Misha Rudnev

On discrete values of bilinear forms.

One of my favourite open questions in discrete geometry is: given a set P of N points in the plane and a non-degenerate bilinear form B, what is the minimum cardinality, in terms of N, of the set – if non-empty – of nonzero values of $B(p,q)$, with p,q in P ? The conjectured answer is N, possibly up to logarithms, but the state of the art is far from it. The question may appear similar to the renown Erdős problem about distinct distances, but at a closer look turns out to be quite different, owing to an inherent degeneracy. So the best we can do so far is the lower bound $N^{9/13}$ over the real and complex field and $N^{2/3}$ in positive characteristic, for a small enough N.