Workshop for young researchers in analysis and mathematical physics

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A Poincaré-Steklov map for the MIT bag model

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In this talk, I will present some study of the Poincaré-Steklov (PS) operator associated with the MIT bag operator on a smooth domain $\Omega \subset \mathbb{R}^3$ with a compact boundary $\Sigma := \partial\Omega$.
This operator can be seen as the analogue of the Dirichlet-to-Neumann mapping, where the free Dirac operator $D_m := -i\alpha \cdot \nabla + m\beta$ plays the role of the Laplace operator, and the Dirichlet and the Neumann traces are replaced by orthogonal projections of the Dirichlet traces along the boundary Σ.
More precisely, this operator is associated with the following boundary value problem
\begin{equation}
(D_m-z)v =0, \quad \text{ in } \Omega, \qquad
P_{\pm}t_{\Sigma}v = g\in H^{1/2}(\Sigma)^{4},
\end{equation}
where $P_\pm$ are the orthogonal projections along the boundary Σ and t_Σ is the classical trace operator.

In the first part of this talk, I will explain how the PS operator fits well into the framework of classical pseudodifferential operators and determine its principal symbol. In the second part, I will discuss the properties of the PS operator when the mass m becomes large enough. Namely, I will show that it is a $1/m$-pseudodifferential operator and I will give its main properties, in particular its semiclassical principal symbol. Then, we apply these results to establish a Krein-type resolvent formula for the Dirac operator $D_M = D_m + M\beta 1_{\mathbb{R}^3\setminus\overline{\Omega}}$ in terms of the resolvent of the MIT bag operator when $M > 0$ is large enough. Finally, we show that the operator D_M in the limit of the large coupling ($M \longrightarrow \infty$) converges to the MIT bag operator in terms of the norm-resolvent with a convergence rate of $\mathcal{O}(M^{-1})$.

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