Workshop for young researchers in analysis and mathematical physics

ID de Contribution: 10

## A Poincaré-Steklov map for the MIT bag model

mardi 10 octobre 2023 14:45 (1 heure)

In this talk, I will present some study of the Poincaré-Steklov (PS) operator associated with the MIT bag operator on a smooth domain  $\Omega \subset \mathbb{R}^3$  with a compact boundary  $\Sigma := \partial \Omega$ .

This operator can be seen as the analogue of the Dirichlet-to-Neumann mapping, where the free Dirac operator  $D_m := -i\alpha \cdot \nabla + m\beta$  plays the role of the Laplace operator, and the Dirichlet and the Neumann traces are replaced by orthogonal projections of the Dirichlet traces along the boundary  $\Sigma$ . More precisely, this operator is associated with the following boundary value problem

\begin{equation}

 $P_{\rm t_{\rm sigma}v = g in H^{1/2}(sigma)^{4},$ 

\end{equation}

where  $P_{\pm}$  are the orthogonal projections along the boundary  $\Sigma$  and  $t_{\Sigma}$  is the classical trace operator.

In the first part of this talk, I will explain how the PS operator fits well into the framework of classical pseudodifferential operators and determine its principal symbol. In the second part, I will discuss the properties of the PS operator when the mass m becomes large enough. Namely, I will show that it is a 1/m-pseudodifferential operator and I will give its main properties, in particular its semiclassical principal symbol. Then, we apply these results to establish a Krein-type resolvent formula for the Dirac operator  $D_M = D_m + M\beta 1_{\mathbb{R}^3\setminus\overline{\Omega}}$  in terms of the resolvent of the MIT bag operator when M > 0 is large enough. Finally, we show that the operator  $D_M$  in the limit of the large coupling  $(M \longrightarrow \infty)$  converges to the MIT bag operator in terms of the norm-resolvent with a convergence rate of  $\mathcal{O}(M^{-1})$ .

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