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On Multiplicative Bases and some Related Problems.

How large can a set of integers be, if the equation $a_1 a_2 \ldots a_h = b_1 b_2 \ldots b_h$ has no solution consisting of distinct elements of this set ? How large can a set of integers be, if none of them divides the product of h others ? The first question is about a generalization of the multiplicative Sidon-sets and the second one is of the primitive sets. In answering the above mentioned questions some lemmas on product representation of integers and extremal combinatorial tools can help. In the results not only the asymptotics are found, but very tight bounds are obtained for the error terms, as well. For example, if the numbers are from the set $\{1, 2, \ldots, n\}$, the precise answer to the second question has both lower- and upper bounds in the form $\pi(n) + cn^{2/(h+1)}/(\log n)^2$ with $c > 0$. Here, n has to be large enough compared to h, but the constants do not depend on h.

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