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René Thom's contribution to linguistics and his semiophysics applied to art and music. A retrospective on behalf of the centenary of his birth¹

Abstract

René Thom made his major contributions to linguistics and semiotics, first, in his book "Stabilité Structurelle et Morphogenèse", in 1972, and second, in his book "Esquisse d'une sémiophysique", 1988. In the first, he presented a list of 16 "morphologies archétypes"; in the second, he proposed to bridge the gap between the sciences of culture and communication and the natural sciences. The present contribution describes how Thom's linguistic conjectures have been further developed and modified in ongoing research until today. His " sémiophysique " program is exemplified in applying his topological and dynamic view to visual art and music.

Although a final evaluation of Thom's journey into linguistics and semiotics cannot be given, its impact on contemporary and future thought in linguistics and semiotics remains highly relevant.

1. Introduction

René Thom and his work have indirectly become part of my scientific biography. In 1977, I sought to extend (mathematized) linguistic theory to the fields of topology (abstract spatiality) and dynamics (language creation and development). Rather, I came across an English version of

¹ Contribution to the conference 20th to 22nd September 2023 at the IHES (Paris) on the occasion of the centenary of René 'Thom's birth. The present text contains changes and elaborations after the conference.

"Stabilité Structurelle et Morphogenèse" (Structural Stability and Morphogenesis) and bought the second revised, corrected, and enlarged French edition from 1977. Like many readers, I was fascinated by the new perspectives and nevertheless disconcerted by the difficulty of reading. However, I decided to place Thom's questioning at the center of my Habilitation thesis entitled *Dynamics of Communication* (germ. "Verständigungsdynamik").²

My first personal contact with René Thom was during the International Semiotics Congress in Vienna in 1979, where he gave a plenary lecture. I met him after his lecture, and he came to listen to my contribution on verbal valences from a catastrophist perspective. He invited me to come and listen to his lecture at the premises of the "Östereichische Studiengesellschaft für Kybernetik" (Austrian Cybernetics Society, Vienna). It was not until 1982, during the Cerisy colloquium "Logos et Catastrophes. À partir de l'oeuvre de René Thom", organized by Jean Petitot, that our paths crossed again. Finally, René Thom invited me to spend the months of November and December 1987 in Bures-sur-Yvette, where I rubbed shoulders with him daily.

The objective of my contribution is to summarize the research I conducted from 1977 to 2023 on the semiotic and linguistic work of René Thom.

2. René Thom's contribution to linguistics (1970-1980)

The book "Structural Stability and Morphogenesis. Outline of a General Theory of Models " (French original 1972) brought René Thom admiration and international fame. However, only subchapter 13.4, p. 309-315, treats language (under the title "Homo loquens". The list of 16 archetypes discussed in this section stands on p.307. In a paper "Topologie et linguistique" published in "Essays on Topology and Related Topics" (1970), he gave a more detailed description of the underlying conjecture on basic topological schemata of sentences, a topic

² A Habilitation grant (from the German research company, DFG), 1977-1979, allowed me to complete my knowledge of differential topology, helped by mathematics professors Klaus Jänich (Regensburg) and Bruno Kramm (Bayreuth). A short version in English was published in Wildgen (1982); major chapters (corrected and augmented) were published in Wildgen (1985; in German). A grammar based on the notions of morphogenesis and topology was put forward in Wildgen (1999; in French)

classically treated regarding logics and the notions of subject and predicate. His conjecture was a provocation for linguists standing in the tradition of structuralism and Ferdinand de Saussure and those believing that logic (either traditional or modern) is the proper frame for questions of semantics and ontology.

Thom's innovative and radical thought has two main sources: On the one hand, his training and research in mathematics, especially in differential topology, and on the other hand, his philosophical reflection, which motivated him to consider the natural sciences (e.g., experimental physics, the ideal of modern sciences) and human sciences from a new perspective. His concepts for linguistics are only a tiny part of this new conception of scientific work.

For Thom, mathematics, particularly the tradition stemming from Poincaré, provides the proper frame of models in general, specifically in biology and linguistics.³ This conceptual frame can make the mass of empirical observations or experiments understandable. His attitude may be compared to Plato's reconstruction of the universe and the human mind in his dialogue "Timaeus"⁴.

³ In the spirit of Kant's epistemology, where the geometry of Euclid is considered as the "synthetic apriori" of knowledge, the classification theorem of catastrophe theory and other results of dynamic systems theory are the presupposition of explanatory models in linguistics and semiotics.

⁴ Socrates' interlocutor, Timaeus, presents in a kind of treatise the view of Greek mathematicians in Plato's time standing in the tradition of the mythical Pythagoras (cf. <u>https://en.wikipedia.org/wiki/Timaeus_(dialogue)</u> and the comparison of platonic solids and catastrophes in Wildgen, 2020).



Figure 1 The five platonic solids (Springer Link)

In his book Structural Stability and Morphogenesis, Thom does not refer to any of his time's expert semioticians or linguists (in the late sixties). He refers only to Lucien Tesnière, whose main posthumous work was published in 1959 (but who had developed his innovative ideas in the 1930s). The list contains 16 verbs in French. However, in the perspective of the universal scope of Thom's models, the proper basis should be all languages, i.e., the archetypal morphologies should be universals of language. The lexicon of verbs in every language amounts to hundreds, if not thousands, of verbs. Therefore, Thom's list of French verbs can only be a suggestion for further work by linguists. Based on the current state of verbal semantics and linguistic typology, we propose to consider four different levels of application:

- Recurrent processual categories in verbal lexemes or the morphological marking of verbs, for instance, the traditional "*Aktionsarten*" (action types): continuative, egressive, ingressive, resultative, perfective, and others.
- The valence of the verb: values 1 to 3; in causative constructions 4 (cf. Tesnière, 1979). Beyond the chemical analogy, one could speak of a force profile in verbal dynamics or an ergativity scale (Greek: ἔργον (érgon) work, labor, task). The central feature is the

restriction of valence to 3 (with extension 4) categories. This limit may be compared to the limits of Platonic solids to five (to three if we consider two pairs of duals).

- 3. Beyond valence, space, time, and manner specifications may accompany the valencebound sentential kernel. These specifications vary strongly with languages (and can be neglected in a model with universal scope)
- 4. Sentences are in many languages organized along a topic profile. Thus, the notion of a subject, which has been central to logical analysis since Aristotle, belongs to the level of topic scales. Therefore, the distinction between subject and predicate is on another level of analysis.⁵

We show in Fig. 2 the list of the French versions in Thom (1972, 1977) because the English translation 1975 contains mistakes and false alignments. Thom's list invited specialists in grammar and semantics to complete it and, if possible, substantiate the claim of a topological/dynamic meaning for these types of verbs.



⁵ The Object (direct and indirect in Tesnière's grammar) has a lower topic rank than the Subject.



Figure 2 Thom's 16 archetypal morphologies in the original French edition are subdivided into six boxes⁶ and some short linguistic commentaries focusing on linguistic generalization.

As a result of this inspection of the examples given by René Thom, we are left with valence-1 processes (*locomotion* and *be* or *become/change*) and a set of action types (first box), the 2-valent types *Capture* and *Emit*, the 3-valent type *Give*, and the 4-valent type *Send*. However, the scope of topological semantics goes further; it advocates the existence of processual images as the content of basic verb types. This claim goes beyond traditional semantics. The schools of Cognitive Semantics (Fillmore, Talmy, Lakoff, Langacker) made a similar claim. However, they could not substantiate it because they did not have the tools to integrate the geometrical/topological aspects of verbal meaning into their modeling. Cf. the critique in

⁶ The partition into six boxes was made by the author to simplify the association of his comments. The ealiest version of Thom's list in Thom (1970: 248) contains 18 elements and in some cases different names. Additional archetypes are: Unir (unite), Séparer (separate), Le Suicide (suicide). Instead of Changer (change) in box one stands Devenir (become), instead of Secouer (stiring) stands Agiter (swing).

Wildgen (1994: chapter 1) and the reviews of Cognitive Semantics in Wildgen (2008: chapters 3 to 5) and Wildgen (2010a: chapter 12).

2.1. Thom's morpho-dynamic turn opposed to mainstream structuralism (1916 – present)

Ferdinand de Saussure (1857-1913), professor of comparative linguistics (mainly Indo-European languages), was the father of classical structuralism, whereas Charles S. Peirce (1839-1914), was the father of modern semiotics, a logician and chemist. He constructed semiotics based on a relational and diagrammatic logic he had devised.⁷ The mathematician René Thom, who started with applications in biology, did not share these traditions and made a fresh and independent start in linguistics, semiotics, and communication studies.

In a talk in Geneva in 1970, René Thom refers to Saussure and outlines his model of "topological linguistics". For René Thom, the morphologies in the human sphere are linked to the morphologies found in animals and the nature surrounding animals. The visual, acoustic, olfactory, and tactile media of communication have their basis in the corporeality and vital functions of living beings, and this is also valid for language that is a late-comer from an evolutionary point of view.

The formal concepts René Thom puts in the foreground differ fundamentally from those advocated and used in linguistic structuralism. In his Theory of Cases (1935), Louis Hjelmslev, the leader of the Copenhagen school of structuralism, postulated a "prelogic", i.e., a kind of "natural" logic underlying mathematical logics in the sense of the "Principia Mathematica" (Whitehead and Russel, 1910), It formed the background of the analysis of case structures, i.e., the typology and case marking of nouns dependent on a verb.⁸ Thom starts from a geometric

⁷ CF. Peirce (1903/1976) and Wildgen (2023b) for the relation of diagram, gestures and archetypal morphologies.

⁸ On the other hand, Chomsky (1957) considered the algebra of free groups (monoïds) as an adequate mathematical background. Since 1965 he introduced feature semantics and eventually after 1981 "thematic roles" to cope with valence restrictions; cf. chapter 9.2 in Wildgen (2010b).

point of view (more geometrico), generalized within the field of differential topology. For René Thom, the spatial and temporal aspects (preconditions of a causal analysis) are primary. His idea of topological linguistics tries to integrate semiotics and other humanistic sciences into the general line of modern science, initiated around 1600 (Kepler and Galilei), brought to a first climax in the 17th century by Descartes, Leibniz, and Newton and responsible for the explosion of knowledge in the next centuries. His proposals ask for a reorientation of questions and scientific strategies in linguistics and an adaptation to the standards established for the other sciences inspired by the development of the natural sciences. It is, therefore, not surprising that linguistic and semiotic circles were reluctant to accept this radical turn by René Thom.

In order to allow for an evaluation of Thom's conjecture on catastrophe theoretical (topological) semantics, I will sketch major traits using later elaborations in my scientific research and related elaborations by Jean Petitot. Cf. the monographs Wildgen (1982, 1985, 1994, 1999. and 2023a) and Petitot (1985, 1992, and 2011).

2.2. Archetypal morphologies. A linguistic elaboration⁹

In my book "Catastrophe Theoretical Semantics" (1982), the basis of applications was the list of simple catastrophes (Arnold, 1972), which contains the "elementary catastrophes" as a subfield (codimension 1 to 4).

⁹ This paragraph summarizes some contents of Wildgen (1982, 1985, 1994, and 2023a, chapter 6).

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Figure 3 Simple germs in the sense of Arnold (1972) (partial picture); screenshot from Wildgen (1982: 7)

The elaboration in my book considered the duals of the cuspoïds (A-series) and neglected the restriction to codimension \leq 4. In the case of the umbilics (Series D), the results of Godwin (1971) and Callahan (1978 and 1980) implied corrections on the four-valued archetypal morphologies. They had to be derived from the compactified umbilics (closing the "Thalweg") in the context of the "Exceptionals" E₆, E₇, E₈, and the double cusp, X₉. Only the "Sending " morphology ("envoyer", in Fig. 2) was derived explicitly from the compactified elliptic umbilic (D₄); cf. Wildgen (1985: 204-208). In the following, we shall not discuss the mathematical details but the interpretation of these archetypes in the context of linguistic analysis.

2.3. Sketch of an empirical interpretation of Thom's conjecture

The category of verbs that logicians traditionally neglected ¹⁰ becomes the theoretical core of a morphogenetic analysis. Thom was motivated in his conjecture on basic sentence schemata by

¹⁰ See for example the logic of Port Royal which wanted to reduce this part of the lexicon to the single verb "to be".

the work of Lucien Tesnière (1893-1954), who had put the verb (and not the subject) at the center of syntactic analysis and who postulated a dynamic relationship between the verb and its dependent nominal complements (called "actants"; in French grammar: subject, direct and indirect complement). The term "valence", which Tesnière introduced, referred to chemical structures, but he and his followers could not complete this reference to chemistry beyond a pedagogical metaphor.¹¹

The first questions that the categorization of a process, an event, or action raises are:

- What structurally stable structures underlie the processes in question such that a lexicon of verbs can refer to them?
- What are the motor controls and perceptual patterns that mentally appropriate such processes?
- Finally, how are perceptual and motor controls correlated with the appropriate linguistic forms?

The motor programs modify the autonomous dynamics of the body's extremities and their contact with objects (e.g., the floor for the feet). These autonomous dynamic structures determine variables for perceiving movements and the cognition that regulates these systems.¹² It can be concluded that the brain reflects the external dynamics (by adding other parameters and distorting them in its metric). However, the question remains: Does this coordination with external physics also control the higher levels of cognition, especially linguistic cognition? We assume an intermediate level, called imaginal or schematic. It applies processes that become increasingly independent from the psycho-physical grounding and more context-dependent (as a consequence, they depend on chance). In what follows, we will start from the psycho-physical level to find schematizations (imaginal representations) that underlie the semantics of verbs. We distinguish three typical levels of organization in the lexicon of verbs and verbal phrases:

¹¹ The link must be sought indirectly via Gibbs phase rule and Euler's formula for regular polyhedra. Cf. Wildgen (1994: 46f.). Peirce had around 1900 already proposed an analogy between chemical valences and basic sentence patterns. However, he argued in the context of structural chemistry and not modern nuclear chemistry.

¹² Cf. the work of Kelso (1995) and Haken (1996).

- Locomotion and its linguistic schematization,
- control of an object by an agent,
- interaction between agents.

In the case of the movement of the body's extremities (e.g., legs, arms), one can use the physical description of the pendulum (or the double pendulum) as a fundamental schema. Fig. 4 shows the correspondence between the double pendulum and the walking motion. The movement of the body supported by the hips is in coordination with the relative movements of the thigh (measured at the knee) and the lower leg (measured at the ankle):



Fig. 4 The movements of the double pendulum and of a leg in walking

Specific movements are directed toward a target (an attractor). This orientation implies a separation of the starting and the ending points. This bimodality corresponds to a transition in space, and its lexical correlates are (in English) *enter/exit* or *come/leave*. The first dichotomy takes up the archetypal morphologies called *Begin* and *End* ("*finir*", *commencer*") in Thom's list. They concern the "Aktionsarten" of verbs and may be realized lexically, morphologically, or syntactically (cf. the comments on 'Thom's list in Figure 2). In the grammar of Greek, Latin, Russian, and other Indo-European languages, they were a major concern of classical grammarians.

In an agent's control of an object, the configurational aspect and the energetic aspect, which introduces the force of the agent and the effect of this force on the object, can be distinguished. A basic type of effect is the creation, destruction, and regeneration of objects (elements of the

environment). It is easy to see the correspondence of this type of action with the emission and capture schemata in Thom's list in Fig. 2: Capture and Emission ("*capturer*, *émettre*"). In the lexicon, the corresponding verbs are, in most cases, 2-valent (for instance, transitive in case systems of the type nominative-accusative.

The process effected by an agent can also influence or alter the quality of an object. In this case, the background is not space/time and motion but a quality space (qualia). All languages provide a lexicon of entities related to bipolar quality scales. In Western languages, the lexicon of adjectives (and participles) demonstrates the field. They are further linked to verbs describing such changes:

Examples: *long – short, strong – weak, good – bad* or *dirty – clean, right – twisted*, and similar ones in other languages. Corresponding verbs of quality change are: *shorten, strengthen, clean,* and twist.

Topologically, the phenomenon of bimodality as a stable pattern in the organization of the quality space is akin to Thom's elementary catastrophe cusp and the archetype *Change* ("changer"); see the list in Fig.2.¹³

An interaction scene that connects several human agents presupposes a very complex perceptual and conceptual analysis of the individual observing the scene. From a repertoire of action controls, one can reconstruct possible patterns of social interaction. However, it turns out that only a small group of these coordinated interactions achieve high stability, allowing schematization and semantic classification.

a) From the angle of the spatial configuration, we can describe the gift, i.e., the scene during which two people exchange an object, by the topology of the attentional focuses. Petitot (2011: 272ff) elaborated on a proposal by Sir Christopher Zeeman (1925 -2016) for a model that uses cognitive algorithms, such as cut locus and diffusion contours. In this elaboration

¹³ Charles Osgood and his team have reconstructed statistically a low-dimensional space underlying hundreds of such quality scales in many languages (The theory is called "Semantic Differential", cf. Osgood (1976).

toward neuro-vision, the catastrophe theoretical model gains more theoretical depth because it shows that the mathematics of differential topology can be used in the specific context of visual scene analysis. The semantic archetype would, in this perspective, be first the result of morphogenesis in visual pattern recognition. Non-human primates and pre-human species (for instance, Australophicines) probably had this faculty. This pattern would have gained social and cultural significance in the transition to language.

b) The 4-valent archetype of Sending ("envoyer") and similar types with a mediating fourth attractor can be derived from the compactified elliptic umbilic (D₄). Three "Thalweg", i.e., vector fields towards - ∞ , can be closed in the context of the embedding Exceptional E₆ (cf. Wildgen, 1985: 204-212).

We shall only show the derivation of the archetype of *Giving*, which may stand for the class of 3-valent processes and actions that involve a transitory object, quality, or instrument, and the archetype of *Sending*, which may also stand for other instrumentally or causally more complex types of processes and actions. The heavy lines stand for the attractors *Peter*, *hammer*, and *Anne*; the encircled are bifurcation, where the middle attractor is emitted, gains dominance, and is captured or eliminated; the arrows point to dominance shifts in the neighborhood of the unstable equality.¹⁴

¹⁴ The sociologist Marcel Mauss (1872-1950) made the "gift exchange" or "ceremonial exchange" a pillar of his theory of society.



Figure 5 The schema of Giving derived from the butterfly (A_6) and comic strip as a representation of the sentence: Peter gives the hammer to Anne (cf. Wildgen, 1994: 146)

The derivation of the archetype of *Sending* (and other 4-valence processes and actions) is based on the elliptic umbilic that has corank 2 (two internal variables x and y), an attractor on one side, and a repellor on the other. The attractor is surrounded by saddles that lead to three valleys of the vector field (called Thalweg). If they are closed (compactified), one obtains three further attractors. The transfer of V₂ (compactified to M₂) from V₁ (compactified to M₁) to V₃ (compactified to M₃) must pass the saddles controlled by the central minimum of the umbilic M₄. In this process, the fourth attractor enables the transfer. Cf. the details in Wildgen (1985: 208 - 2012).¹⁵



Figure 6 (Left)The vector field around the central minimum in the elliptic umbilic, the saddles, and the valleys V_1 , V_2 , and V_3 that can be closed by compactification and are the basis of the 4valence process called Sending. (Right) Dynkin diagram of the compactified situation; (+) represents a minimum.

2.4. The validity of 'Thom's conjecture after 50 years (1972-2022)

Evaluating Thom's conjecture on verbal valences (deep cases) and basic sentence constructions after 50 years is difficult for several reasons. First, there are different layers of interpretation: spatial motion, action, interaction, and qualitative changes, even mental processes (virtual ontologies). Second, the case patterns in classical Indo-European languages, e.g., Sanskrit, Greek, and Latin, do not represent the corpus of languages worldwide. There are languages without case classification and marking. Word order, stress patterns, or contextual cues may provide comparable information (e.g., Mandarin Chinese). One distinguishes two basic types of

¹⁵ There are more mathematical difficulties in the transition to corank 2 unfoldings that should be further clarified in mathematical research. Thom gave some hints but the task of finding and classifying proper paths in the bifurcation plane of the catastrophes remains open.

languages with explicit case marking: *nominative-accusative* and *ergative-absolutive*. Many Western languages prefer the accusative-nominative type (except the Basque language). However, case marking with flectional suffixes has disappeared in many European languages, e.g., Romance languages (e.g. French and Spanish) and some Germanic languages (e.g. English). Contrary to the pre-eminence of synchronic analysis in classical structuralism, such processes ask for new dynamic access, for instance, in the line of modern grammaticalization theory. In this perspective, the morphogenetic models proposed by Thom in 1972 gain a new relevance. They can describe processes in language change as continuous or abrupt (catastrophic) transitions in the space of choices. The author proposed a catastrophe theoretical model for the transition between different case systems based on a set of paths in the bifurcation plane of the cusp (A₄). Cf. Wildgen (2017).



Figure 7 A "cusp "- model of the typology of case marking and underlying historical processes

In summary, empirical research could only partially match Thom's conjecture with facts in the world's languages. However, his hypothesis opened a window to understanding the morphogenesis of case marking and "meaning building" in sentences. Moreover, it introduced a spatial and dynamical representation of case concepts and verbal meanings that are cognitively

more realistic than representations in terms of predicate calculus that dominated formal semantics in the 20th century.¹⁶

3. The semiophysics of Thom (1980-2000) and the dynamics of meaning (in art and music)

A second phase of Thom's research in semiotics and linguistics began around 1978 and led to his book "Sketch of a Semiophysics" (Thom, 1988). He generalized the notions of salience ("saillance") and pregnancy/vital relevance ("prégnance")¹⁷ to circumscribe the forces at work in the creation and stable use of semiotic patterns. Salience relates to the sense organs (sight, hearing, smell, touch, and others) and the affordances in the surrounding world. Pregnancy considers the biological and mental functions of action and communication linked to perceiving salient features and perceptually controlled action. The coordination of both enables "meaning building", i.e., the morphogenesis of meaning. In the case of language, it surfaces on very different temporal and spatial scales:

- The origin of language,
- Language change,
- Pidgin or Creole language, they witness the capacity to form spontaneous linguistic patterns.
- The infant's discovery of lexica meaning and syntax,
- Lexical and syntactic innovation in poetic creativity.

Semiophysics builds a bridge between the traditionally separated fields of humanistic and social sciences and the fields of the natural sciences. The mathematics of "dynamic systems theory" (catastrophe theory, synergetics, chaos theory, stochastic dynamic, and other subfields) may catalyze new ideas linking both sides. Thus, semiophysics is rather a program of research and not yet an established discipline. In the following, we shall consider with priority applications of

¹⁶ The proposals of quasi-dynamic gestalts in Cognitive Semantics in the style of Lakoff, Langacker, and Talmy has been criticized in Wildgen (1994: chapter 1) as a purely didactic short hand of intuitions in early Gestalt theory (e.g. Kurt Lewin's topological psychology, 1936).

¹⁷ The notions proposed by Thom are related to similar ones in ethology (Konrad Lorenz) and Gestalt theory (valence, symbolic pregnancy); cf. Wildgen (2010a)

semiophysics beyond language, i.e., in visual and musical communication.¹⁸ In the case of visual meaning, we take a departure from regular, symmetrical shapes and consider the basic force effects that are visually accessible. The analysis of Leonardo's mural "Last Supper" may represent this strategy. In the case of music, the salience patterns are given by acoustic laws and their physiological implementation in the cochlea and the cortical centers. Motion patterns underlie the patterns of rhythm in music. The pregnancy aspect is governed by the memory of forces in musical performance and associated emotional processes. For details, cf. Wildgen (2013, 2018 and 2023a).

3.1. The case of visual art

Leonardo da Vinci (1452-1519) was a new type of artist in Renaissance times connected to contemporary science and philosophy. He illustrated Luca Pacioli's treatise "De divina proportione" (composed 1497, published 1509) and devised the first skeletal representation of a platonic solid.

¹⁸ The psychophysical resources are clearer in the visual domain (e.g. art) and the domain of music (hearing and performing music). In the case of language, the "arbitraire du signe" (arbitrariness of the correlation of sign shape and meaning) emphasized by de Saussure and linguistic structuralism has the effect of a stochastic noise masking the operation of basic principles founded in the nature of the human environment and the human body. In the following examples we have chosen cases where cultural (or academic) traditions are less prominent and the creativity of the artist in his confrontation with the human mind/body and nature is prominent.



Figure 8 Leonardo's picture of the (regular) dodecahedron in the treatise "De divina Proportine" by Luca Pacioli (printed 1509)

The following section discusses dynamic and topological features in Leonardo's mural "Last Supper" (1495-1498) as an example of semiophysics in art. Cf. for basic materials Marani (2001) and for semiotic analyses Wildgen (2004, 2010c, and 2013: 66-102).



Fig. 9: Leonardo da Vinci The Lord's Supper in Milan, Santa Maria Delle Grazie (1495-1498) (Wikipedia Commons)

In Leonardo's mural "Last Supper", the central figure, Jesus, represents the type of an equilateral triangle (in three dimensions, a tetrahedron). The refectory is a box similar in shape to a cube or if one follows the lines of perspective one half of the octahedron (adapted to the rectangular shape of the refectory in Milan). The four groups of apostles also have a triangular shape, but they are agitated by a perturbation with its origin in Jesus saying, "One of you will betray me". The group with John, Judas, and Peter is the most agitated one; it even creates a triangular void on the right side of Jesus. John is leaning back to listen to Peter, whereas Judas resists the shock wave issued by Jesus (Peter points with the knob of his knife to Judas). Leonardo uses the mathematics of regular polygons and polyhedra to organize the traditional topic of the " Last Supper". As Slodowy (1988) showed, there is a link between regular polygons and the cuspoïds and of regular polyhedra and the unfoldings E_6 , E_7 , and E_8 : the umbilics are the topological correspondent to the intermediate Kleinian Dihedra.

For Leonardo, natural dynamics is the source of bodily dynamics and, finally, the dynamics of the mind. Therefore, his art starts from the physics of objects and bodies in space/time, resulting in a morphogenesis of meaning in a semiotic/symbolic space.

Repetitions with deformations dominate the dynamic of traditions in art. This copy and variation feature in the works of art can be demonstrated in the series of paintings with the topic "Last Supper" since the 11th century (cf. Wildgen, 2004 and 2010c). Elements of Leonardo's mural reappear in modern paintings. For instance, Andy Warhol selects major motifs, reducing the colors and the magnitude. Geometrically, the picture is a semi-periodic tessellation of the rectangular surface with triangular and rectangular pictures extracted from 'Leonardo's mural. Cf. Wildgen (2018: 86-89.¹⁹

¹⁹ For tessellations, cf. Penrose (1974) and https://en.wikipedia.org/wiki/Penrose_tiling.



Figure 10 Andy Warhol, 1987, Last Supper (regarding Leonardo's mural in Milan)

Concerning the mathematics of non-linear and chaotic dynamics, Pollock's action paintings correspond to new geometrical and dynamic principles, as did Leonardo's painting, relative to the mathematics of the 16th century. Jackson Pollock (1912-1956) leaves the static configuration of the first abstract paintings (e.g., Kandinsky "Komposition" VII, 1913) and makes a radical move towards motion and action in paintings. In his dipping technique, color flows out of containers or is sprayed over the canvas.²⁰ If we look at Pollock's painting "Grey and Red" (1948), we can understand the thin threads as the trace of quick motion and the thick lines or knots as halting points (assuming a constant paint flow).

²⁰ Cf.for more details Karmel (1999) and Wildgen (2013 : 144-156).



Figure 11: "Grey and Red ", 1948, 57,5 x 78,4 cm, Frederik R. Weisman Foundation, Los Angeles

Basic Gestalt laws of composition in paintings, e.g., up and down, left and right, or diagonals linking the corners, are disregarded. Taylor, Nicolich, and Jonas (1999) showed that Pollock's paintings have a fractal order. Specifically, two fractal measures are characteristic: The first is related to rapid hand motion on a scale of 2.5 to 5 cm, and the second scale (5 cm to 2.5 m) corresponds to his arm and body motion (into and around the canvas). Moreover, the fractal dimension increased when he developed his new painting style (*action painting*). In 1943, it was still linear (dimension 1). Almost ten years later, the fractal dimension is 1,72, thus nearer to a line filling the given surface.²¹ The principles of visual salience exploit the psycho-physical potential of humans, although the profile and preferences in this use change historically. The

²¹ The measurement uses the box counting technique. Boxes (i.e. quadratic surfaces) on the canvas are made smaller and smaller and one calculates how many steps of reduction are necessary to make the distinction between line and void surface disappear. If it does not disappear (no void) the dimension is 2, i.e., the plane is *full*.

dimension of pregnancy (meaning, cf. Thom, 1988) also changes. If, in the Middle Ages, religious texts gave the preferred contents that artists had to represent, mythological contexts came to the fore in the Renaissance. Eventually, the court life and later everyday life in the civic society, industrial contexts, and the artists' privacy became prominent. In abstract art, all these variable contents were eliminated or reduced to purely visual effects in the eye of the spectator (eventually, it triggers individual memories and emotions). This development shows that the dimension of *denotation* in semantics (in Peirce or "mental image" in de Saussure) is not a necessary ingredient of "meaning" in general. However, it is important in language and may be implemented in art and music.

3.2. The semiophysics of music

The physics (acoustics) of overtones in the human voice and musical instruments is essential to human perception and control of musical forms. The overtones have a regular physiological equivalent in the inner ear (cochlea). At the same time, this natural grid can be stretched in time, i.e., the resonating overtones can be "spelled out" as elements of a melody. As a result, the sequential order in music inherits the natural ordering system of the overtones.²² Memory effects impose the first severe restriction because memory works like a hedge in the cerebral recognition and assembling of complexes of musical signs.

As a consequence of these conditions of sound perception in humans emerges the phenomenon of categorical perception, i.e., the transitions from continuous scales (in physics) to boundaries imposed on this continuum, together with a radical reduction of the space of choices in musical perception. Eventually, the instruments devised and used by a population and later written music codes led to a wealth of musical forms and artificial "meanings" beyond natural forms of music.²³

²² The space of possible auditory distinctions is vast at this first level. Sundberg (1991: 62) computes a maximum of 1400 pitch distinctions and 280 distinctions of loudness.

²³ These meanings presuppose a learned knowledge of specific traditions of music. As in the case of language, a detailed and rich understanding of the message intended by the composer remains inaccessible if this prior learning

The human gait in walking and running is a major source of rhythm in music. Walking and dancing are subject to coordination dynamics, i.e., the motion of the right and the left leg are coordinated, and the motion of one dancer is coordinated with the motion of other dancers. The gesture is a particular form of movement that emerges from the field of physical movement and contains an intentional momentum, such as nodding, pointing, or eye-gazing. In music, gestures are a basic type of musical gestalt. A melody may be decomposed into several gestures, some of which are within the boundaries of beats, some exceeding them. Thus, musical communication can be understood to emerge from the physics/physiology of hearing, the motor dynamics of limbs in locomotion and dance, and the articulatory dynamics in singing. The handling of musical instruments is a derivative of these dynamic sources. In the history of musical theory, mathematicians since the legendary Pythagoras, later Leonard Euler, and more recently Mazzola (2002) propose a pertinent space in which music is embedded.



Figure 12 The torus of tierces in Mazzola (2002). The twelve tones (0...11) are arranged according to the distance between a major third (meridian circles), e.g. (2), (6), (10) and a minor third (equatorial circles), e.g. (2), (5), (8), (11). The spiral movement on the torus (see the arrows) corresponds to the steps in seconds, i.e. the sequence of 12 semitones

has not been successful. However, there lies a continuum between the meaning of music for experts and for laymen/women.

Music theorists in different historical phases proposed practical rule systems for music composition (for instance, the rules of counterpoint in the 17th century, the paradigm of classical tonality, and atonal music in the 19th and 20th centuries). We take the simple case of a song and its melody. The course of the melody has a center that can be used as a zero expectation of melody progression, and the melody is the deformation of this standard. Leyton (2004) assumes a "memory" as the specific meaning of a melody. It contains information about forces that have become effective and influenced the melody's design. The forces that shape the melody push the melodic line up or down concerning the middle (comfortable) level. These force patterns and their memory traces are natural candidates of musical pregnancy (meaning) not dependent on literary or other figural attributions.



Fig. 13 The melody course as a result of forces acting on a medium level

These movements are perceived as meaningful sign shapes. Downward movement may express sadness or resignation; an ascending movement, on the other hand, enthusiasm or hope. We take as an example the force memory of musical meaning in the song "Hallelujah" by Leonard Cohen. It moves in a flat curve around G/A in the first ten beats. After "It goes like this", it climbs up in steps until it reaches the E'. The repeated "Hallelujah" descends in a wave-like movement until it reaches the lower and final C, the resting point (the song is written in C).

If we apply Leyton's concept of memory of forces, we can draw the following schema of applied forces (Fig. 14) (Video link: <u>http://www.leonardcohen.com/</u>).



Fig. 14 Motion and force in the song "Hallelujah" by Leonard Cohen

Mathematical considerations were already central to musical thinking and composing in antiquity. In the 20th century, with synthesizers and computer-aided music, mathematically sophisticated types of music became typical in the avant-garde scene. Muso-mathematical considerations have played a role in musical composition since Claude Debussy (1862-1918) and Olivier Messiaen (1908-1992). The topological network of tunes was discovered at the end of the 19th century and gave rise to the "Tonnetz"-architectures of music based on mathematical group theory. The next generations of composers took up the suggestions of Thom's morphogenesis (cf. Andreatta, 2018) and the concepts introduced by Chaos Theory and Fractal Geometry.²⁴ Pagano (2018) analyzed Ligeti's piano studies: Vertigo, L'Escalier du diable, and Columnia Infinita (written around 1991). She writes: "The Devil's Staircase is a mathematical function, and the piece effectively describes it" (ibid. 96).

²⁴ Physiological processes, such as the heartbeat, seem to be regular, but they show fractal patterns Therefore it is not surprising that fractal patterns also are present in music. A team from the Max Plank Institute for Dynamics and Self-Organization in Göttingen, for example, discovered fractal patterns and self-similarity in the music of a well-known drummer (Jeff Porcaro). The fractal character arises when small deviations are generated and multiply in an iterative process, spread out, and finally overshoot the original pattern.

These hints may be sufficient to show that mathematical models of music are not only a possible means to understand music; musicians also use them to compose music and open new musical performance and understanding fields.

4. Conclusions

Fifty years after the publication of "Stabilité structurelle et morphogenèse" (1972) and twenty years after Thom's death (2002), the question arises if beyond his important research in mathematics (Fields medal 1958; cf. the posthumous volumes of his mathematical works, Thom (2017-2022) his scientific engagement in biology, linguistics/semiotics, and philosophy will have permanent effects in these fields In the case of his "topological grammar" (Thom, 1970 and 1972), his conjecture on the topology and dynamics of case frames (deep cases, verbal valence) received relevant corrections and elaborations between 1980 and 2010 and increased the approximation to linguistic facts and the integration of current theoretical advances in cognitive semantics and neuro-geometry. Nevertheless, Thom's ideas were so far from the priorities in linguistic structuralism and its self-restriction to synchrony (or to self-contained systems without natural determinations) that most linguists in the seventies of the last century did not dare to integrate these proposals into their current research. The morphogenetic perspective of Thom asks for natural (physical, biological, cognitive) explanations of linguistic facts beyond the "arbitraire du signe" established as the backbone of linguistic theories. Language creativity, language learning and change, and even the evolution of language become central questions in a morphogenetic analysis of language. The complicated, rule-governed linearization of human thought in specific languages is only of a subordinated scientific interest. Due to the poor response to Thom's conjectures in linguistics, major questions in the morphogenesis of language remained unexplored, and the topological and dynamic modeling techniques have not been substantially advanced. However, Thom's conjectures and sketches of models have opened up a vast field of theorizing and empirical approaches. The rapid development of the mathematics concerned (see chaotic, fractal, and stochastic dynamics), the turn of cognitive sciences towards naturalistic modeling (neuronal or biological), the expansion of semiotics in zoo-semiotics and

inter- and intracellular communication prepared the ground for an application of the morphodynamic paradigm.

Thom's second major contribution, his theory of salience and pregnancy, has been expanded towards multimodal communication (cf. Wildgen, 2023a). In the applications beyond language, the psycho-physical basis of communication is less obscured by sociocultural variation and enrichment. In the second part of this article, it is shown that innovative artists since the Renaissance (Leonardo da Vinci) and in modernity (Warhol, Pollock) reduced the impact of figurative (literary and traditional) content in art, such that the principles of visual morphogenesis (physical and corporeal determinants) come to the fore. Similar tendencies can be observed in music. It becomes evident if rule-governed compositions are neglected, and creative and innovative strategies are focused. We could only give a few examples of visual semantics in art and a simple example of "meaning building" in music. For more details, cf. Wildgen (2013, 2018; in German) and (2023a, chapters 3 and 4; in English).

In the case of language, the morphogenesis and creativity of meaning-making are less easy to access. Classical structuralism in linguistics and semiotics has made this impediment a central virtue and thus obstructed the further explanation of language and language use beyond the rules of usage. A major difficulty has been the adequate and general definition of "meaning," including visual, musical, olfactory, and other symbolic forms.²⁵

²⁵ Fillmore (1977) proposed a classification of scenarios and frames underlying sentences, Langacker (1987) introduced the key terms "trajectory" and "landmark" (cf. classical Gestalt-terms "foreground" and "background"). In Cognitive Semantics, linguists applied basic notions of Gestalt-psychology (introduced in the first half of the 20th century) but they did not dare to look beyond these psychological reflexes of the affordances in the eco-sphere of humans nor to the kinematics and dynamics in nature. In the spirit of structuralism questions of evolution, morphogenesis and development were neglected.

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