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On the Brun-Titchmarsh inequality.

A given interval length N being fixed, the maximal number of primes $\rho(N)$ that are in an interval of this length may contain is a major quantity already investigated by Hardy & Littlewood in 1922. The Brun-Titchmarsh inequality in the form given by Montgomery & Vaughan in 1973 provides the upper bound $2N/\log N$ for $\rho(N)$; a first unsolved question is to decide whether this factor 2 is tight or not. A similar bound holds for primes in arithmetic progressions and breaking the factor 2 is known to be equivalent to showing that no Siegel zero exists, or equivalently that enough small primes split in some quadratic extensions. This talk will present a history of the problem and conclude with a new result. By using a functional inequality that mixes the Selberg sieve and the Large sieve, we prove, together with Soroosh Yazdani, that $\rho(N) \leq 2N/(\log N + 5.66 + o(1))$. The method gives ground to the conjecture that the constant 5.66 can be replaced by an arbitrary large constant.