BMS algebra and supertranslation-invariant Lorentz charges

Marc Henneaux
(Université Libre de Bruxelles and Collège de France)

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or “BMS_4” group if one wants to emphasize the spacetime dimension.

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In addition to the homogeneous Lorentz group,

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and transform in an infinite-dimensional non-trivial representation of the Lorentz group.

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where T_i and S_α are respectively the generators of the standard translations and of the pure supertranslations.

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These features were first pointed out at null infinity but are equally present at spatial infinity.

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At the same time, because the pure supertranslations do not form an ideal on account of the preceding point, they cannot be meaningfully quotientized out to get the Poincaré algebra as a quotient algebra.

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A similar construction for supertranslations appears to be more intricate for the full BMS algebra.

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Work done in collaboration with Oscar Fuentealba and Cédric Troessaert, e-Prints 2211.10941 [hep-th] and 2305.05436 [hep-th]

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(Dirac, Arnowitt-Deser-Misner, Regge-Teitelboim)

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This puts strong and interesting restrictions.

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$$\int d^3x \pi^{ij} \dot{g}_{ij} \sim \ln r.$$

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The asymptotic symmetry reduces then to the Poincaré group (and not more).

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The idea is to allow a "parity-twisted component" of a specific form in the leading orders in the asymptotic expansions of the metric and its conjugate momentum.

This parity-twisted component takes the form of a diffeomorphism generated by $\mathcal{O}(1)$ diffeomorphisms (rewritten in Hamiltonian form).

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The twisting is thus characterized by two functions of the angles, one odd (U) and one even (V).

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Introduction : structure of the BMS_4 algebra

Ordinary supertranslations

Logarithmic supertranslations

The logarithmic BMS_4 algebra

Nonlinear redefinitions

Conclusions and comments

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The answer is positive. One can further enlarge the symmetry in this manner, which then includes in addition “logarithmic supertranslations” conjugate to supertranslations.

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The physical logarithmic supertranslations are thus parametrized by the same class of functions as the pure supertranslations. We denote them L_α.

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with $\mathscr{B}$ being the abelian algebra of the logarithmic supertranslations.

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[If p is a constant of the motion, p^2 or any function of p is also a constant of the motion but it is clearly not independent from p even though not a multiple of p.]

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($f^A dQ_A$ is exact, “integrability”)

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[If f_{AB} in $\{Q_A, Q_B\} = f_{AB}(Q)$ is invertible, one can go to Darboux coordinates where $f_{AB} = \delta_{AB}$.]

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There are many examples where the Poisson bracket of two asymptotic symmetry charges is non-linear (extended supergravity models in 3D, higher spins in 3D, asymptotically flat spacetimes in higher dimensions).

Even when the Poisson brackets are linear, one can consider non-linear redefinitions of the charges.

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The logarithmic supertranslations are canonically conjugate to the pure supertranslations, and a Darboux-like procedure enables one to "decouple" them from the other generators.

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One can also check

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so that the Poincaré algebra is unchanged.

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Integrability of the charges associated with these field-dependent transformations is not an issue since we are working with the generators throughout.

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THANK YOU !