Optimal control of quantum systems: Applications to the robust control of Bose-Einstein Condensates and to quantum speed limit with piecewise constant control

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This presentation will focus on two recent applications of optimal control techniques to quantum systems.

In the first part, the goal is to manipulate the motional states of an atomic Bose-Einstein condensate (BEC) in a one-dimensional optical lattice. This study is a joint work wit the experimental group of Pr. D. Guéry-Odelin in Toulouse University (France). The protocols operate on the momentum comb associated with the lattice through its amplitude and phase [3]. A precise and versatile control for a wide variety of targets has been demonstrated. However, in order to improve the agreement between theory and experiment, it is important to design control processes that are robust with respect to experimental uncertainties (Figure 1). One limitation of the experimental setup under study is the value of the quasi-momentum which is not exactly zero as assumed in [3]. Due to the large dimension of the Hilbert space, numerical algorithms such as GRAPE [1] have to be used. In the case of the simultaneous control of an ensemble of systems, we propose a new formulation of GRAPE with a sequential-update of the control. We show numerically the superiority of this sequential approach with respect to the standard one [2].

In the second part, we apply a recent extension of the Pontryagin Maximum Principle (PMP) [4] to piecewise constant control. PMP, on which optimal control theory is based, was originally formulated for continuous functions. In practice, experimental implementations resort to piecewise constant signals. In [5], the authors have mathematically proved that for general non-linear dynamics, the PMP should be modified in a non-trivial way. We have transposed this result to quantum systems and applied it to the time-optimal control of simple quantum systems with two levels (Figure 2). In these systems, we derive exact quantum speed limits accounting for this key experimental constraint [6]. This should be an important step toward understanding the role of technological limitations in the manipulation of quantum systems, a key issue in quantum control.

References

[1] Glaser S. J., Boscain U., Calarco T., Koch C., Kockenberger W., Kosloff R., Kuprov I., Luy B., Schirmer S., Schulte-Herbrüggen T., Sugny D. and Wilhelm F. 2015 *Eur. Phys. J. D* **69**, 279

[2] Dionis E. and Sugny D. 2022 *J. Phys. B: At. Mol. Opt. Phys.* **55**, 184006

[3] Dupont N., Chatelain G., Gabardos L., Arnal M., Billy J., Peaudecerf B., Sugny D. and Guéry-Odelin D. 2021 *PRX Quantum* **2**, 040303

[4] Boscain U., Sigalotti M. and Sugny D. 2021 *PRX Quantum* **2**, 030203

[5] Bourdin L. and Trélat E. 2017 *Automatica* **79**, 273

[6] Dionis E. and Sugny D. 2023 *Physical Review A* **107**, 032613

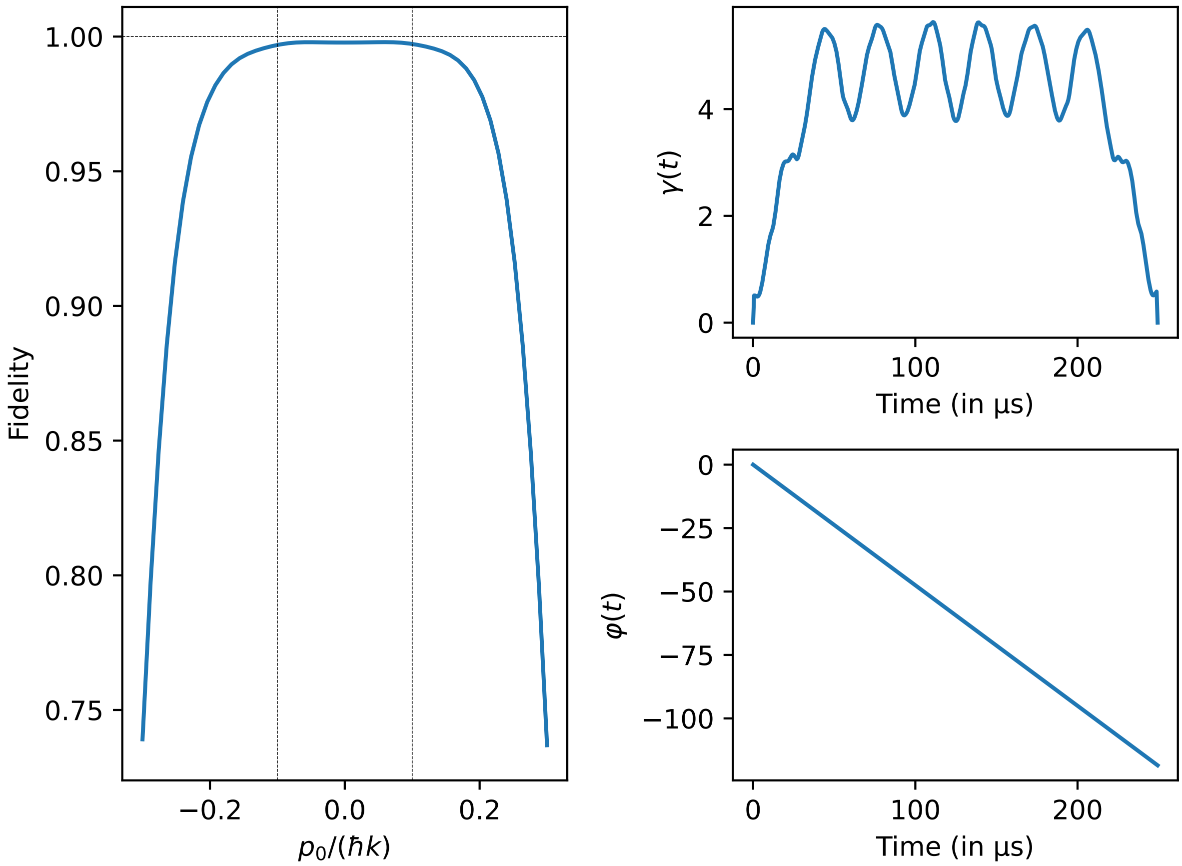

Figure 1: *Robust optimal control of BEC. Left panel, fidelity with respect to quasi-momentum. Top right panel, amplitude control. Bottom right panel, phase control.*

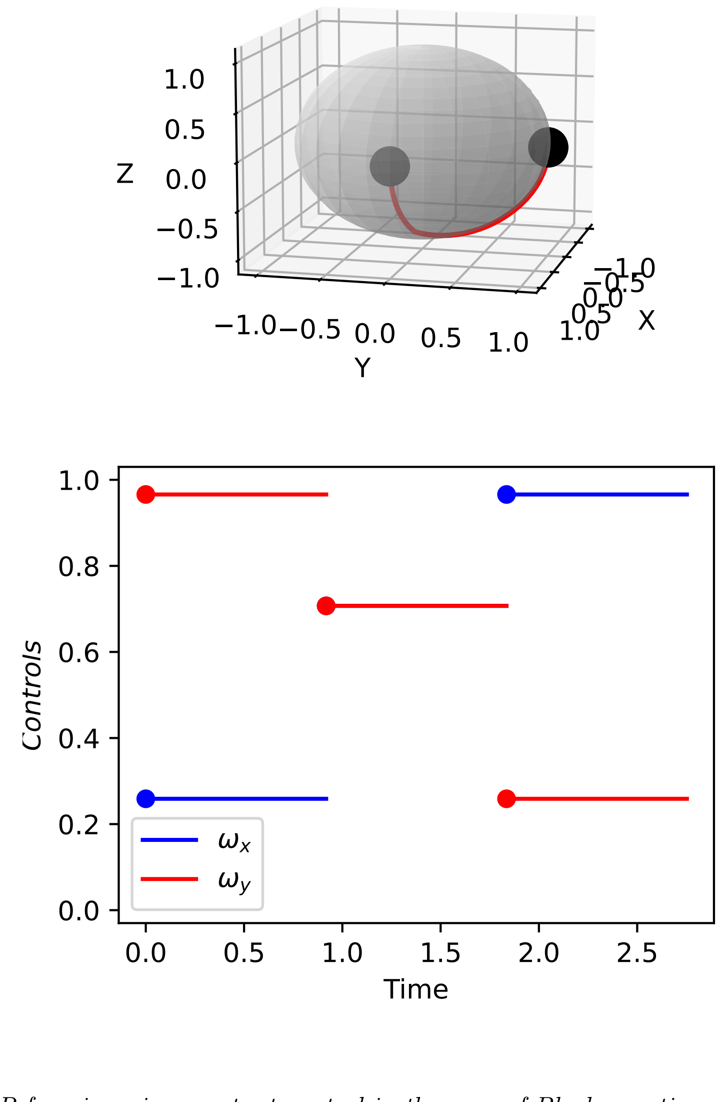

Figure 2: *PMP for piecewise constant control in the case of Bloch equations. Top panel, trajectory on Bloch sphere. Bottom panel, optimal controls.*