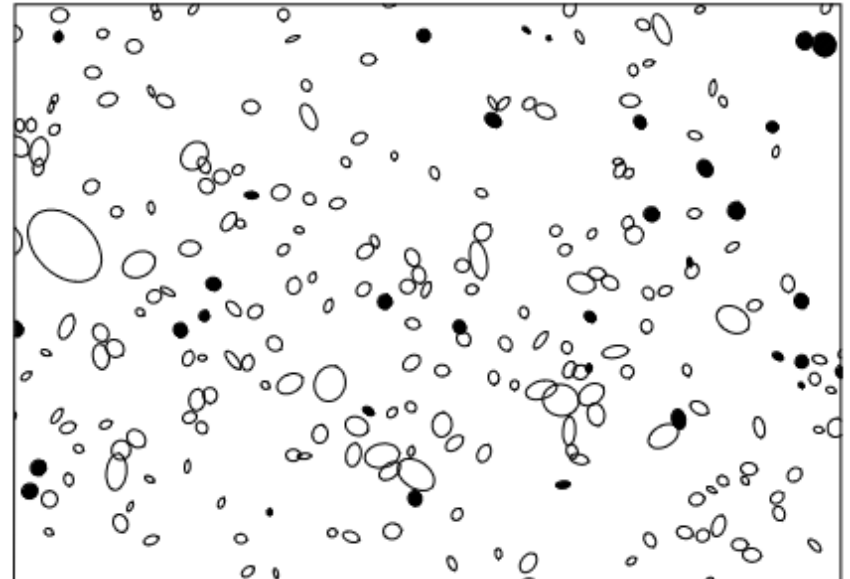
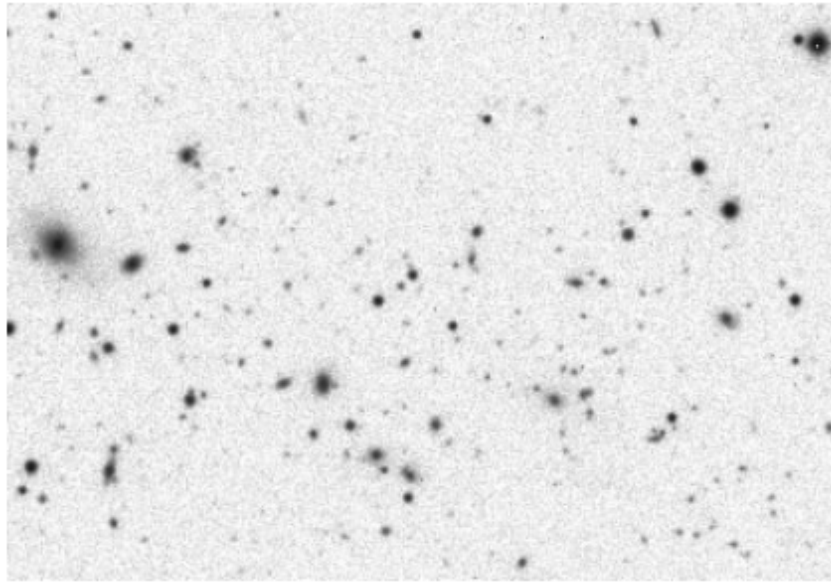
# □ What about datasets coverage ?

- MOCs seem to address the problem, but...
- Can I get general coverage properties ?
  - e.g. sky\_fraction [0..1]  
(and attach these metadata to the dataset to facilitate filtering in discovery)
- What is the optimal HEALPix order ?
- What about coverage of astronomical catalogues ?

# □ Astronomical catalogues

- Extraction of a list of sources from images

Bertin et al. 1996

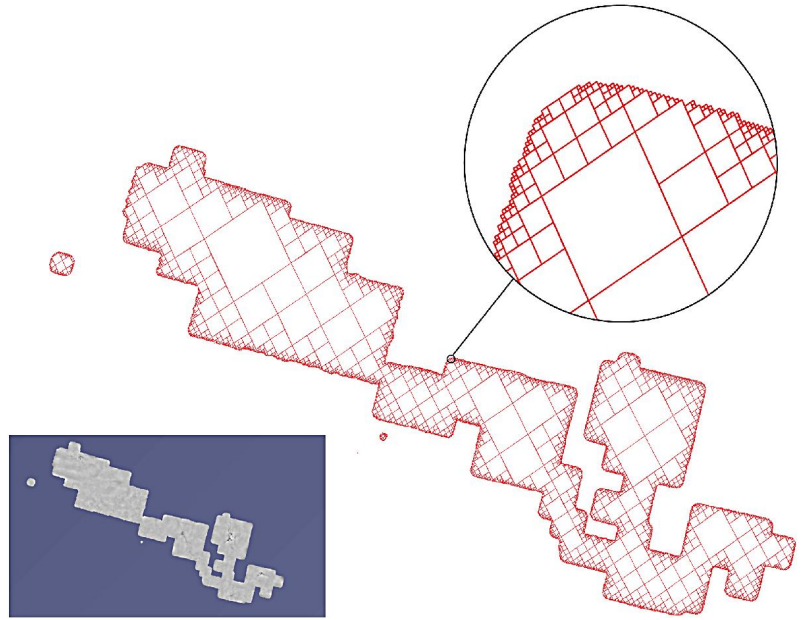
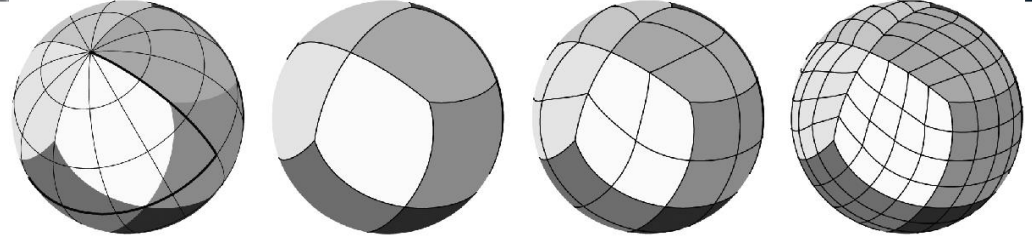


065.014917	+52.334263	0.07	0.07	90	04200358+5220033	15.621	0.068	0.070	18.4	14.980	0.095	0.096	13.2	14.923	0.126	0.126	8.8	AAB	222	221	ccc	665616	<a href="#">4.2</a>	279	798312760
065.013035	+52.334438	0.06	0.06	135	04200312+5220039	14.232	0.038	0.040	66.0	13.789	0.047	0.048	39.7	13.642	0.049	0.050	28.5	AAA	222	221	cco	665556	<a href="#">4.2</a>	99	798312758
065.003712	+52.330948	0.33	0.29	169	04200089+5219514	16.580	0.147	0.147	7.6	16.271	0.272	0.273	4.0	<b>15.681</b>				BDU	220	110	000	060600	<a href="#">10.7</a>	7	798312754
065.004288	+52.333900	0.06	0.06	90	04200102+5220020	14.073	0.033	0.035	76.4	13.137	0.038	0.039	72.3	12.818	0.029	0.030	60.8	AAA	222	111	000	666666	<a href="#">5.8</a>	262	798312752
065.001656	+52.333672	0.23	0.22	108	04200039+5220012	16.442	0.133	0.133	8.6	15.859	0.192	0.192	5.9	<b>15.003</b>				BCU	220	110	cco	061600	<a href="#">5.8</a>	82	798312754
065.005662	+52.338749	0.06	0.06	90	04200135+5220194	11.717	0.021	0.024	669.3	10.729	0.028	0.029	664.2	10.412	0.022	0.023	558.0	AAA	222	111	000	666655	<a href="#">7.6</a>	78	798312779
065.009027	+52.339203	0.06	0.06	90	04200216+5220211	13.832	0.029	0.031	95.4	13.381	0.039	0.040	57.7	13.181	0.041	0.042	43.6	AAA	222	111	ccc	666655	<a href="#">7.6</a>	73	798312781
065.012324	+52.339809	0.21	0.19	45	04200295+5220233	16.607	0.160	0.161	7.4	15.859	0.178	0.178	5.9	<b>15.630</b>				CCU	220	110	cco	060500	<a href="#">7.6</a>	253	798312779
065.018176	+52.337391	0.27	0.27	135	04200436+5220146	16.448	0.127	0.128	8.6	<b>16.087</b>				15.541	0.209	0.209	5.0	BUD	202	101	000	060006	<a href="#">6.4</a>	205	798312766

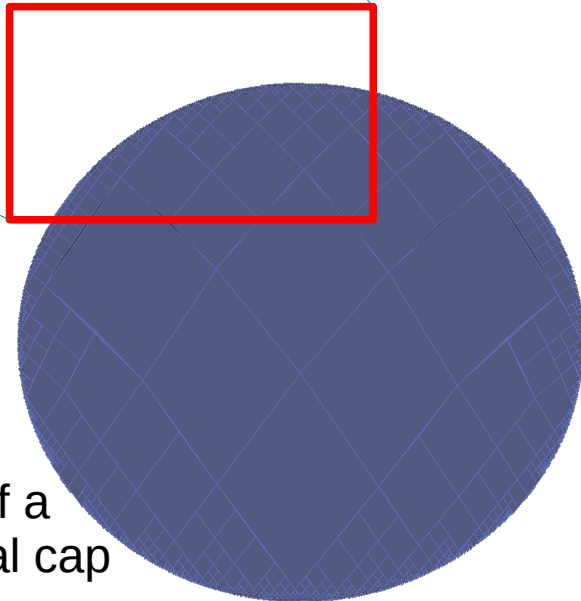
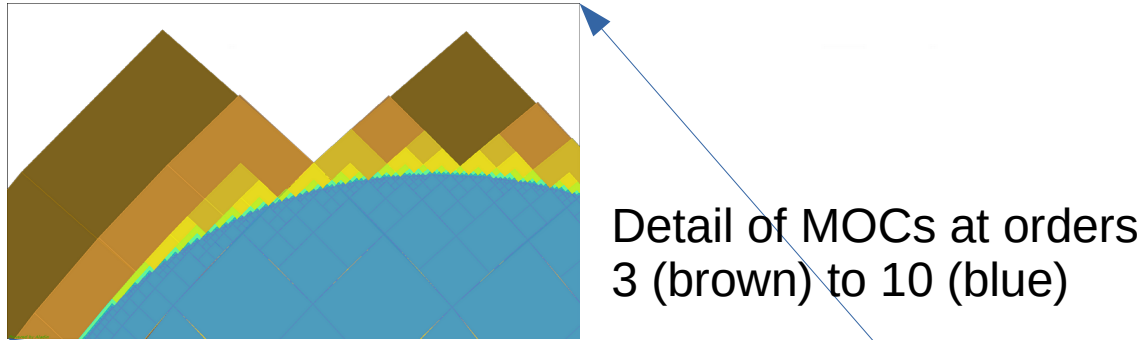


# □ Optimal HiPS & MOC order ?

$k$	$N_{\text{side}} = 2^k$	$N_{\text{pix}}$	$\theta_{\text{pix}}$	$k_{\text{tile},512}$	$N_{\text{tile},512}$	$\theta_{\text{tile},512}$
0	1	12	58°6			
1	2	48	29°3			
2	4	192	14°7			
3	8	768	7°33			
4	16	3072	3°66			
5	32	12 288	1°83			
6	64	49 152	55'0			
7	128	196 608	27'5			
8	256	786 432	13'7			
9	512	3 145 728	6'87	0	12	58°6
10	1024	12 582 912	3'44	1	48	29°3
11	2048	50 331 648	1'72	2	192	14°7
12	4096	201 326 592	51'5	3	768	7°33
13	8192	805 306 368	25'8	4	3072	3°66
14	$2^{14}$	$3.22 \times 10^9$	12'9	5	12 288	1°83
15	$2^{15}$	$1.29 \times 10^{10}$	6'44	6	49 152	55'0
16	$2^{16}$	$5.15 \times 10^{10}$	3'22	7	196 608	27'5
17	$2^{17}$	$2.06 \times 10^{11}$	1'61	8	786 432	13'7
18	$2^{18}$	$8.25 \times 10^{11}$	0'81	9	3 145 728	6'87
19	$2^{19}$	$3.30 \times 10^{12}$	0'40	10	12 582 912	3'44
20	$2^{20}$	$1.32 \times 10^{13}$	0'20	11	50 331 648	1'72
21	$2^{21}$	$5.28 \times 10^{13}$	0'10	12	201 326 592	51'5
22	$2^{22}$	$2.11 \times 10^{14}$	50.3 mas	13	805 306 368	25'8
23	$2^{23}$	$8.44 \times 10^{14}$	25.1 mas	14	$3.22 \times 10^9$	12'9
24	$2^{24}$	$3.38 \times 10^{15}$	12.6 mas	15	$1.29 \times 10^{10}$	6'44
25	$2^{25}$	$1.35 \times 10^{16}$	6.29 mas	16	$5.15 \times 10^{10}$	3'22
26	$2^{26}$	$5.40 \times 10^{16}$	3.15 mas	17	$2.06 \times 10^{11}$	1'61



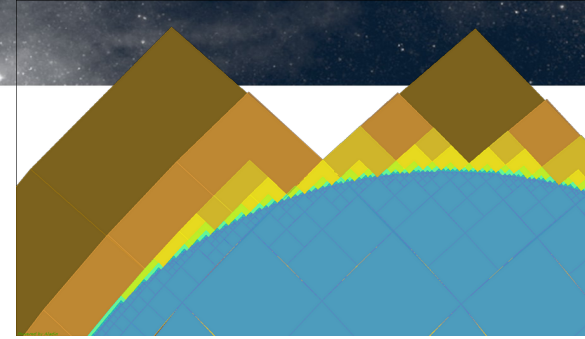
# □ Optimal HiPS & MOC order ?



- MOCs can be used to estimate the sky coverage, by adding the areas of all HEALPix pixels.
- The total MOC area is overestimating the true area, because pixels spread on the perimeter.

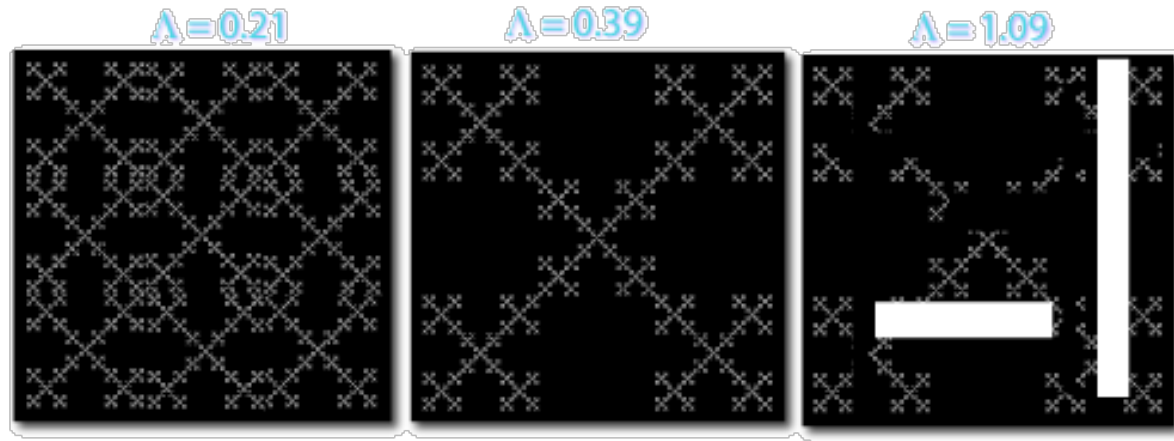
# □ Optimal HiPS & MOC order ?

- The highest the order of the MOC, the better the precision.
- The area of one HEALPix pixel at order  $k$  is  $\Omega_{\text{pix}} = (\pi/3) 4^{-k}$
- The estimated area from a MOC at order  $k$  is related to the true area by a relation of the form  $S_k = S_{\text{true}} (1 + L \cdot 2^{-k})$  where  $L$  is a parameter related to the perimeter of the MOC
- More complex MOCs will have larger values of  $L$
- We define  $\mathcal{L} = \log(L)$  as the **lacunarity** of the dataset



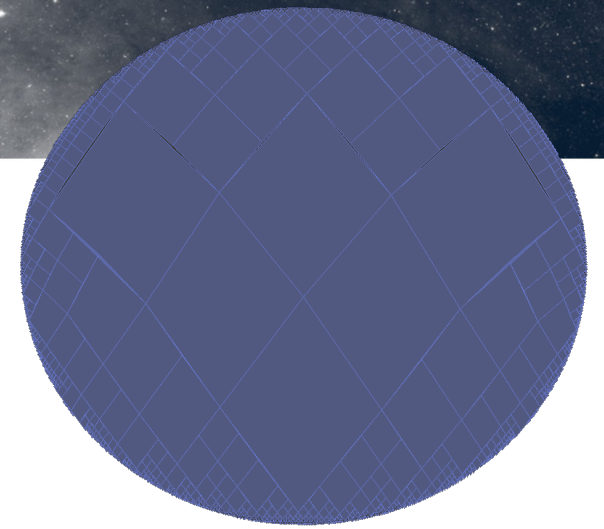
# □ Lacunarity

- Lacunarity (Latin lacuna, meaning "gap") was introduced by Mandelbrot to describe the texture of fractals

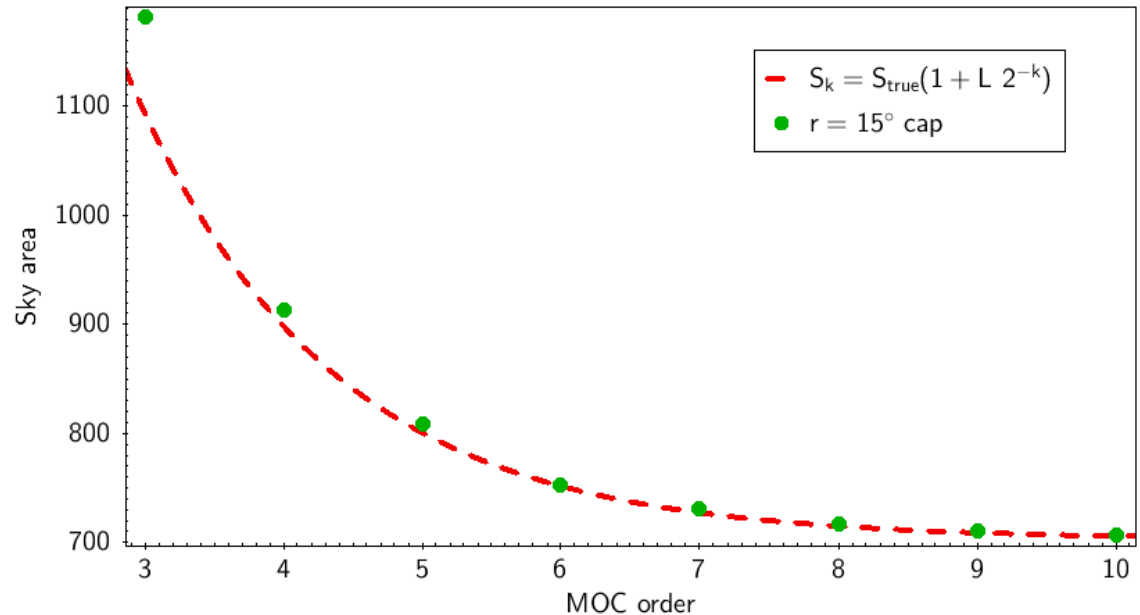


- Extended (see for example Dong (2000)) to describe the distribution of gap sizes in geometric objects :
  - homogeneous objects have low lacunarity
  - heterogeneous objects have high lacunarity

# □ Example on spherical cap



- Fitting the formula  $S_k = S_{\text{true}}(1 + L \cdot 2^{-k})$  over orders  $k=7$  to  $10$  gives an estimated  $S_{\text{true}} = 704.5 \text{deg}^2$  (exact area  $S = 702.8 \text{deg}^2$ ) with  $\mathcal{L} = 0.648$ .

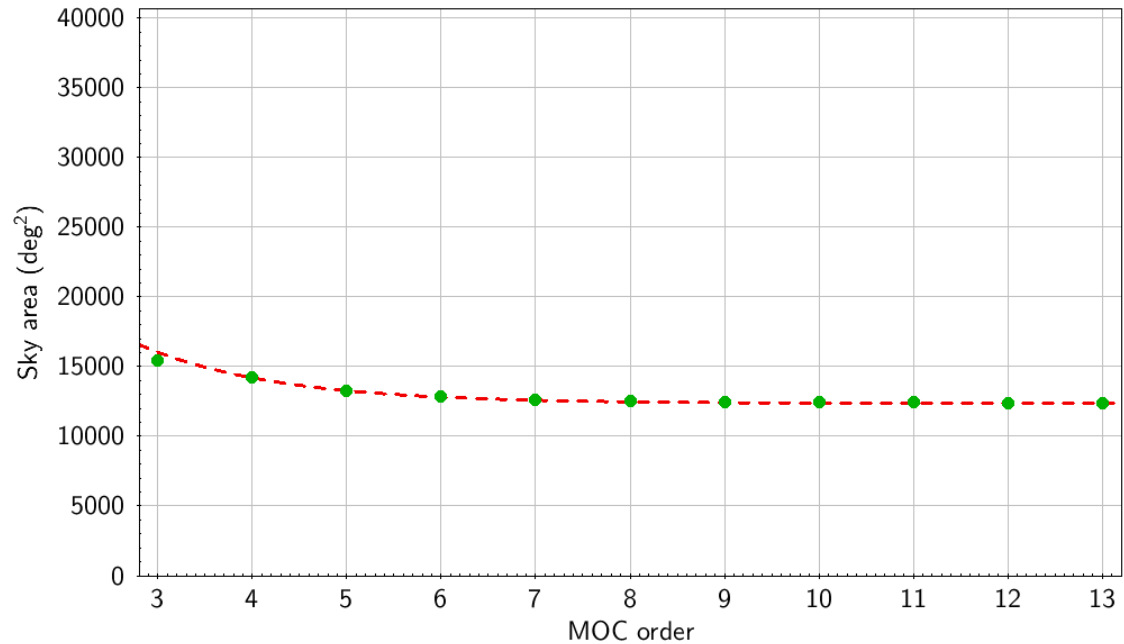
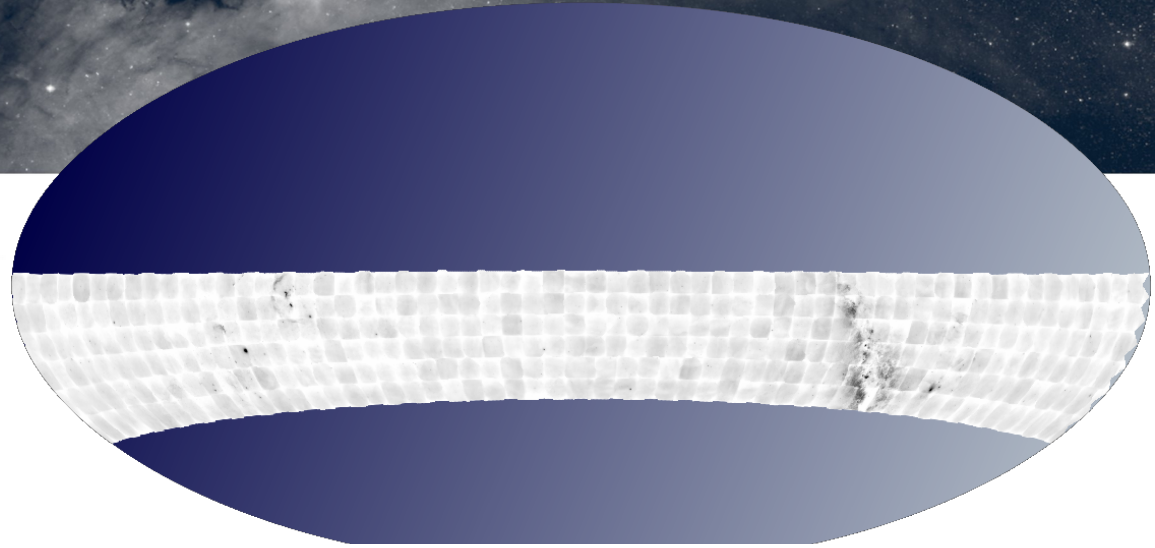


# □ Coverage of catalogues ?!

- More difficult than for image surveys.
- Beware of oversampling !!
  - If the density of sources per HEALPix pixel becomes too low (at high orders  $k$ ), empty pixels cause an underestimation of the area.
- At very high orders  $k$ , asymptotic limit (1 source per pixel)  
 $S_k = N(\pi/3) 4^{-k}$   
with  $N$  the number of catalogue sources

# □ Example

- CDS/P/MAMA/posse
- $S_{\text{true}} = 12358 \text{ deg}^2$
- $\mathcal{L} = 0.377$

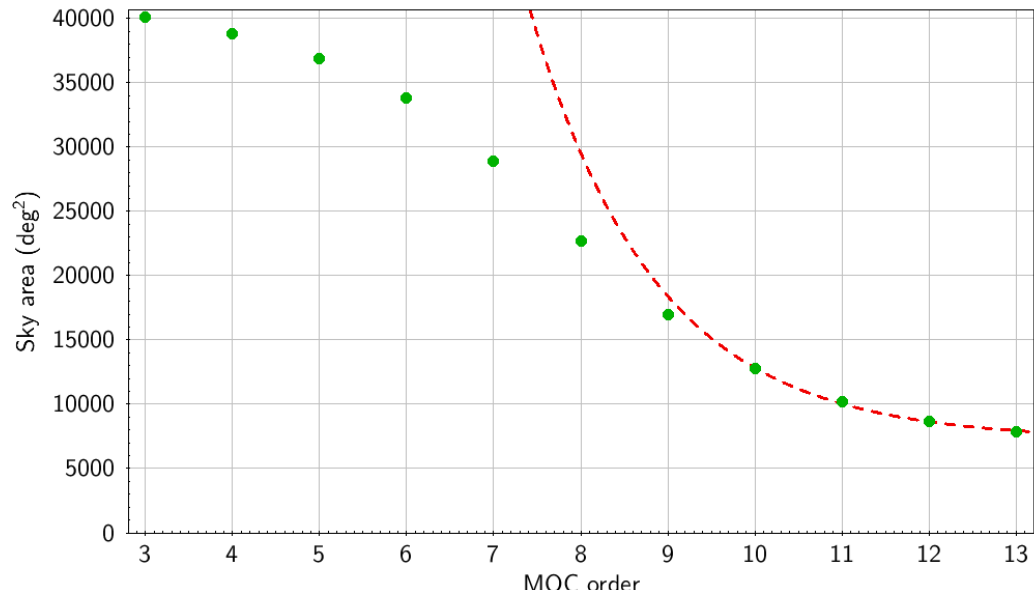
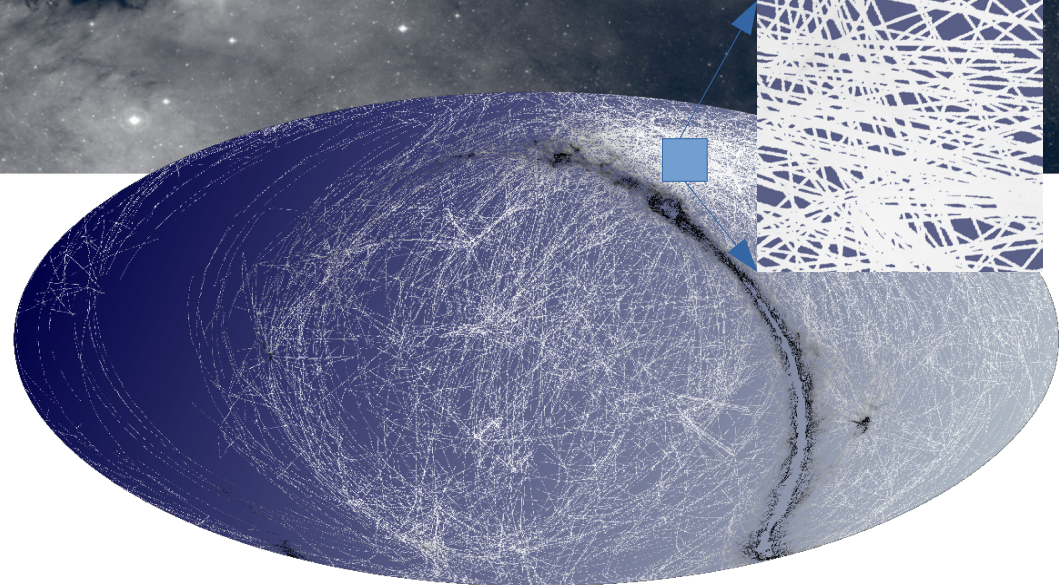


# □ Example

- CDS/P/ISOPHOT/170

- $S_{\text{true}} = 7252 \text{ deg}^2$

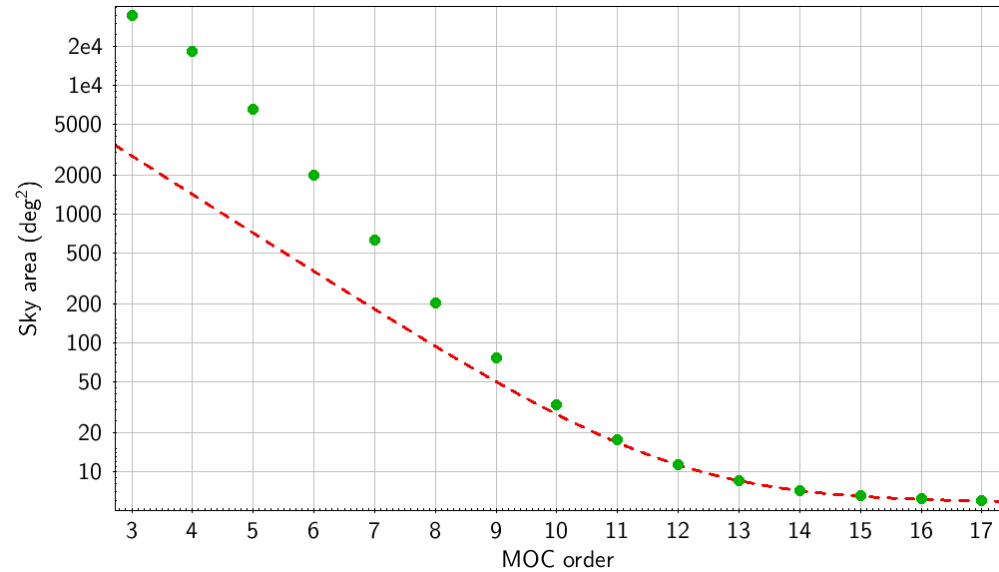
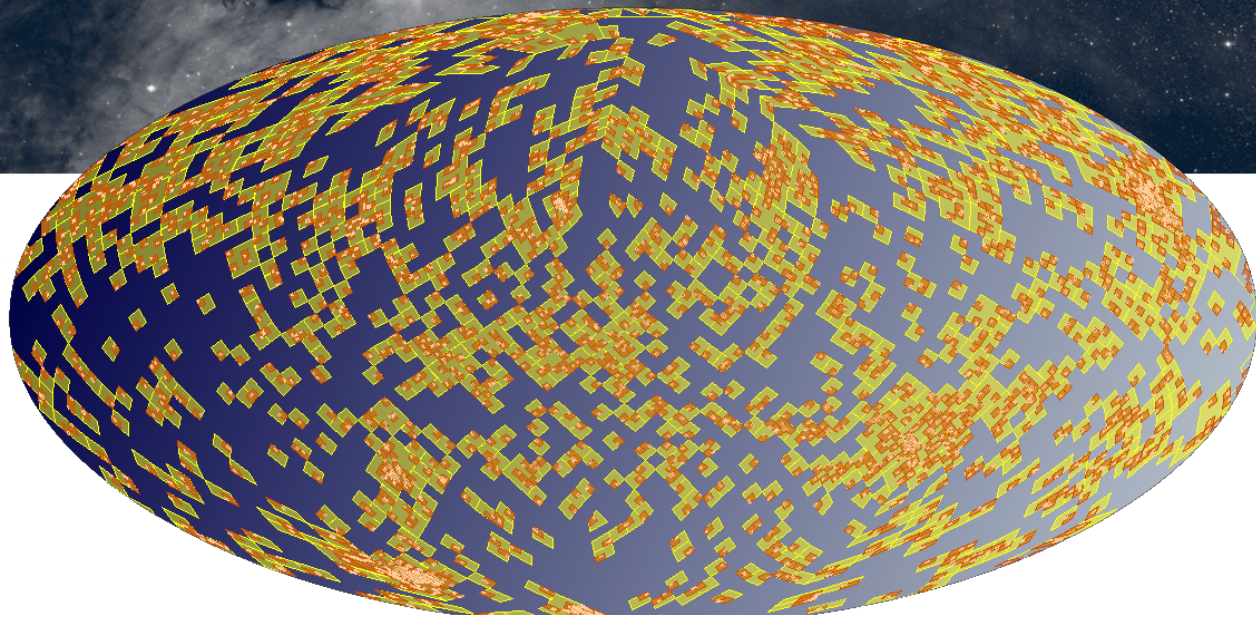
- $\mathcal{L} = 2.896$





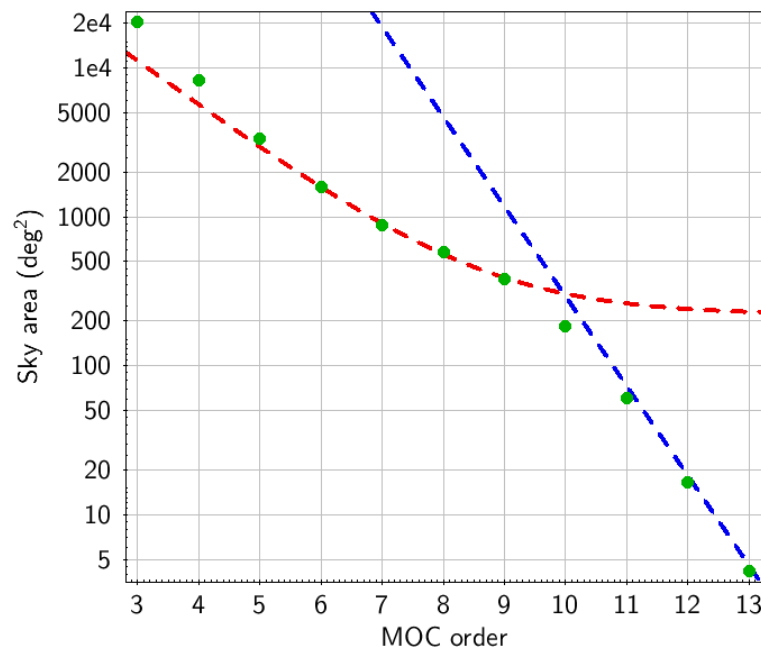
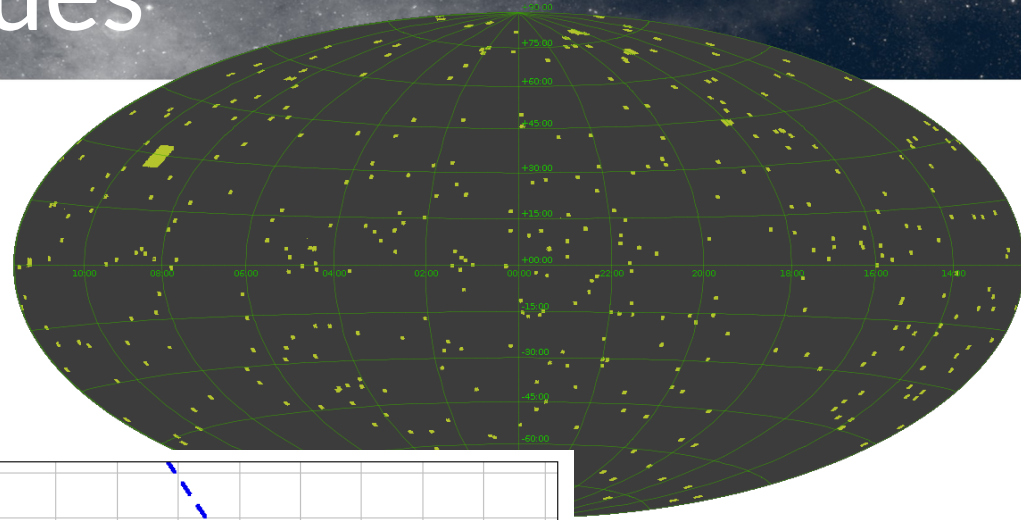
# Example

- CDS/P/HST/V
- $S_{\text{true}} = 5.745 \text{ deg}^2$
- $\mathcal{L} = 3.595$



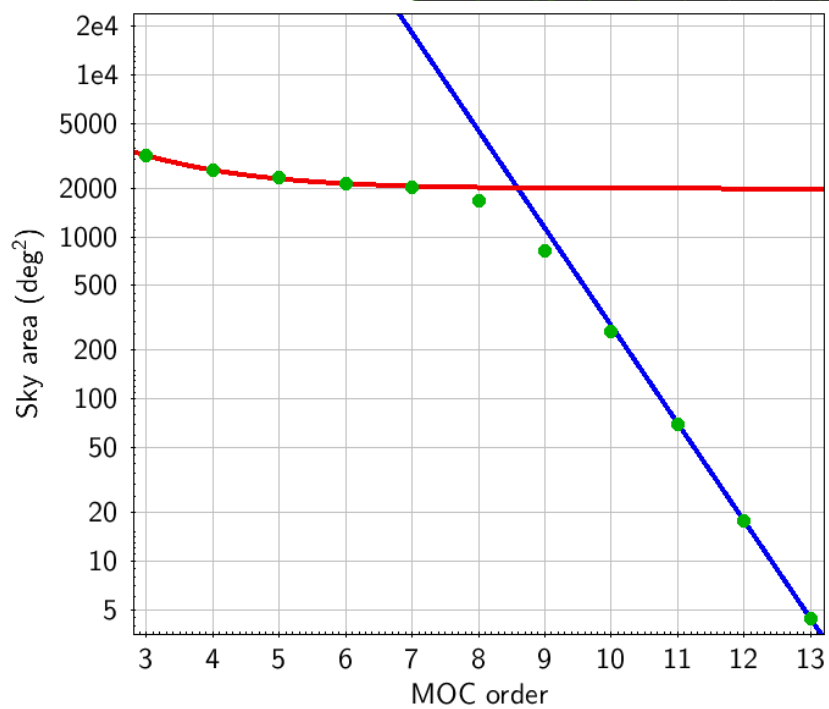
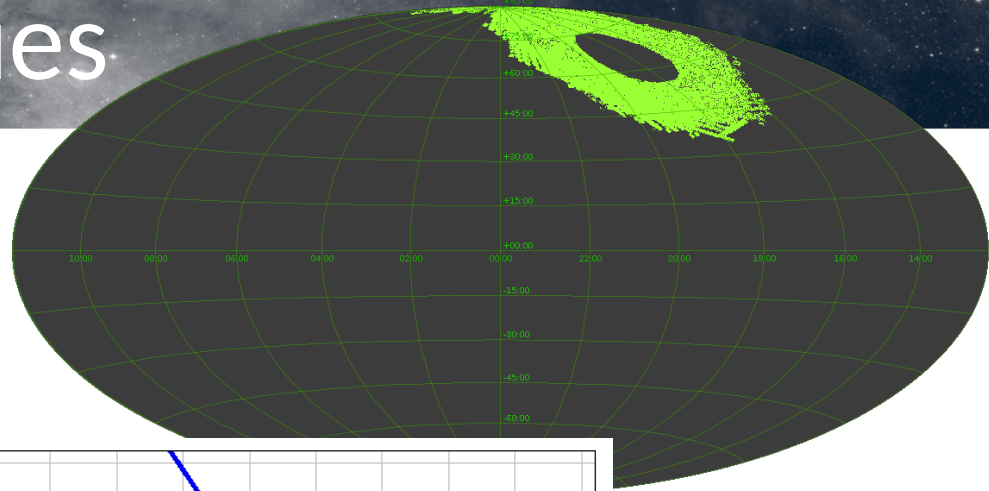
# Example for catalogues

- CDS/I/273A/erlcat  
(de Vegt et al., 2001)
- $N=89\,422$  sources
- $S_{\text{true}} = 219.5 \text{ deg}^2$
- $\mathcal{L} = 2.6$



# Example for catalogues

- CDS/I/335/table1 (Men et al., 2016)
- N=86 467 sources
- $S_{\text{true}} = 1984 \text{ deg}^2$
- $\mathcal{L} = 0.683$



# □ Summary

- MOC offer a convenient description of the coverage of astronomical image surveys or catalogues
- Global parameters such as total area (sky fraction) or lacunarity can provide additional insight on the data distribution on the sky
- Combining spatial MOCs with time (or frequency) coverage enables to easily solve difficult problems