WELL-POSEDNESS AND QUASI-ADIABATIC CONTROL OF THE SCHRÖDINGER EQUATION BY DEFORMATIONS OF THE DOMAIN

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We consider the Schrödinger equation

$$i\partial_t u(t) = -\Delta u(t) \quad \text{on } \Omega(t) \tag{$*$}$$

where $\Omega(t) \subset \mathbb{R}^N$ is a moving domain depending on the time $t \in [0, T]$. We discuss how to provide a meaning to the solutions of such an equation defined on a time-dependent Hilbert space and how to control it.

In the first part of this talk, we ensure the well-posedness of the equation. We assume the existence of a bounded reference domain Ω_0 and a specific family of unitary maps $h^\sharp(t) : L^2(\Omega(t), \mathbb{C}) \longrightarrow L^2(\Omega_0, \mathbb{C})$. The conjugation by $h^\sharp$ provides a transposed equivalent equation of $(*)$ of the form

$$i\partial_t v = h^\sharp(t)H(t)h_\sharp(t)v \quad \text{on } \Omega_0 \tag{$**$}$$

where $h_\sharp = (h^\sharp)^{-1}$. The Hamiltonian $H(t)$ is a magnetic Laplacian operator:

$$H(t) = -(\text{div}_x + iA) \circ (\nabla_x + iA) - |A|^2$$

where A is an explicit magnetic potential depending on the deformation of the domain $\Omega(t)$. The formulation $(**)$ allows us to ensure the well-posedness of $(*)$ on $\Omega(t)$ endowed with suitable boundary conditions.

The second part of the talk presents the global approximate controllability of the Schrödinger equation $(*)$. The result is obtained by constructing suitable quasi-adiabatic deformations of the domain $\Omega(t)$ such that $\Omega(0) = \Omega(T) = \Omega$. The controllability follows from the Hamiltonian structure of the transposed equation $(**)$.