The Nuclear Engineering MOdelling (NEMO) group at Politecnico di Torino: A focus on reactor physics studies and research

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Politecnico di Torino

First of all … thanks

- Thanks to the organizers of this event, prof. Olga Mula and prof. Nicolas Seguin, for their kind inviation to participate and give a presentation on some of the research activities that I am coordinating at Politecnico di Torino
- Thanks in particular for the support in the organizational aspects, allowing me to be present here, considering that
  - I have 2 courses to teach in PoliTO during this semester
  - Unpredicatable events, such as landslides, are making VERY difficult to reach France from Italy these days
- I am enjoying very much being here and participating to this event

- I made some last second modifications to the slides this morning to make it more consistent with what you heard already


The NEMO group
http://www.nemo.polito.it/

The Nuclear Engineering MOdelling group

http://www.nemo.polito.it/

- Research group at Politecnico di Torino, in the Department of Energy (DENERG), working on mathematical and numerical modelling for applications related to innovative fission and fusion applications:
- A synthesis of the research topics:
  - Physics and engineering of current and innovative fission nuclear systems
  - Multiphysics modelling of next-generation reactors
  - Deterministic and Monte Carlo methods for neutral and charged particles transport
  - Superconducting magnets and cryogeny for nuclear fusion reactors
  - Analysis of high thermal flux components in nuclear fusion reactors
  - Plasma-wall interactions in fusion reactors
  - Computational thermal Fluid Dynamics (CFD) applications to nuclear and systems
  - Probabilistic Risk Asssessment (PRA) and reliability analysis for fission and fusion plants

Reactor physics and neutron transport activities

- We are a rather large and diverse research group, where the larger fraction of people is focused on fusion-related topics. As full professor in nuclear reactor physics, I can extract the topics «closer to my heart»:
  - **Physics** and engineering of current and **innovative fission nuclear systems**
  - **Multiphysics modelling** of next-generation reactors
  - **Deterministic and Monte Carlo methods** for neutral and charged particles transport
  - Superconducting magnets and cryogeny for nuclear fusion reactors
  - Analysis of high thermal flux components in nuclear fusion reactors
  - Plasma-wall interactions in fusion reactors
  - Computational thermal Fluid Dynamics (CFD) applications to nuclear systems
  - Probabilistic **Risk Asssessment** (PRA) and reliability analysis for fission and fusion plants

EUROfusion

DTT

Modelling activities in the NEMO group

SAMOSAFER

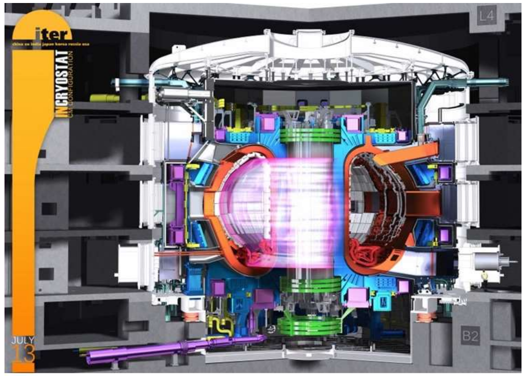
PASCAL

GRE@T-PIONEER

FUSION

FISSION

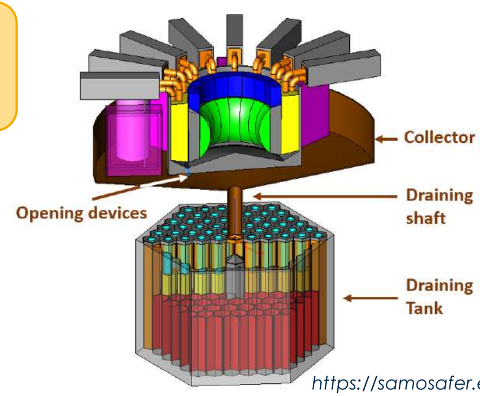
Superconducting magnets (high and low T) - Cryogenics



https://www.iter.org/mach/tokamak

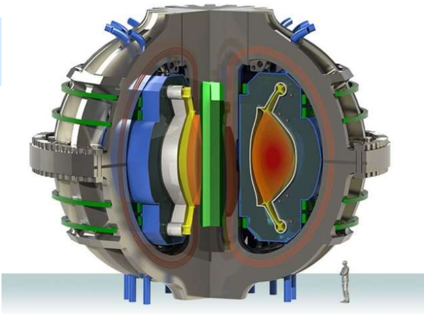
Neutronics and multiphysics

Uncertainty Quantification



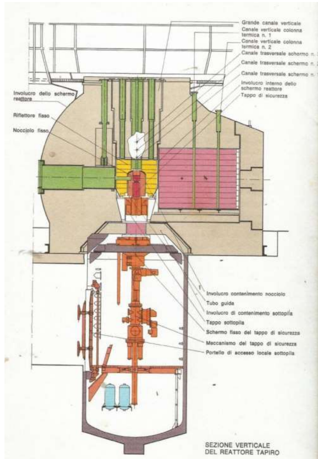
https://samosafer.eu/project/

Breeding blanket



https://news.mit.edu/2015/small-modular-efficient-fusion-plant-0810

Safety and risk analysis



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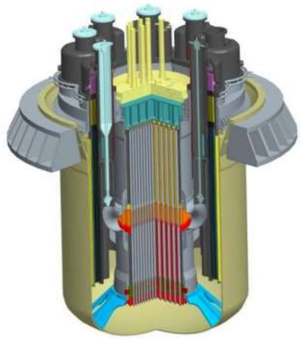


https://aris.iaea.org/PDF/ALFRED.pdf

Plasma and power exhaust

Decommissioning

A focus on reactor physics studies

- Reactor physics research focused mainly on innovative nuclear systems
- Generation IV reactors:
    - molten salt reactors: neutronic core modelling, core multiphysics, functional safety analysis and fauld decection modelling, UQ
    - liquid-metal cooled fast reactor: core multiphysics, subchannel thermal-hydraulics modelling, UQ
- Fusion reactors:
    - ARC design: neutronic modelling of the breeding blanket (BB), BB multiphysics, UQ
- Common features
    - Heavy involvment of PhD students and post-doc
    - Development of modelling approaches and application to engineering «interesting» cases
    - Cross-fertilization among topics and among people
- In the following slides I will try to design and illustrate a path accross different topics studied in the latest year

A theoretical problem: the eigenvalue formulation of the neutron balance equation

- Classic problem in reactor physics: $\hat{L}\varphi + \hat{C}\varphi = \hat{S}\varphi + \hat{F}\varphi$

streaming ($\hat{L}$), *collision* ($\hat{C}$), *scattering* ($\hat{S}$), *fission* ($\hat{F}$)

that requires an eigenvalue formulation for its actual solution

- The eigenvalue problem is
    - of relevant interest for *technological applications* (e.g. the criticality problem for a nuclear reactor or the determination of the asymptotic behavior)
    - of relevant interest as a *physico-mathematical problem* (e.g. study of the eigenvalue spectrum, which in turn may lead to applications such as ω-modes)
- Various formulations can be identified, with different mathematical characteristics and practical implications

The eigenvalue formulations - I

- k-eigenvalue problem (effective multiplication constant)

$$\hat{L}\varphi + \hat{C}\varphi - \hat{S}\varphi = \frac{1}{k}\hat{F}\varphi$$

  - Focus on the multiplication phenomenon
  - Widely adopted in nuclear eng. community (present in any code)
  - Solution has physical meaning only if highest eigenvalue (k0) strictly equal to 1
- α-eigenvalue (inverse of the stable period)

$$\frac{\alpha}{v}\varphi + \hat{L}\varphi + \hat{C}\varphi = \hat{S}\varphi = \hat{F}\varphi$$

  - Time eigenvalue, recently a subject of large interest in the nuclear eng. community
  - Physical significance of the eigenvalue (inverse of stable period) and of the eigenfunction (asimptotic evolution)
  - Complete formulation (with delayed neutrons) has a more complex dependence on the eigenvalue

$$\frac{\alpha}{v}\varphi + \hat{L}\varphi + \hat{C}\varphi = \hat{S}\varphi = \left(\hat{F}_p + \hat{F}_d(\alpha)\right)\varphi$$

  - Implementation in deterministic and MC codes is under way (e.g. TRIPOLI)

The eigenvalue formulations - II

- γ-eigenvalue (effective multiplication factor per collision)
  - Suggested by Davison (sometimes referred with letter c)
  - «applies» to scattering and fission

$$\hat{L}\varphi + \hat{C}\varphi = \frac{1}{\gamma}\left(\hat{S}\varphi + \hat{F}\varphi\right)$$

- δ-eigenvalue (effective density factor)
  - «applies» to all collision terms of the equation
  - Can be interpreted referring to material density

$$\hat{L}\varphi = \frac{1}{\delta}\left(-\hat{C}\varphi + \hat{S}\varphi + \hat{F}\varphi\right)$$

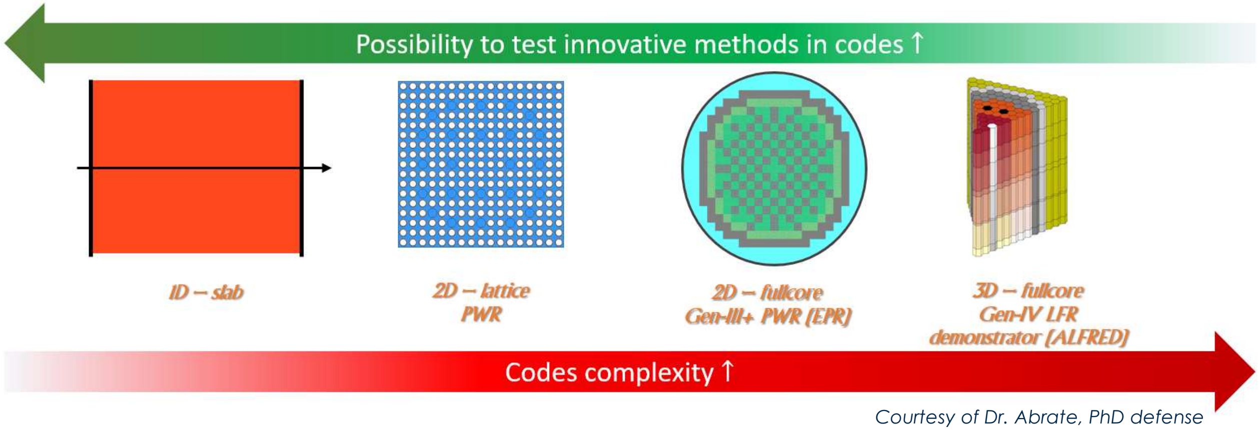
- These formulations, although based on the same logic as k, have less intuitive nuclear engineering meaning
- *Theoretical interests*:
  - Perform a comparative assessment of the different formulations of the neutron balance equation eigenvalue problem
  - Focus on the angular convergence of transport model approximations → convergence of P_N formulation

N. Abrate, M. Burrone, S. Dulla, P. Ravetto, P. Saracco, Eigenvalue formulations for the Pn approximation to the neutron transport equation, *J. Comp. Theoret. Trans.*, **50** (5), 407-429, 2021.

N. Abrate, S. Dulla, P. Ravetto, P. Saracco, On some features of the eigenvalue problem for the P_N approximation of the neutron transport equation, *Ann. Nucl. En.*, **163**, 108477, 2021.

How to perform such assessment of methods NOT already available in existing codes?

- Methods development and assesments performed in simplified configurations, with ad-hoc devised codes
    - Full control of implementation
    - Easier interpretation of results
    - Possibility to tackle «fundamental» issues, such as the comparison odd-even P_N



Courtesy of Dr. Abrate, PhD defense

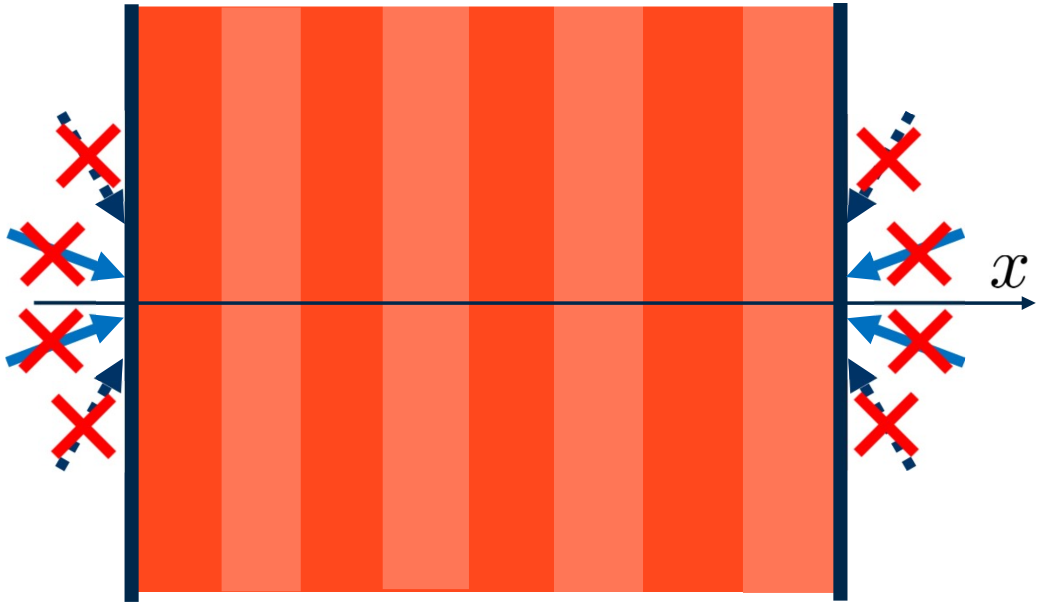
When outcomes are promising, the step towards large-scale implementation can be discussed

- In the next slides some results of this study are shown, which led to the idea of an ***innovative approach to the eigenvalue formulation*** itself, with a practical engineering implications

P_N model in 1D slab

- Simple geometry, still possible to account for heteroegeneity and multigroup treatment
- Convergence study of
  - N order in P_N,
  - anysotropy order of scattering
- Comparision of performance of different boundary conditions (Mark vs. Marshak)
- Focus on the use of even-order P_N

Idea of 1D slab: see slide 18 of Schlottbom presentation this morning



BC: see slide 15 of Schlottbom presentation this morning

Small detour on what the P_N model is

cfr slide 21 Schlottbom

- Expansion of angular flux on spherical harmonics → in 1D slab become Legende polynomials

$$\varphi(x,\mu)=\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\phi_\ell(x)P_\ell(\mu)$$

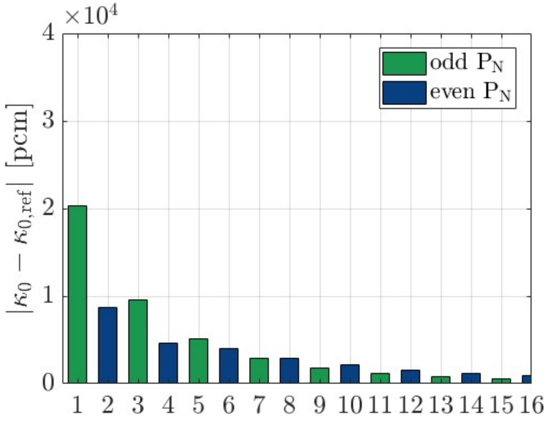
- Infinite set of 1st order coupled differential equations for the flux moments

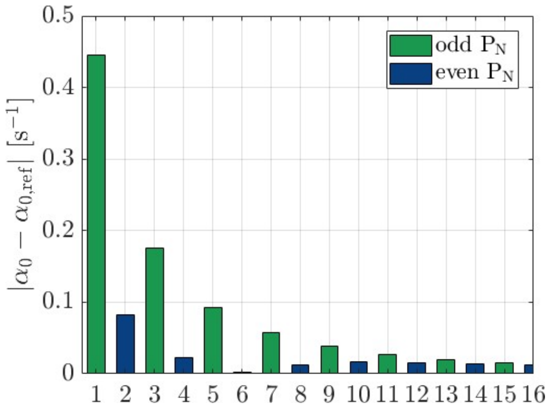
$$\frac{\ell}{2\ell+1}\frac{d\phi_{\ell-1}(x)}{dx}+\frac{\ell+1}{2\ell+1}\frac{d\phi_{\ell+1}(x)}{dx}+\Sigma(x)\phi_\ell=\Sigma_s(x)\eta_\ell\phi_\ell+\nu\Sigma_f(x)\eta_\ell\phi_\ell\delta_{\ell,0}+S_\ell$$

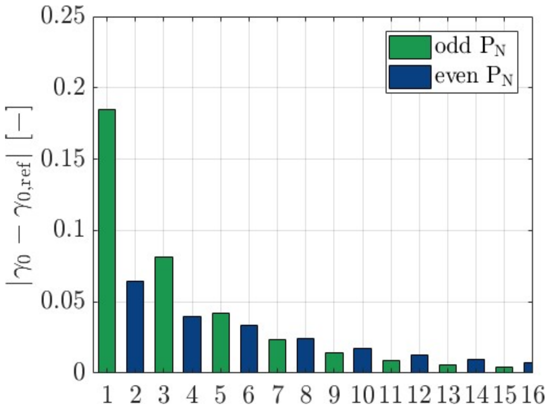
- Sequence is obviously truncated at some points
  - N=order or truncation
  - N+1 BCs, physically based: Mark imposes zero incoming flux in the discrete values of μ corresponding to the zeroes of P_{N+1}, Marshak imposes zero incoming odd angular moments
- N is classically chosen odd, as *even-order P_N is mathematically ill-posed*
  - Possibility to reduce it to PN-1 with linear combination of equations
  - Grey area in the definition of directions when adopting Mark BC

cfr slide 34 Schlottbom

Comparison of even and odd-order P_N - I

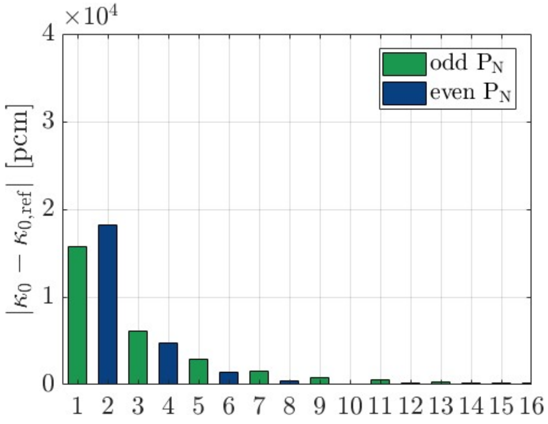


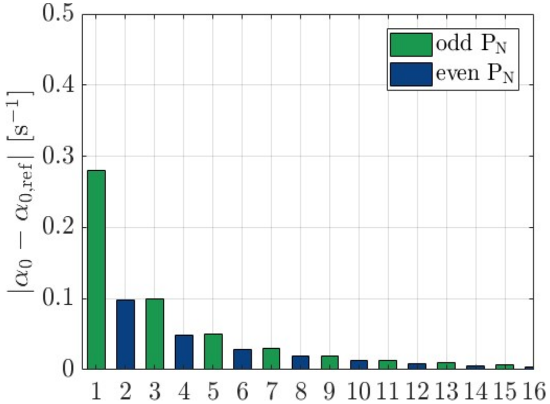


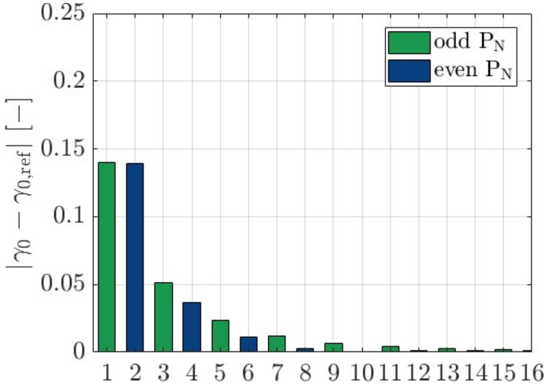


Mark BC

H = 1.79613 cm
(optically small system)
2-group
Pu-239 based fuel





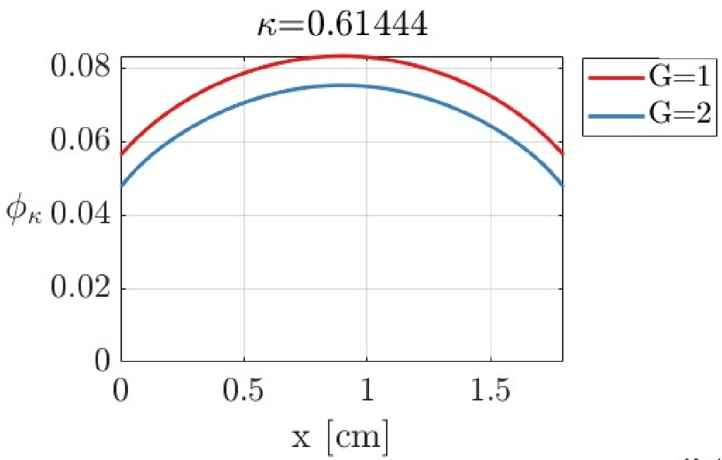


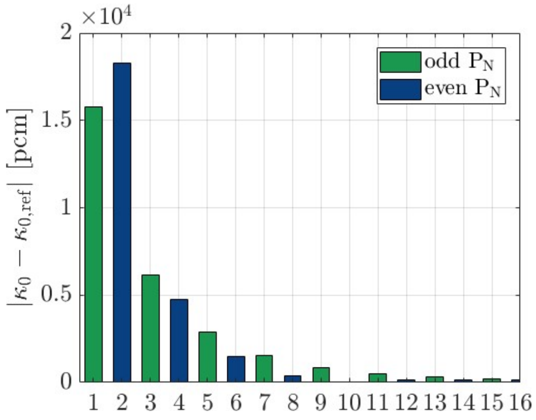
Marshak BC

NOTE: P_2 reduces to P_1, which in turn is diffusion, but with a different value for D – see backup slides

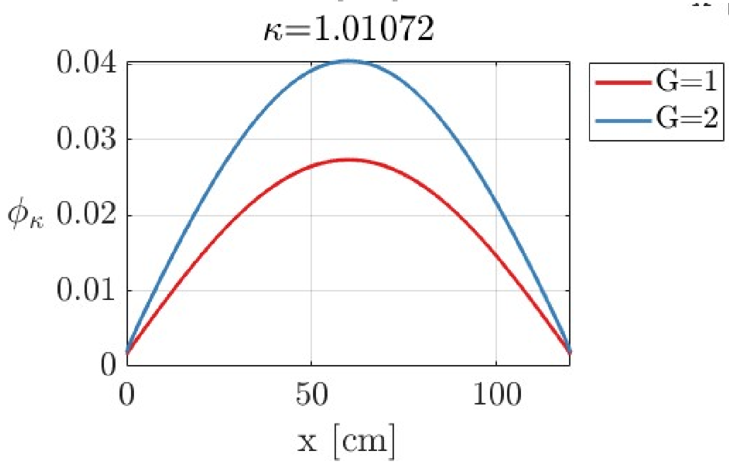
Optically large vs. small systems

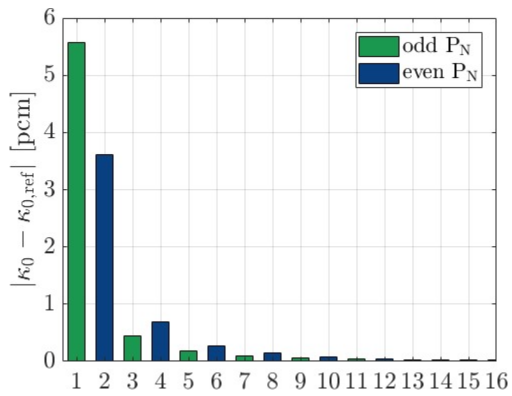
Same system of previous slides



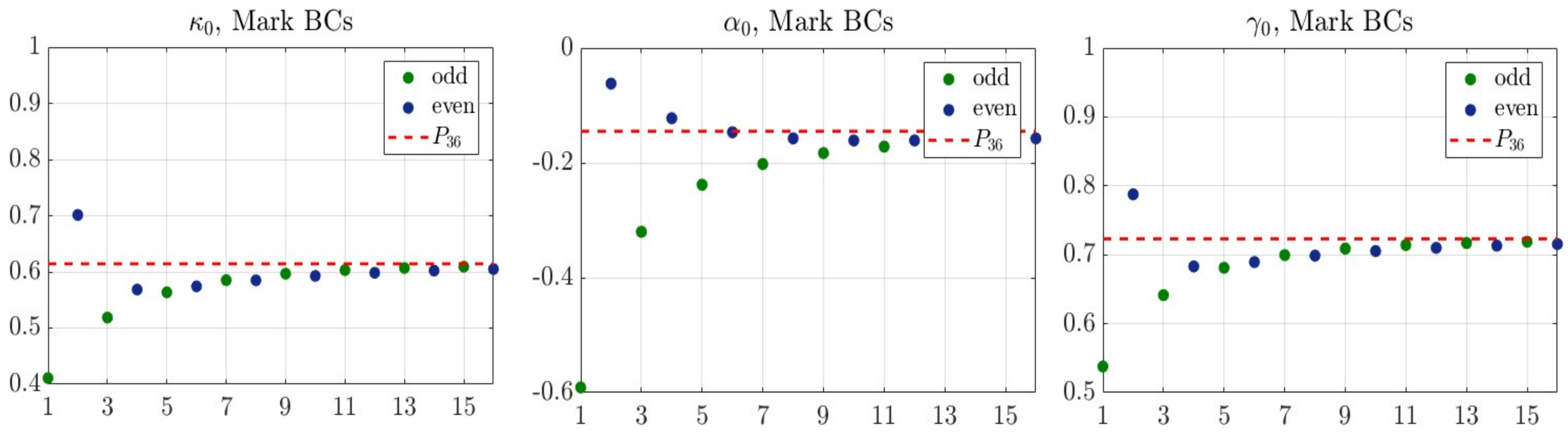


Cross sections adjusted to have more diffusive system,
H = 120 cm



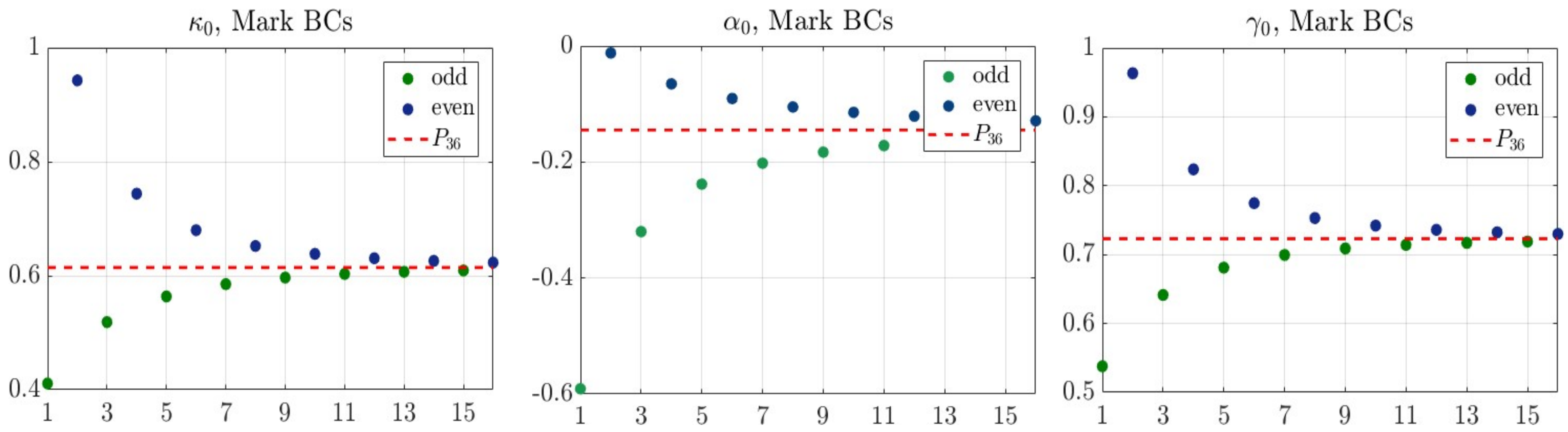


Comparison of even and odd-order P_N - II



Mark BC (roots P_N)

H = 1.79613 cm
(optically small system)
2-group
Pu-239 based fuel



Mark BC (roots P_{N+1})

NOTE: no results for the «density» eigenvalue, as its convergence was showing issues… but there's always a physical reason …

The δ–eigenvalue formulation amd beyond

$$\hat{L}\varphi = \frac{1}{\delta}\left(-\hat{C}\varphi + \hat{S}\varphi + \hat{F}\varphi\right)$$

- Physical interpretation
  - Action on medium atomic density (as part of macroscopic Xsection)
  - Contributes to competing phenomena (absorption vs. scattering vs. fission) → possibility NOT to find a solution
  - Can also be seen as spatial scaling (when multiplied by the streaming term)
- Interest in exploiting this effect
  - Eigenvalue definition focusing on some specific physical phenomenon (as k for fission) … but also on a specific isotope and/or a certain domain in space
  - Definition taylored to the specific application of interest
    - Definition of moderation ratio (a classic!)
    - Critical boron concentration search
    - Control rod design
    - Fuel concentration diluted in FLiBe in MSR

Engineering problems tipically solved iteratively

Generalized ζ eigenvalue formulation - I

A generalised eigenvalue for a specific isotope m^*, located in the region $\mathcal{V}_{m^*}$,

N. Abrate, S. Dulla, P. Ravetto, P. Saracco, A Generalized Eigenvalue Formulation for Core-Design Applications, *Nucl. Sc. Eng.*, **197**:8, 2047-2071, 2023

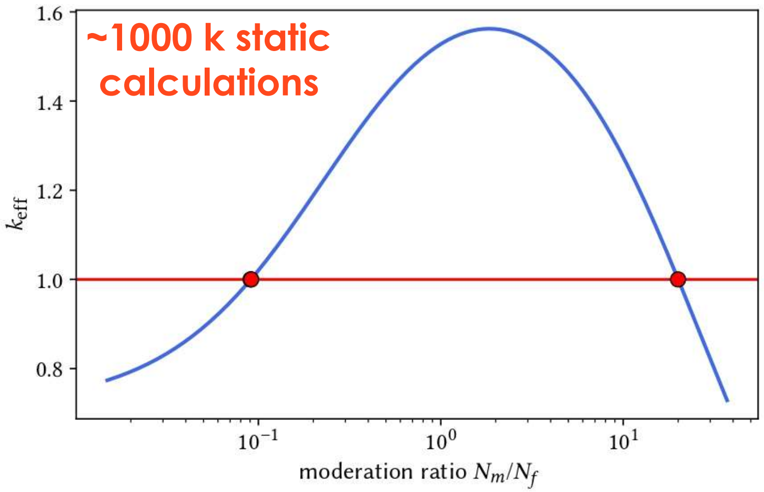
$$\begin{cases}
\underbrace{\vec{\Omega}\cdot\nabla}_{\text{Streaming}}\phi(\vec{r},E,\vec{\Omega}) + \sum_{\substack{m=1\\ m\neq m^*}}^{M} [N_m(\vec{r})\underbrace{\sigma_{t,m}(E)}_{\text{Removal}}]\phi(\vec{r},E,\vec{\Omega}) + \frac{1}{\zeta}[N_{m^*}(\vec{r})\underbrace{\sigma_{t,m^*}(E)}_{\text{Removal}}]\phi(\vec{r},E,\vec{\Omega}) = \\
\sum_{\substack{m=1\\ m\neq m^*}}^{M} \int dE' \oint d\vec{\Omega}' [N_m(\vec{r})\underbrace{\sigma_{s,m}(E')}_{\text{Scattering}}] f_{s,m}(\vec{r},E'\to E,\vec{\Omega}'\cdot\vec{\Omega})\phi(\vec{r},E',\vec{\Omega}') + \\
\frac{1}{\zeta}\int dE' \oint d\vec{\Omega}' [N_{m^*}(\vec{r})\underbrace{\sigma_{s,m^*}(E')}_{\text{Scattering}}] f_{s,m^*}(\vec{r},E'\to E,\vec{\Omega}'\cdot\vec{\Omega})\phi(\vec{r},E',\vec{\Omega}') + \sum_{\substack{m=1\\ m\neq m^*}}^{M} \int dE' \oint d\vec{\Omega}' [N_m(\vec{r})\nu_m(E')\underbrace{\sigma_{f,m}(E')}_{\text{Fission}}]\frac{\chi_m(E)}{4\pi}\phi(\vec{r},E',\vec{\Omega}') + \\
\frac{1}{\zeta}\int dE' \oint d\vec{\Omega}' [N_{m^*}(\vec{r})\nu_{m^*}(E')\underbrace{\sigma_{f,m^*}(E')}_{\text{Fission}}]\frac{\chi_{m^*}(E)}{4\pi}\phi(\vec{r},E',\vec{\Omega}') \qquad \vec{r}\in\mathcal{V}_{m*} \\
\\
\underbrace{\vec{\Omega}\cdot\nabla}_{\text{Streaming}}\phi(\vec{r},E,\vec{\Omega}) + \sum_{m=1}^{M} [N_m(\vec{r})\underbrace{\sigma_{T,m}(E)}_{\text{Removal}}]\phi(\vec{r},E,\vec{\Omega}) = \sum_{m=1}^{M} \int dE' \oint d\vec{\Omega}' [N_m(\vec{r})\underbrace{\sigma_{s,m}(E')}_{\text{Scattering}}] f_{s,m}(\vec{r},E'\to E,\vec{\Omega}'\cdot\vec{\Omega})\phi(\vec{r},E',\vec{\Omega}') + \\
\sum_{m=1}^{M} \int dE' \oint d\vec{\Omega}' [N_m(\vec{r})\nu_m(E')\underbrace{\sigma_{f,m}(E')}_{\text{Fission}}]\frac{\chi_m(E)}{4\pi}\phi(\vec{r},E',\vec{\Omega}') \qquad \vec{r}\notin\mathcal{V}_{m*}
\end{cases}$$

Generalized ζ eigenvalue formulation - II

- Direct implementation in existing, k-based, codes is not trivial
- Proof of concept and applications to significant problems performed in a in-house code developed for the purpose (*extension of the previous one…*)
- A few examples of applications are given

k-search in homogeneous fuel-moderator mix

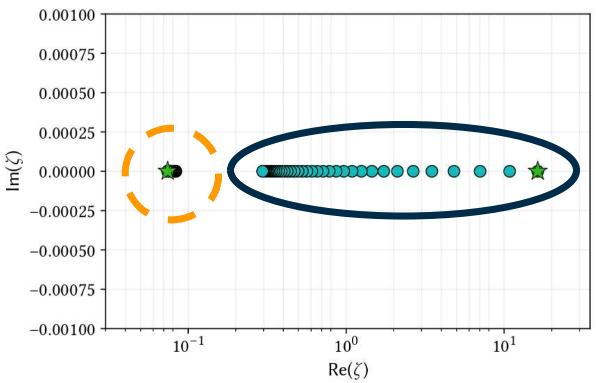
- Which is the critical water atomic density that has to be mixed to a given quantity of fissile material?
- Usually, we do this iteratively!



Introducing ζ in front of the moderator density → one eigenvalue problem

ζ formulation		iterative approach	
N_m/N_f	k_{eff}	N_m/N_f	k_{eff}
0.09096	1.00000	[0.09060, 0.09132]	[0.99918, 1.00082]
20.01752	1.00000	[19.98347, 20.14059]	[1.00074, 0.99734]

When looking at the eigenvalue spectrum …



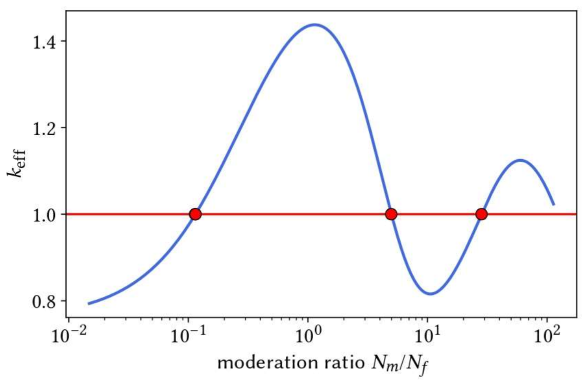
Undermoderated → larger eigenvalue separation → stability

Overmoderated → smaller ES

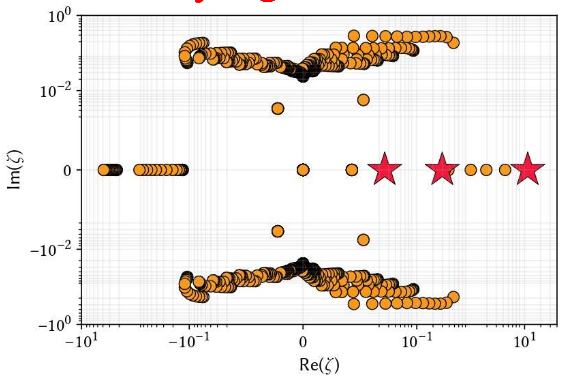
k-search in more complex arrangements

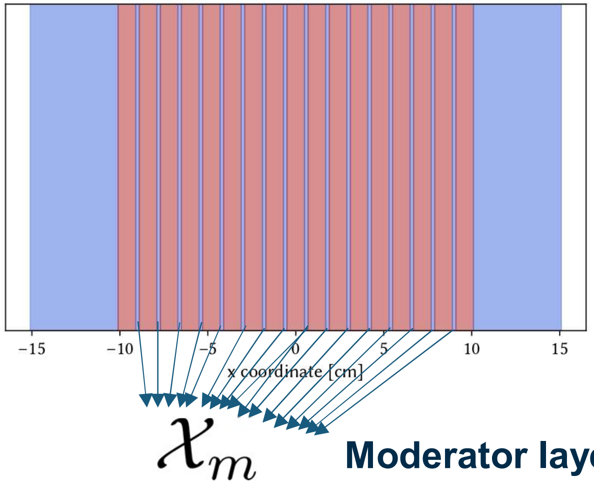
- A similar case is made for a more complex configuration (1D section of a core)

iterative



ζ eigenvalue

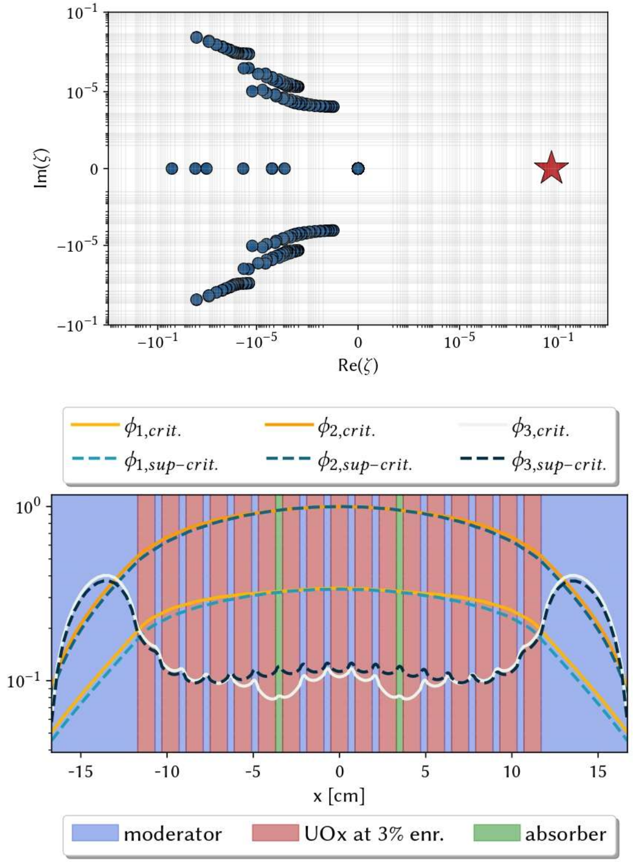




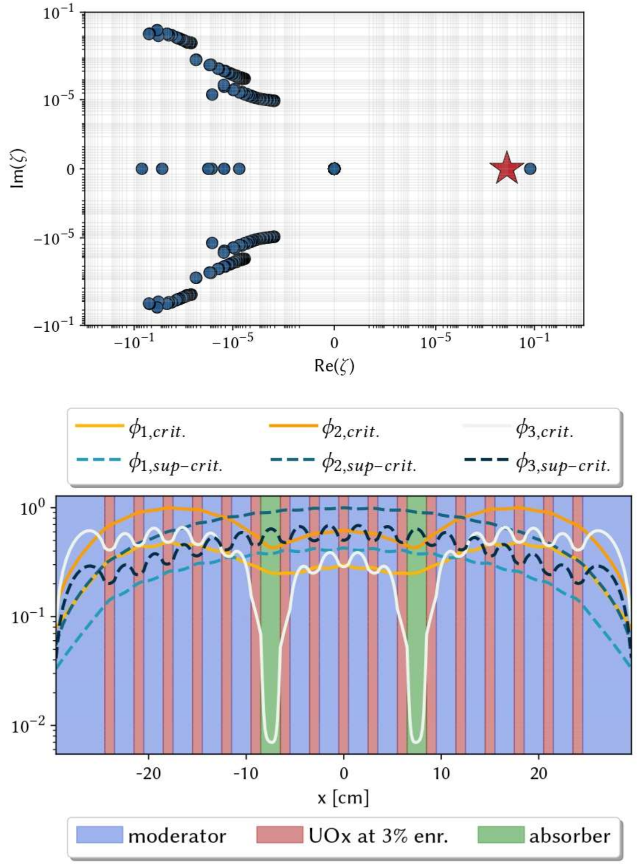
- All the three solutions are found in one single eigenvalue calculation

Control rod design

What is the critical concentration of absorber for the control rods?



Different moderation ratio

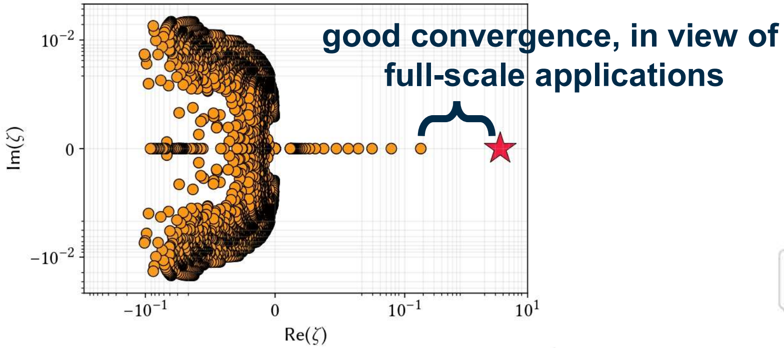


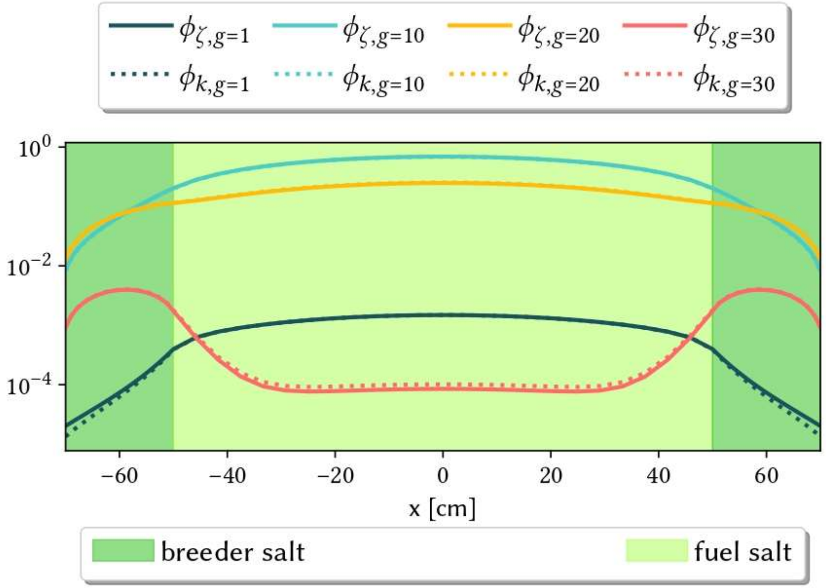
NOTE: Still, it is possible NOT to find a solution (positive eigenfunction) for this problem, why?

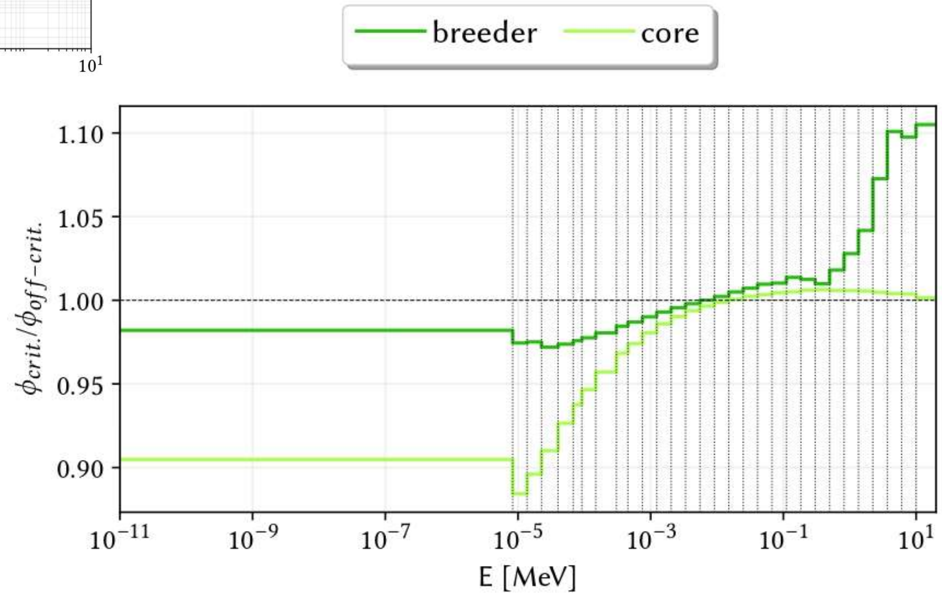
Fissile concentration in salt in MSFR

What is the critical concentration of U-233 to be diluted in the salt?

30-group, P_1 model







Some remarks on the ζ eigenvalue formulation

- The ζ eigenvalue equation allows to estimate the effect of a specific nuclide and its location in the reactor on the total balance, in a self-consistent way
  - $\mathcal{V}_{m^*}$ does not need to be simply-connected
  - ζ is a scaling factor for the atomic density of the isotope m* in the region $\mathcal{V}_{m^*}$
- The generalised ζ problem does not always yield a positive eigenfunction: it is not guaranteed that the steady state can be attained acting on the isotope m* in a certain region of the phase space
- More realistic reactor configurations (2D, 3D) will be considered in the future … that would be the plan …

- Is there any other use for these eigenvalues/eigenfunctions?

Alternative weighting functions for group collapsing of nuclear data

- Core-design and safety calculations are typically based on a deterministic approach to the solution of neutron balance, thus requiring multi-group (from ≈70 to ≈ 3 groups) nuclear data
- Group collapsing is (still) a challenging problem
  - Definition of energy intervals
  - Treatment of resonances
  - Weighting function to be adopted

$$\overline{\Sigma}_{y,g,i}(\vec{p}) = \frac{\int_{V_i} d\vec{r} \int_{E_g}^{E_{g+1}} dE\, \Sigma_y(\vec{r},E,\vec{p})\, \psi(\vec{r},E,\vec{p})}{\int_{V_i} d\vec{r} \int_{E_g}^{E_{g+1}} dE\, \psi(\vec{r},E,\vec{p})}$$

K. Dugan, R. Sanchez, and I. Zmijarevic, Cross section homogenization for transient calculations in a spatially heterogeneous geometry, *Ann. Nucl. En.*, 116 (2018)

- Standard choice is the k-eigenfunction
- Recent interest in the possible adoption of alternative routes (e.g. time eigenfunction)
- Focus on the application to time-dependent simulations in LFR