LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

SCALE INVARIANT EINSTEIN-CARTAN GRAVITY AND FLAT SPACE CONFORMAL SYMMETRY

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Institut Denis Poisson, Tours, 19 October 2023

Mainly based on

- G.K., Mikhail Shaposhnikov, Andrey Shkerin, Sebastian Zell: 2106.13811 [hep-th]
- G.K., Mikhail Shaposhnikov, Andrey Shkerin, Sebastian Zell: 2106.13811 [hep-th]
- G.K., Mikhail Shaposhnikov, Sebastian Zell: 2203.09534 [hep-ph]
- G.K., Mikhail Shaposhnikov, Sebastian Zell: 2307.11151 [hep-th]
- G.K., Mikhail Shaposhnikov, Sebastian Zell: to appear soon!

as well as

- G.K., Alexander Monin: 1510.08042 [hep-th]
- G.K., Javier Rubio: 1606.08848 [hep-ph]
- G.K., Vladimir Kazakov, Mikhail Shaposhnikov: 1908.04302 [hep-th]
- G.K., Marco Michel, Javier Rubio: 2006.11290 [hep-th]

Introduction and (phenomenological) motivation

The Standard Model (SM) of particle physics (circa '70s) is THE success story

- Description of a plethora of phenomena in the microcosm
- Its last missing piece, the Higgs boson was observed in July 2012, $\sim$ 11 years ago!
- So far no convincing deviations from the SM have been observed at particle physics experiments
- Moreover, the SM could be a self-consistent effective field theory up to very high energies ($\sim M_P$)

Introduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Introduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Compelling indications that the answer is negative!

Introduction and (phenomenological) motivation

Experimental point of view

The SM (plus gravity) fails to accommodate in its context well established observational facts

- Neutrino physics
- Dark matter
- Baryon asymmetry of the Universe
- Homogeneity and isotropy of the Universe at large scales

Introduction and (phenomenological) motivation
Theoretical point of view

The SM suffers from

- Landau Pole(s) associated with the $U(1)$ & Higgs sectors, but **@** energies $\gg M_P$, so usually swept under the "quantum gravity carpet"
- Strong-CP problem
- Cosmological Constant issue
- Hierarchy issue (incredible smallness of Higgs mass M_H as compared to M_P)

Not a threat to its self-consistency
$\Rightarrow$ some pieces of the puzzle are not understood.

Various attempts to go beyond the SM

- (low-energy) Supersymmetry [Fayet '75, '77 & Witten '81 & Dimopoulos, Georgi '81 & Ibanez, Ross '81]
- Compositeness [Weinberg '76, '79 & Susskind '79]
- Large extra dimensions [Arkani-Hamed, Dimopoulos, Dvali '98 & Randall, Sundrum '99]

Distinct experimental signatures right above the electroweak scale differentiate them from the SM

So far no convincing deviations from the SM have been observed at particle physics experiments, yet

Where to look?

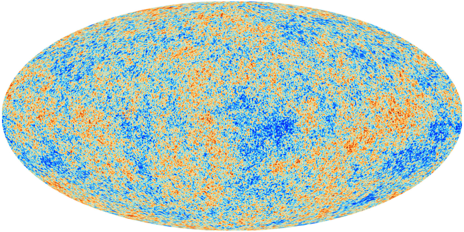
I am going to be very modest here

Put at use the fact that Nature shows a tendency toward being liberated from scales

See what this implies for phenomenology if taken at face value

Possible relevance of scale or conformal invariance

• Almost flat, scale-invariant, CMB spectrum



• The SM at the classical level contains only one dimensionful parameter, the Higgs mass M_H (in the absence of gravity).

Scale- & conformally- invariant for $M_H = 0$

Possible relevance of scale or conformal invariance

Could it be that CFTs play a fundamental role in Nature?

When this symmetry is exact it has some "peculiar" implications:

- Forbids the presence of dimensionful parameters
- No particle interpretation—the spectrum is continuous

But Nature (SM) has:

- dimensionful parameters
- particles

The central role of gravity

In one way or another, the symmetry needs to be broken for the picture I am trying to paint be phenomenologically acceptable.

In addition, gravity has to enter the game for this picture to be complete.

The mere presence of gravity necessarily breaks the symmetry.

The central role of gravity

Gravity-induced conformal symmetry breaking may be effectuated in:

A. maximally brute-force manner, i.e. couple conformal SM to gravity such that scale (& conformal) transformations are broken

B. maximally "compatible" with the symmetry manner, i.e. couple conformal SM to gravity in a scale-invariant manner *

* When talking about gravity we have to be careful and differentiate between conformal and Weyl ($\equiv$ gauged dilatations) [Karananas, Monin '15]

A. An almost scale-invariant Universe

Constructing the action

Selection rules: [Karananas, Shaposhnikov, Shkerin, Zell '21]

- The purely gravitational part of the action contains operators of mass dimension not greater than 2 $\leftrightarrow$ only massless graviton in the gravity spectrum
- The matter Lagrangian comprises the SM with $M_{\text{Higgs}} = 0$.
- The coupling of matter to gravity only happens through operators of mass dimension not greater than 4 $\leftrightarrow$ "logical" to impose, but may be relaxed

Constructing the action

Naively simple

$$S \sim \int (M^2 + \xi h^2) R - \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4} h^4 + \dots \ ,$$

with $R = \text{Ricci scalar(metric)}$, $h =$ Higgs field in unitary gauge and … stand for the rest of the SM.

For $M^2 \ll \xi h^2$, nontrivial modification to the dynamics[1]

Nonminimal coupling is actually a kinetic mixing between Higgs & graviton operative at high-energies relevant in the early Universe → Higgs inflation [Bezrukov, Shaposhnikov '07]

[1] For $M^2 \gg \xi h^2$, standard SM & gravity, but range of validity lowered to M_P/ξ instead of M_P

The success of Higgs inflation is inevitable

Cast gravity to its usual, Einstein-Hilbert, form (via Weyl rescaling). In the "Einstein frame" the approximate conformal symmetry is nonlinearly realized

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2}\frac{M_P^2}{\kappa}\frac{(\partial_\mu h)^2}{h^2} - \frac{\lambda M_P^4}{4\xi^2}\left(1 - \frac{2M_P^2}{\xi h^2} + \dots\right)$$

To make this statement explicit:

- take flat spacetime limit $g_{\mu\nu} \to \eta_{\mu\nu}$
- under the (nonlinear) conformal transformation

$$h \mapsto h' = e^{\sigma} h \ , \quad \sigma = \partial_\mu \varepsilon^\mu \ , \quad \varepsilon^\mu = \text{conformal Killing vector} = \mathscr{O}(x^2)$$

- the kinetic term is manifestly invariant since $\partial^2 \sigma = 0$
- the potential breaks the conformal symmetry explicitly down to diffs **@** $M_P/\sqrt{\xi}$

The success of Higgs inflation is inevitable

Highly suggestive form - canonicalize via exponential map

$$h = M_P \mathrm{e}^{\sqrt{\kappa}\phi/M_P} \ , \quad \kappa = \frac{\xi}{1+6\xi}$$

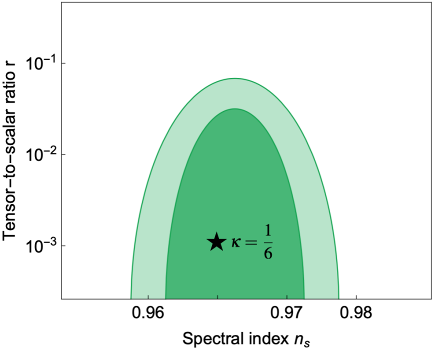
In terms of ϕ

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda M_P^4}{4\xi^2} \left(1 - 2\mathrm{e}^{-\sqrt{\kappa}\phi/M_P} + \ldots \right)$$

Deviation from exact de Sitter is exponentially small

The success of Higgs inflation is inevitable

approximate scale symmetry, broken spontaneously → Higgs is the pseudo Nambu-Goldstone boson → approximate shift symmetry → exponentially flat potential → excellent agreement with observations

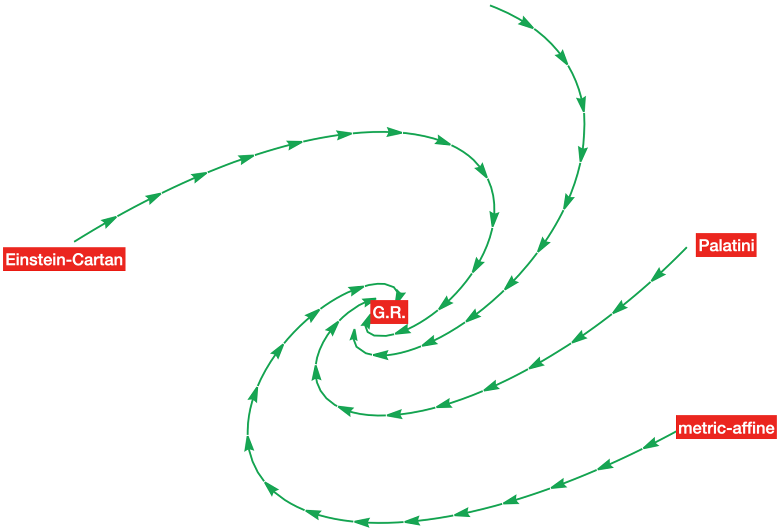


CMB normalization → $\xi \sim 10^4$

amount of gravitational waves $r(\kappa) \sim \frac{10^{-4}}{\kappa}$ $\quad \kappa = \frac{\xi}{1+6\xi} = \frac{1}{6}$

The non-uniqueness (?) of Higgs inflation
Rethinking GRavity

But, *which* gravity?
or, better ask *which formulation* of gravity?



G.R. (metrical gravity) is the attractor in this “landscape” of formulations

But, *which* gravity?

or, better ask *which formulation* of gravity?

- purely metrical
- Palatini
- Einstein-Cartan (EC)
- metric-affine

Does it really matter? e.g. [Shaposhnikov, Shkerin, Zell '20 & Karananas, Shaposhnikov, Shkerin, Zell '21]

- Only massless graviton & absence of matter: the above *completely equivalent*
- Only massless graviton & presence of matter: the above *not equivalent anymore*

Einstein-Cartan(-Sciama-Kibble) theory

In what follows I'll only discuss EC gravity $\rightarrow$ interaction follows from the gauge principle $\rightarrow$ as close as it gets to particle physics

Gauge shifts & Lorentz transformations, by introducing the tetrad e and connection ω with their corresponding field strengths

$$\text{torsion:} \quad T \sim \partial e + \omega e$$
$$\text{curvature:} \quad F \sim \partial \omega + \omega^2$$

Pure EC gravity

The aim is to construct a gravitational sector that propagates only a massless spin-2 field

Start by writing all possible terms—there are ten of them—up to two derivatives of the fields

Schematically:

$$S_{\text{gr}} \sim \int \text{cosm. const.} + 2 \times \text{curvature scalars} + 2 \times \partial(\text{torsion}) + 5 \times \text{torsion}^2$$

Appearances are (very) deceiving...

The connection and thus torsion is not dynamical

Pure EC gravity

Everything becomes transparent by obtaining the equivalent metric theory:

1. vary the action wrt the nondynamical connection ω
2. Solve its algebraic eom (easy)

$$\delta_\omega S_{\rm gr} = 0 \quad \leftrightarrow \quad \omega \sim \partial e$$

3. Plug the above back into the action to get

$$S_{\rm gr} \sim \int \text{cosm. const.} + \text{Ricci scalar(metric)}\ ,$$

which is nothing more than the Einstein-Hilbert action.

Logic is the same with matter, simply $\exists$ more "ingredients"

The consequences are not the same though

SM & EC gravity

The interaction of fields, specially Higgs with gravity is modified as compared to the metrical gravity [Shaposhnikov, Shkerin, Timiryasov, Zell '20 & Karananas, Shaposhnikov, Shkerin, Zell '21]

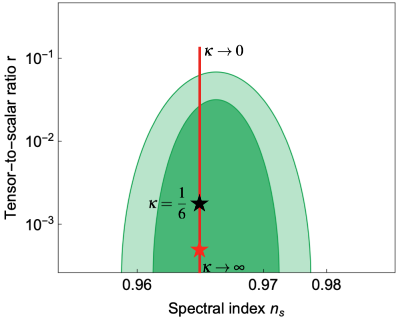
$$S \sim \int (M^2 + \xi h^2)\text{curvature scalars} \quad + \quad \zeta h^2 \; \partial(\text{torsion})$$
$$+(M^2 + \eta h^2)\text{torsion}^2 \quad - \quad \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}h^4 + \dots$$

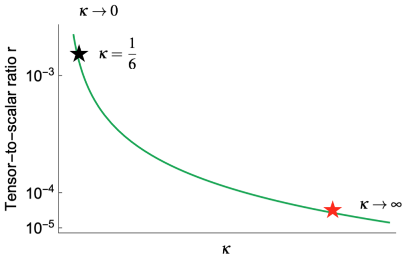
At the same time, the principle underlying the inflationary dynamics is still there $\rightarrow$ approximate scale/shift symmetry

$$S \sim \int \frac{M_P^2}{2}R - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{\lambda M_P^4}{4\xi^2}\left(1 - 2\mathrm{e}^{-\sqrt{\kappa}\chi/M_P} + \dots\right) \,, \quad \kappa = \kappa(\xi, \zeta, \eta)$$

The slope of the potential is controlled by κ, and so does the production of gravitational waves

Higgs inflation in EC gravity





B. A scale-invariant Universe

Why bother?

So far I discussed conformality being broken in a brute-force manner, but most statements are applicable to the symmetry-consistent manner too

Now I will focus on the situation where gravity enters the picture in a scale-invariant manner

This is where things become even more interesting

Scale invariant EC gravity + SM

The need for an additional dilaton

First of all, the action must be liberated from any explicit scales, i.e. $M_H = 0$ and $M_P = 0$; in other words, my starting point is

$$S \sim \int \xi h^2 \text{curvature scalar} \quad + \quad \zeta \partial h^2 \text{ torsion}$$
$$+ \eta h^2 \text{torsion}^2 \quad - \quad \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4} h^4 + y h \bar{\psi} \psi \; + \dots$$

Unsatisfactory for particle physics & cosmological phenomenology, or how I managed to ruin SM & early Universe in one try! ☺

Literally, it's the EC version of induced gravity

Scale invariant EC gravity + SM

The need for an additional dilaton

Untangle the "mess" by finding the equivalent metric theory

- Integrate out the connection (still nondynamical!); in particular

$$\omega \sim \partial e + \partial h + \dots$$

- Go to Einstein frame by Weyl rescaling the metric
- Make the kinetic term canonical in terms of $\phi = M_P e^{\frac{h}{M_P}}$

The result is

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda M_P^4}{4\xi^2} + y \frac{M_P}{\sqrt{\xi}} \bar{\psi} \psi + \dots$$

Minimally coupled massless scalar field interacting with matter derivatively and gravitationally...

The origin of scales

A viable scale-invariant embedding of the conformal SM requires the introduction of a massless scalar field, the dilaton χ

This is the scale donor: the Planck mass is generated dynamically via spontaneous symmetry breaking

$$< \chi > \rightarrow \; M_{\text{Planck}} \; \rightarrow < h >$$

The dilaton may play interesting role in the late Universe, being responsible for the present-day accelerated expansion

Vanishing vacuum energy in SSB CFTs

The vacuum energy in such constructions is automatically zero, in spite of the fact that scales have been generated

Nontrivial statement, but it literally follows from dimensional analysis: the potential is a homogeneous function of the fields, or in other words

$$V(\chi,h) \propto \chi\frac{\partial V}{\partial \chi}+h\frac{\partial V}{\partial h}$$

SSB means $<\chi>,<h>\neq 0$, thus

$$\left.\frac{\partial V}{\partial \phi}\right|_{<\chi>,<h>}=0 \quad \rightarrow \quad V(<\chi>,<h>)=0$$

Scale invariant EC gravity + SM

Higgs-dilaton model

The starting point

$$S \sim \int (\xi_h h^2 + \xi_\chi \chi^2) \text{curvature scalar} + (\zeta_h \partial h^2 + \zeta_\chi \partial \chi^2) \text{ torsion}$$
$$+ \quad (\eta_h h^2 + \eta_\chi \chi^2) \text{torsion}^2 - \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{\lambda}{4} h^4 + y h \bar{\psi}\psi + \ldots$$

As usual, integrate out the connection and go to Einstein frame. This results into

$$S \sim \int \frac{M_P^2}{2} R - \frac{M_P^2}{2(\xi_h h^2 + \xi_\chi \chi^2)} \begin{pmatrix} \partial\chi & \partial h \end{pmatrix} \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \partial\chi \\ \partial h \end{pmatrix} - M_P^4 V \ ,$$

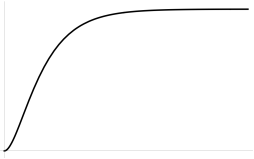
with $\gamma_{ij} = \gamma_{ij}(h/\chi)$, $V = V(h/\chi)$. In other words, the kinetic terms of the scalars span a nontrivial two-dimensional manifold.

All the information is encoded into the scalar curvature κ of this manifold

Inflationary dynamics = practically single-field

Predictions in a wide class of models appear almost universal and independent of the details [Karananas, Rubio '16 & Karananas, Michel, Rubio '20]
Exponentially flat plateau with

$$n_s = 1 - \frac{2}{N}$$



Intricate link between "geometry" & production of primordial gravitational waves

$$r = r\left(M_P^2|\kappa|\right)$$

The "arbitrariness" is due to the nontrivial gravitational dynamics encoded into 12 couplings...

Flat space conformal symmetry

A way towards constraining the dynamics

We can do (much) better though!

A straightforward computation shows that the action is not invariant under the nonlinearly realized conformal symmetry

$$h \mapsto h' = e^{\sigma} h \ , \quad \chi \mapsto \chi' = e^{\sigma} \chi \ , \quad \sigma = \partial_\mu \varepsilon^\mu \ .$$

Contrary to the single-field case, the potential is conformally invariant now, since it depends on ratios of the fields

$$V(h/\chi) \mapsto V(h'/\chi') = V(e^{\sigma} h / e^{\sigma} \chi) = V(h/\chi) \ .$$

Flat space conformal symmetry

A way towards constraining the dynamics

The breaking, however, has propagated to the kinetic sector

$$K(h,\chi) \sim \int \frac{M_P^2}{2(\xi_h h^2 + \xi_\chi \chi^2)} \begin{pmatrix} \partial\chi & \partial h \end{pmatrix} \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \partial\chi \\ \partial h \end{pmatrix} ,$$

since

$$K(h',\chi') \neq K(h,\chi) .$$

Remember, there is a nontrivial scalar-gravity mixing due to nonminimal couplings...

Scale of conformality-breaking may well be much below M_P

How to proceed?

Flat space conformal symmetry

The field-space curvature

Requiring that conformality is broken by gravity @ M_P is in one-to-one with requiring that the scalar curvature κ of the field manifold satisfies

$$M_P^2|\kappa| \lesssim 1$$

Why? Consider the following toy model:

$$S \sim \int d^4x \left[-\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\left(1+\frac{\tilde{\kappa}\phi_1^2}{2M_P^2}\right)\partial_\mu\phi_2\partial^\mu\phi_2 \right] .$$

Two canonical fields + a higher-dimensional operator that explicitly breaks conformal invariance at energies $M_P/\sqrt{\tilde{\kappa}}$
The field-space curvature in the limit $\phi_1 \to 0$ is

$$M_P^2|\kappa| = \tilde{\kappa}$$

κ sets the scale of conformal symmetry breaking

Example 1

$$S = \int d^4x\sqrt{g}\left[\frac{1}{2}\left(\xi_h h^2 + \frac{1}{\xi_h^2}\chi^2\right)\mathring{R} - \frac{(\partial_\mu h)^2}{2} - \frac{(\partial_\mu \chi)^2}{2}\right.$$

$$+ \partial_\mu \chi^2 v^\mu + (\xi_h \partial_\mu h^2 + \partial_\mu \chi^2) a^\mu + \frac{1}{2\xi_h}\left(\frac{8\xi_h^3}{\xi_h - 2}h^2 + \frac{\chi^2}{\xi_h^2}\right) v_\mu v^\mu$$

$$\left. + \frac{1}{\xi_h}\left(\frac{h^2}{\xi_h} + \frac{\chi^2}{\xi_h^2}\right) v_\mu a^\mu + \frac{1}{2\xi_h}\left(\frac{h^2}{\xi_h} + \frac{\chi^2}{\xi_h^2}\right) a_\mu a^\mu - \frac{\lambda}{4}h^4\right] .$$

This theory has

$$M_P^2|\kappa| < 1 \ ,$$

for all field values. Observables

$$n_s \approx 1 - \frac{2}{N} \ , \quad r \approx \frac{13}{N^2} \ .$$

Example 2

$$S=\int d^4x\sqrt{g}\left[\frac{1}{2}\left(\xi_h h^2+\frac{1}{\xi_h^2}\chi^2\right)\mathring{R}-\frac{(\partial_\mu h)^2}{2}-\frac{(\partial_\mu \chi)^2}{2}\right.$$
$$+\partial_\mu\chi^2 v^\mu+\partial_\mu h^2 a^\mu+\frac{1}{\xi_h^2}\left(\xi_h h^2+\frac{\chi^2}{\xi_h^2}\right)v_\mu a^\mu$$
$$\left.+\frac{\xi_h^3+12\xi_h-1}{18\xi_h^3}\left(\xi_h h^2+\frac{\chi^2}{\xi_h^2}\right)a_\mu a^\mu-\frac{\lambda}{4}h^4\right]\ .$$

As before

$$M_P^2|\kappa|<1 \quad \forall \quad h,\chi$$

Observables

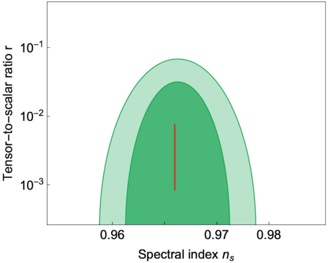
$$n_s\approx 1-\frac{2}{N}\ ,\quad r\approx\frac{12}{N^2}\ .$$

A conjecture

- conformality is preserved up to the Planck scale, i.e. $M_P^2|\kappa| \lesssim 1$ for all field values,
- slow-roll inflation is possible,
- the observed amplitude of CMB perturbations is reproduced,
- the Higgs self-coupling fulfills $\lambda \geq 10^{-12}$,

then the inflationary predictions are really universal

$$n_s = 1 - \frac{2}{N} \, , \quad r = \frac{4}{M_P^2\,|\kappa|\,N^2} \gtrsim \frac{12}{N^2}$$



Is model bulding unnecessary?!

More to come (soon), a comprehensive numerical study is ongoing
[Karananas, Shaposhnikov, Timiryasov, Zell 'XX]

Recap

- Conformal symmetry may be the key for the harmonic symbiosis of gravity & SM
- If inflation is the mechanism responsible for the isotropy and homogeneity of our Universe, then the Higgs field is responsible for inflation
- Gravity plays an important role, being responsible for breaking conformality
- Einstein-Cartan gravity is as close as it gets to the particle physicist's mindset
- Combining gravity & SM in a scale-invariant manner offers(?) insights on the cosmological constant problem
- The central role is played by the field-space curvature κ
- Flat space conformal symmetry fixes it
- Predictions universal and close to the ones of single-field Higgs inflation (conjecture)

Thank you!