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Duistermaat index and eigenvalue interlacing for perturbations in boundary conditions

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Eigenvalue interlacing is a tremendously useful tool in linear algebra and spectral analysis. In its simplest form, the interlacing inequality states that a rank-one positive perturbation shifts the eigenvalue up, but not further than the next unperturbed eigenvalue. For different types of perturbations, this idea is known as the "Weyl interlacing" (additive perturbations), "Cauchy interlacing" (for principal submatrices of Hermitian matrices), "Dirichlet-Neumann bracketing" and so on.

We discuss the extension of this idea to general "perturbations in boundary conditions", encoded as interlacing between eigenvalues of two self-adjoint extensions of a fixed symmetric operator with finite (and equal) defect numbers. In this context, even the terms such as "signature of the perturbation" are not immediately clear, since one cannot take the difference of two operators with different domains. However, it turns out that definitive answers can be obtained, and they are expressed most concisely in terms of the Duistermaat index, an integer-valued topological invariant describing the relative position of three Lagrangian planes in a symplectic space. Two of the planes describe the two self-adjoint extensions being compared, while the third one corresponds to the distinguished Friedrichs extension.

We will illustrate our general results with simple examples, avoiding technicalities as much as possible and giving intuitive explanations of the Duistermaat index, the rank and signature of the perturbation in the self-adjoint extension, and the curious role of the third
extension (Friedrichs) appearing in the answers.

Based on a work in progress with Graham Cox, Yuri Latushkin and Selim Sukhtaiev.

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