Weighted zero-sum problems and codes

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Zero-sum problems in finite abelian groups

Let $(G, +, 0)$ be a finite abelian group.
Let $S = g_1 \dots g_n$ be a sequence of elements of G.
Simple fact: If n is large enough, there exists a non-empty $I \subset [1, n]$ such that

$$\sum_{i \in I} g_i = 0.$$

'If S is sufficiently long, then it has a zero-sum subsequence.'
Question (Davenport, 66): What does 'sufficiently long' mean precisely?
Note: Numerous variants of this problem; e.g., imposing a restriction on the length of the subsequence (Harborth; Erdős–Ginzburg–Ziv).

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Davenport constant

For $(G, +)$ finite abelian group. Let $\mathsf{D}(G)$ denote the Davenport constant, i.e.,

- the smallest ℓ such that each sequence $g_1 \dots g_\ell$ over G has a (non-empty) zero-sum subsequence, i.e., $\sum_{i \in I} g_i = 0$ for some $\emptyset \neq I \subset \{1, \dots \ell\}$.
- equivalently, 1 plus the maximal length of a zero-sum free sequence.
- equivalently, the maximal length of a minimal zero-sum sequence, i.e., $\sum_{i=1}^{\ell} g_i = 0$ yet $\sum_{i \in I} g_i \neq 0$ for $\emptyset \neq I \subsetneq \{1, \dots \ell\}$.

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Motivated by questions of Baayen, Davenport, Erdős.
Various applications: Number Theory (Carmichael numbers, Factorizations), Graph Theory

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Some results on D(G)

Let $G = C_{n_1} \oplus \cdots \oplus C_{n_r}$ with $n_i \mid n_{i+1}$. Then,

$$D(G) \geq 1 + \sum_{i=1}^{r} (n_i - 1) = D^*(G).$$

Equality holds for (Olson, Kruyswijk, van Emde Boas, 1969)

- p-groups (group rings, later polynomial method).
- groups of rank at most 2 (inductive method, reduction to p-groups).

In some other cases, e.g.,

- $C_2^2 \oplus C_{2n}$ (van Emde Boas).
- $C_3^2 \oplus C_{3n}$ (Bhowmik, Schlage-Puchta).
- $C_4^2 \oplus C_{4n}$ and $C_6^2 \oplus C_{6n}$ (S.).

But, not always. For example (Baayen), for odd n,

$$D(C_2^4 \oplus C_{2n}) > D^*(C_2^4 \oplus C_{2n}).$$

Numerous other examples (but none for groups of rank three or C_n^r).

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Upper bounds

Balasubramanian–Bhowmik and Bhowmik–Schlage-Puchta

$$\mathsf{D}(G) \leq \frac{|G|}{k} + k - 1$$

for k not 'too large' relative to $|G| / \exp(G)$.

$$\mathsf{D}(G) \leq \exp(G)\left(1 + \log \frac{|G|}{\exp(G)}\right)$$

Kruyswijk/van Emde Boas (later Meshulam)

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Other zero-sum constants

Let $(G, +)$ fin. ab. group. Let $j \in \mathbb{N}$ with $j \geq \exp(G)$.
$\mathsf{s}_{\leq j}(G)$ denotes the smallest ℓ in $\mathbb{N}$ such that for each sequence $g_1 \dots g_\ell$ there exists $\emptyset \neq I \subset \{1, \dots, \ell\}$ such that $\sum_{i \in I} g_i = 0$ with $|I| \leq j$.

$\eta(G) = \mathsf{s}_{\leq \exp(G)}(G)$.
Similarly $\mathsf{s}_{=j}(G)$ denotes the smallest ℓ in $\mathbb{N}$ such that for each sequence $g_1 \dots g_\ell$ there exists $\emptyset \neq I \subset \{1, \dots, \ell\}$ such that $\sum_{i \in I} g_i = 0$ with $|I| = j$.
$\mathsf{s}(G) = \mathsf{s}_{=\exp(G)}(G)$ (Erdős–Ginzburg–Ziv constant).

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Weighted zero-sum constants

Introduced by about a decade in a series of papers by Adhikari, Balasubramanian, Chen, Friedlander, Konyagin, Pappalardi, Rath.

For $(G, +)$ finite abelian group and $W \subset \mathbb{Z}$. Let $\mathsf{D}_W(G)$ denote the W-weighted Davenport constant, i.e.,

- the smallest ℓ such that each sequence $g_1 \dots g_\ell$ over G has a (non-empty) W-weighted zero-sum subsequence, i.e., $\sum_{i \in I} w_i g_i = 0$ for some $\emptyset \neq I \subset \{1, \dots \ell\}$ and $w_i \in W$.

Analogously, one defines $\mathsf{s}_{=j,W}(G)$ and $\mathsf{s}_{\leq j,W}(G)$.

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The j-wise Davenport constant

If n is large enough, there exists $I_1, \dots, I_j \subset [1, n]$ disjoint such that

$$\sum_{i \in I_j} g_i = 0$$

for each j.

'If S is sufficiently long, then it has j (disjoint) zero-sum subsequence.'

Question (Halter-Koch, 92): What does 'sufficiently long' mean precisely?

I.o.w: Determine the smallest $\mathsf{D}_j(G)$ such that each sequence of length at least $\mathsf{D}_j(G)$ has a j disjoint zero-sum subsequence.

Equivalently: determine the maximum length of a sequence in G without j disjoint zero-sum subsequence (few zero-free subsequences).

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A motivation, Recasting the Inductive Method

Delorme, Ordaz, Quiroz showed:
Let G be a finite abelian group and H a subgroup, then

$$\mathsf{D}(G) \leq \mathsf{D}_{\mathsf{D}(H)}(G/H).$$

Results on $\mathsf{D}_j(G)$

Exact value:
Known for groups of rank at most two and in closely related situations (Halter-Koch; Delorme, Ordaz, Quiroz).
Yet, in contrast to the standard Davenport constant, not known for general p-groups.
For elementary p-groups its is known (for all j) for:

- C_2^3 (Delorme, Ordaz, Quiroz)
- C_3^3 (Bhowmik, Schlage-Puchta)
- C_2^4, C_2^5 (Freeze, S.)

For specific j, in particular $j = 2$, known for some more C_2^r.

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Some classical bounds on $\mathsf{D}_j(G)$

Lower bound:
Let $G = H \oplus C_n$ with $n = \exp(G)$. Then,

$$\mathsf{D}_j(G) \geq j \exp(G) + \mathsf{D}(H) - 1.$$

Sharp for groups of rank ≤ 2, and some other cases; but this is/should be a rare phenomenon.

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Clearly, $\mathsf{D}_j(G) \leq j\,\mathsf{D}(G)$; this is only sharp for cyclic groups.

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Elementary 2-groups, small j

Theorem (Plagne and S.)

For each sufficiently large integer r we have

$$\begin{aligned}
1.261\, r &\leq \mathrm{D}_2(C_2^r) \leq 1.396\, r, \\
1.500\, r &\leq \mathrm{D}_3(C_2^r) \leq 1.771\, r, \\
1.723\, r &\leq \mathrm{D}_4(C_2^r) \leq 2.131\, r, \\
1.934\, r &\leq \mathrm{D}_5(C_2^r) \leq 2.478\, r, \\
2.137\, r &\leq \mathrm{D}_6(C_2^r) \leq 2.815\, r, \\
2.333\, r &\leq \mathrm{D}_7(C_2^r) \leq 3.143\, r, \\
2.523\, r &\leq \mathrm{D}_8(C_2^r) \leq 3.464\, r, \\
2.709\, r &\leq \mathrm{D}_9(C_2^r) \leq 3.778\, r, \\
2.890\, r &\leq \mathrm{D}_{10}(C_2^r) \leq 4.087\, r.
\end{aligned}$$

For $j = 2$, Komlós and Katona–Srivastava; in a different context.

Elementary 2-groups, small j, II

Theorem (Plagne and S.)

When j tends to infinity, we have the following:

$$\log 2 \left(\frac{j}{\log j}\right) \lesssim \liminf_{r \to +\infty} \frac{\mathsf{D}_j(C_2^r)}{r} \leq \limsup_{r \to +\infty} \frac{\mathsf{D}_j(C_2^r)}{r} \lesssim 2 \log 2 \left(\frac{j}{\log j}\right).$$

Link to coding theory

(Cohen–Zémor)
Let $g_1 \dots g_n$ sequence in C_2^r. Consider $g_i = (a_i^1, \dots, a_i^r)^T$ with $a_i^j \in C_2$.
Then $\sum_{i \in I} g_i = 0$ if and only if

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Intersecting codes

A code is called intersecting if each two non-zero codewords do not have disjoint support. (Studied by Katona, Miklós, Cohen–Lempel,...)

The following are (essentially) equivalent [Cohen–Zémor]:

- Determine for which n, k intersecting $[n, k]$-codes exist.
- Determine $\mathrm{D}_2(C_2^r)$.

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Argument for the upper bounds

Delorme, Ordaz, and Quiroz:

$$D_{j+1}(G) \leq \min_{i \in \mathbb{N}} \max\{D_j(G) + i, s_{\leq i}(G) - 1\}.$$

Need/want knowledge on $s_{\leq i}(C_2^r)$; then apply repeatedly.

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Some ad-hoc terminology

Let $f : [0,1] \to [0,1]$ (non-increasing, continuous, and) each $[n,k,d]$ code (binary linear) satisfies

$$\frac{k}{n} \leq f\left(\frac{d}{n}\right).$$

I.o.w., the functions in the upper bounds of the rate of a code by a function of its normalized minimal distance. Call it "upper-bounding function"; and "asypmtotically upper-bounding function" if holds for all large n.

E.g. Hamming bound:

$$f(\delta) = 1 - h\left(\frac{\delta}{2}\right).$$

with

$$h(u) = -u \log_2 u - (1-u)\log_2(1-u)$$

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Key lemma

Lemma

Let f be an [asymptotic] upper-bounding function. Let d, n, and r be three positive integers [n sufficiently large] satisfying $2 \leq d \leq n-1$ and

$$\frac{n-r}{n} > f\left(\frac{d+1}{n}\right),$$

then

$$s_{\leq d}(C_2^r) \leq n.$$

Upper bounds, summary

- Use DOQ to reduce to $s_{\leq i}(C_2^r)$.
- Reduce $s_{\leq i}(C_2^r)$ to "bounds on codes."
- Use bounds from coding theory (small j, McEliece, Rodemich, Rumsey, and Welch; asymt. Hamming)
- Perform some computations and assemble the pieces.

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Lower bounds

Let j be a positive integer. Then

$$\mathrm{D}_j(C_2^r) \geq \log 2 \frac{j}{\log(j+1)} r$$

as r tends to infinity.

Proved via a counting argument similar to argument of Cohen–Lempel for intersecting codes, $j = 2$.

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True value?

Extrapolating a Conjecture of Cohen–Lempel:
For any positive integer j,

$$\lim_{r \to +\infty} \frac{\mathrm{D}_j(C_2^r)}{r} \sim \log 2 \left(\frac{j}{\log j} \right).$$

That is the lower bound.

Weighted Davenport constant, recall

For $(G, +)$ finite abelian group and $W \subset \mathbb{Z}$. Let $\mathsf{D}_W(G)$ denote the W-weighted Davenport constant, i.e.,

- the smallest ℓ such that each sequence $g_1 \dots g_\ell$ over G has a (non-empty) W-weighted zero-sum subsequence, i.e., $\sum_{i \in I} w_i g_i = 0$ for some $\emptyset \neq I \subset \{1, \dots \ell\}$ and $w_i \in W$.

Multiwise weighted Davenport constant

Let $\mathsf{D}_{W,j}(G)$ denote the W-weighted j-wise Davenport constant, i.e.,

- the smallest ℓ such that each sequence $g_1 \dots g_\ell$ over G has a j disjoint (non-empty) W-weighted zero-sum subsequence, i.e., $\sum_{i \in I_k} w_i g_i = 0$ for some disjoint $\emptyset \neq I_k \subset \{1, \dots \ell\}$ and $w_i \in W$ (for $k = 1, \dots, j$).

(Marchan, Ordaz, Santos, S.)

Which sets of weights?

We focus on:

- $\{-1, 1\}$ (plus-minus weighted)
- $A = \{1, 2, \ldots, \exp(G) - 1\}$ (fully weighted)

But there are plenty of other options (see next talk).

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Equivalences

Lemma

Let p be an odd prime, and let $r \geq 3$ and $n \geq 4$ be integers. Let $g_1, \ldots, g_n \in C_p^r \setminus \{0\}$ and assume the g_i's generate C_p^r. The following statements are equivalent.

1. *The sequence $g_1 \ldots g_n$ has no A-weighted zero-subsum of lengths at most 3.*
2. *The $[n, n-r]_p$-code with parity check matrix $[g_1 \mid \cdots \mid g_n]$ has minimal distance at least 4.*
3. *The set of points represented by the g_i's in the projective space of dimension $r-1$ over the field with p elements is a cap set of size n, that is there are no three points on a line.*

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In particular, the following integers are equal.

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Weighted zero-sum problems and codes

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LAGA, Université Paris 8

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