Dilates and
Baumslag-Solitar groups

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Subsets of the set of the integers of the form

$$n \star A = \{rx : x \in A\},$$

where r is a positive integer and A is a finite subset of the set of the integers are called r-dilates. The Baumslag-Solitar groups are defined as follows:

$$BS(m,n) = \langle a, b \mid b^{-1}a^m b = a^n \rangle$$

where m, n are integers.

We obtain new direct and inverse results for sums of two dilates. Then we apply them to solve certain direct and inverse problems in Baumslag-Solitar groups. We concentrate on the groups $BS(1,n)$ and their subsets of the type

$$S = \{b^r a^{x_1}, b^r a^{x_2}, \ldots, b^r a^{x_k}\} = b^r a^A$$

where r is a positive integer and $A = \{x_1, x_2, \ldots, x_k\}$ denotes a finite sequence of integers.

We also investigate the structure of arbitrary subsets of $BS(1,2)$ satisfying small doubling properties.

References

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