High frequency analysis: from operator algebras to PDEs

August 28th - Sept 1st, 2023

Université d'Angers

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Location:
Laboratoire angevin de recherche mathématique
Faculté des Sciences, Bât. I - Salles 001
2 Boulevard Lavoisier
Campus de Belle-Beille
49045 Angers cedex 01

How to come ?
Bus Line 2 direction Beaucouzé-Hauteroche, stop IUT.
ram C, direction Belle-Beille Campus, last stop.

Maps and more information on:
`https://math.univ-angers.fr/nous-visiter/`

Schedule of the talks:

Lundi	Mardi	Mercredi	Jeudi	Vendredi
	9h - 9h45 Robert Yuncken	9h - 9h45 David Krejcirik	9h - 9h45 Ewert Eske	9h - 9h45 Serena Federico
	10h15 -11h Victor Arnaiz	10h15 -11h Marcello Seri	10h15 -11h Soren Mikkelsen	10h15 -11h Vladimir Georgescu
	11h15 - 12h Discussions	11h15 - 12h Discussions	11h15 - 12h Discussions	11h15 - 12h Discussions
14h30 -15h15 Véronique Fischer	14h30 - 15h15 Lisette Jager	14h30 - 15h15 Davide Barilari	14h00 - 14h45 Discussions	
15h45 - 16h30 Patrick Gérard	15h45 - 16h30 Léo Morin	15h45 - 16h30 Cyril Letrouit	15h00 - 15h45 Roman Schubert	
16h45 - 17h25 Lancement des sessions de discussions				

Organisers:
Clotilde Fermanian Kammerer (`clotilde.fermanian@univ-angers.fr`)
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Abstracts

- **Victor Arnaiz Solorzano (Université de Nantes) - Quantum limits of some perturbed sub-Laplacians.**

In this talk I will present some recent results obtained independently in collaboration with Gabriel Rivière and Chenmin Sun on the spectral study of sub-elliptic operators. In the particular cases of the Baouendi-Grushin operator on the torus and certain perturbations of sub-Riemannian contact Laplacians in dimension three, we will describe the quantum limits associated with these operators.

- **Davide Barilari (University of Padova) - Spectral summability for 1D oscillators and Fourier Analysis in Carnot groups**

In this talk, I will address some questions concerning spectral properties of the sublaplacian $-\Delta_G$ on Carnot groups. The attention will focus on the Engel group, which is the main example of a Carnot group of step 3.

Thanks to Fourier analysis on the Engel group in terms of a frequency set, we give fine estimates on the convolution kernel satisfying $F(-\Delta_G)u = u \star k_F$, for suitable scalar functions F, proving an interesting summation formula for the spectrum of the sublaplacian.

This analysis requires a summability property on the spectrum of the quartic oscillator, which is of independent interest. If time permits we will discuss possible questions and generalization of this result to more general Carnot groups.

This is a joint work with H.Bahouri, I.Gallagher and M.Léautaud

- **Ewert Eske (Hannover University) - Fredholm operators on graded Lie groups**

A graded Lie algebra has a decomposition which is compatible with the Lie bracket. This allows to define a differential calculus on the corresponding group G in which an element of the Lie algebra can have order higher than one when viewed as a left-invariant differential operator. This notion of order is implemented in the pseudodifferential calculi by Fischer–Ruzhansky (for graded Lie groups) or van Erp–Yuncken (for general filtered manifolds). They generalize operators belonging to Hörmander's symbol classes. In this talk, I will discuss how global pseudodifferential calculi on $\mathbb{R}^n$, like the Shubin calculus, can be generalized to graded Lie groups using appropriate groupoids. In particular, we study when differential operators with polynomial coefficients on G define Fredholm operators. This relates to a Rockland type condition in terms of the representations on G. This is joint work with Philipp Schmitt and Ryszard Nest.

- **Serena Federico (University of Bologna) - Weyl calculus on graded groups.**

In this talk we will discuss a class of symmetric pseudo-differential calculi on graded nilpotent Lie groups using the Hörmander symbol classes introduced by V. Fisher and M. Ruzhansky. Among the quantizations generating these calculi, we shall identify a candidate Weyl quantization on general graded nilpotent Lie groups by comparison with the well-know Weyl quantization on $\mathbb{R}^n$.

Finally, we will see that in the case of the Heisenberg group our candidate Weyl quantization coincides with the only possible one.

- **Véronique Fischer (University of Bath) - Analysis in sub-elliptic and sub-Riemannian settings: challenges and recent approaches.**

  In this talk, we will discuss various approaches that have emerged over the last decades to tackle different questions related to the analysis of sub-elliptic and sub-Riemannian settings. For instance, the study of hypoellipticity in these contexts has made considerable progress using groupoid methods from operator algebra while spectral properties of certain sub-Laplacians have been analysed with PDE and micro-local techniques. We will also describe the development of a systematic notion of phase-space in these highly non-commutative contexts together with the associated semiclassical and microlocal analysis for subelliptic operators.

- **Vladimir Georgescu (CY University) - Field C^*-algebra and spectral analysis of quantum many channel Hamiltonians.**

  This talk concerns the field C^*-algebra associated to a symplectic space (in a representation, this is the C^*-algebra generated by the field operators) and the spectral theory of the self-adjoint operators affiliated to it. The field algebra is graded by the semilattice of finite dimensional subspaces of the symplectic space and this fact has deep consequences in the spectral analysis of the self-adjoint operators affiliated to it, which turns out to be a broad generalization of N-body Hamiltonians. We also briefly mention some results and difficulties in the case of infinite dimensional symplectic spaces, where the field algebra seems to be too small.

- **Patrick Gérard (Université Paris-Saclay) - The zero dispersion limit for the Benjamin-Ono equation on the line.**

  Zero dispersion problems for nonlinear evolution equations are known to create very strong oscillations so that there are very few examples for which it is possible to describe the limit. In this talk I will discuss the case of the Benjamin-Ono equation on the line, for which the Lax pair structure provides valuable tools leading to a complete answer to this problem.

- **Lisette Jager (Université de Reims) - Wick symbol of evolution operators for operators acting on the Fock space or on the Wiener space.**

  In the context of an infinite dimensional analogue of the Weyl pseudodifferential calculus, we have to work with the Fock space and with the Wiener space. This talk aims at giving a characterization, in terms of the Fock space, of a concept (a set of test functions) initially defined on the Wiener space. The second part is concerned with the explicit computation of the Wick symbol of evolution operators. More precisely, we consider a multiplication operator on a space of square integrable functions. Its second quantization is a self-adjoint operator (on the Fock space) and remains

self-adjoint if one adds a Segal field. Both operators give rise to groups of unitary operators. We compute the Wick symbols of operators of this kind.

- **David Krejcirik (University of Praha) - Magnetic Hardy inequalities in the Heisenberg group**

  We introduce a notion of magnetic field in the Heisenberg group and we study its influence on spectral properties of the corresponding magnetic (sub-elliptic) Laplacian. We show that uniform magnetic fields uplift the bottom of the spectrum. For magnetic fields vanishing at infinity, including Aharonov-Bohm potentials, we derive magnetic improvements to a variety of Hardy-type inequalities for the Heisenberg sub-Laplacian. In particular, we establish a sub-Riemannian analogue of Laptev and Weidl sub-criticality result for magnetic Laplacians in the plane. This is joint work with Biagio Cassano, Valentina Franceschi and Dario Prandi.

- **Cyril Letrouit (Université Paris-Saclay & CNRS) - Maximal multiplicity of Laplacian eigenvalues in negatively curved surfaces**

  I will present a joint work with Simon Machado (IAS) in which we make progress on a longstanding conjecture by Colin de Verdière: we obtain a sublinear bound on the maximal multiplicity of first Laplacian eigenvalues for negatively curved surfaces. For the proof, we take inspiration from arguments developed recently in the context of graphs of bounded degree.

- **Soeren Mikkelsen (University of Bath) - Schrödinger evolution in a low-density random potential: convergence to solutions of the linear Boltzmann equation**

  It is a fundamental problem in mathematical physics to derive macroscopic transport equation from the underlying microscopic transport equations. In this talk, we will consider such a problem. To be precise we will consider solutions to a time-dependent Schrödinger equation for a potential localised at the points of a Poisson point process. For these solutions we will present a result stating that the phase-space distribution converges in the annealed Boltzmann-Grad limit to a semiclassical Wigner measure which solves the linear Boltzmann equation.

- **Léo Morin (University of Aarhus) - Semiclassical normal forms for magnetic Laplacians**

  The semiclassical analysis of magnetic Laplacians is closely related to the analysis of hypoelliptic sums of squares. I will present some semiclassical normal forms for the magnetic Laplacian, which provide precise description of its spectrum. I will especially emphasize on the geometry of the underlying phase space. One purpose of this talk is to open discussions to understand better the links with results on subriemannian / hypoelliptic operators.

- **Roman Schubert (University of Bristol) - Decoherence time scales and the Hörmander condition**

  Decoherence is the suppression of interference effects in quantum mechanics due to the coupling of a system to an environment. The evolution in an open quantum system is typically described by the Lindblad equation and I will describe how semiclassical analysis of the Lindblad equation leads to a classical diffusion equation in the Hörmander sum of squares form and how hypoellipticity of that equation is related to decoherence. I will not assume any background knowledge on the Lindblad equation.

- **Marcello Seri (University of Groningen) - A panorama of singular sub-Laplacians and their spectra.**

  Laplace-Beltrami operators on rank-varying sub-Riemannian structures have been recently gaining interest due to their exotic properties. In this talk we will start from the 0th property of their analysis: self-adjointness. In a large number of cases, and in contrast with the Riemannian case, the sub-Riemannian setting presents large families of operators which are essentially self-adjoint even though the manifold is non-complete. We will then move on to present a panoramic view of what little is known about their spectral properties, with a particular emphasis on sub-Riemannian Weyl laws. Throughout the talk we will touch upon a number of simple-to-state open questions to stimulate participant's interest.

- **Robert Yuncken (Université de Lorraine) - Hypoellipticity, pseudodifferential operators, tangent groupoids, and the Helffer-Nourrigat conjecture.**

  Résumé : In 1979, Helffer and Nourrigat made a very broad conjecture about the hypoellipticity of differential operators which are polynomials in a family of vector fields. Their conjecture generalises a vast number of results — eg, the elliptic regularity theorem, Hörmander's sums-of-squares theorem, and Rockland's Theorem (proven by Helffer-Nourrigat) on hypoellipticity for left invariant vector fields on graded nilpotent Lie groups. Helffer and Nourrigat proved several cases of the conjecture, but it has become newly accessible thanks to a beautiful observation by Debord and Skandalis which characterizes classical pseudodifferential operators in terms of Connes' tangent groupoid. We will discuss how groupoidal methods can be used to resolve the Helffer-Nourrigat conjecture. This talk is based on joint work with E. Van Erp, with I. Androulidakis and O. Mohsen, and with N. Couchet.

Participants

Name	Affiliation
Victor Arnaiz	Universite de Nantes
Hajer Bahouri	Sorbonne Université
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Serena Federico	University of Bologna
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