Gradient flow face to face 3

Université Lyon 1

11-14 September 2023

Schedule

	Monday 11	Tuesday 12	Wednesday 13	Thursday 14
9:00 - 9:10	*Welcome address*			
9:10 - 9:40	Mazon	Esposito	Laurençot	Schlichting
9:50 - 10:20	Volzone	Radici	Lisini	Hraivoronska
10:30 - 11:00	**Coffee** ☕	**Coffee** ☕	**Coffee** ☕	**Coffee** ☕
11:00 - 11:30	Stephan	Iorio	Schmidtchen	During
11:40 - 12:10	Venel	Portinale	Di Francesco	Di Marino
12:30 - 13:30	**Lunch**	**Lunch**	**Lunch**	**Lunch**
13:30 - 15:30	**Free time**	**Free time**	**Free time**	
15:30 - 16:00	**Coffee** ☕	**Coffee** ☕	**Coffee** ☕	
16:00 - 16:30	Yoldas	Gentil	David	
16:40 - 17:10	Erbar	Bonforte	Simonov	
17:20 - 17:50		Kreusser		
19:30		**Social Dinner**		

Social Dinner

The Social Dinner will take place on Tuesday evening, at the **Brasserie Georges**, 30 Cours de Verdun. The easiest way to go is to use the **Metro A** until **Perrache**.

Titles and abstracts for Monday 11

9:10 - 9:40. José Mazón: Weak solutions to gradient flows in metric measure spaces

Analysis of evolution equations in metric measure spaces requires very different methods from the Euclidean setting. The reason is that, in general, there is no notion of a gradient of a function. Instead, one can use the minimal upper gradient, which roughly corresponds to the length of the gradient. Here, we show how to introduce the notion of weak solutions in metric measure spaces in the model case of the p-Laplacian evolution equation (including the borderline case $p = 1$, i.e., the total variation flow). For $p > 1$, it has been previously studied as the gradient flow in L^2 of the p-Cheeger energy. Using the first-order differential structure on a metric measure space introduced by Gigli, we characterize the subdifferential in L^2 of the p-Cheeger energy. This leads to a new definition of solutions to the p-Laplacian evolution equation in metric measure spaces, in which the gradient is replaced by a vector field (defined via Gigli's differential structure) satisfying some compatibility conditions. Existence of solutions in this sense is obtained indirectly, using the Fenchel-Rockafellar duality theorem. For $p = 1$, we additionally encounter the problems known already in the Euclidean case, i.e. that the solutions for any given time do not lie in any Sobolev space, but only in the space of functions of bounded variation. For this reason, we need to extend Gigli's differential structure to this case, and prove a Green-Gauss formula similar to the one by Anzellotti for Euclidean spaces. Then, we use it to characterize the 1-Laplacian operator and introduce weak solutions to the total variation flow.

9:50 - 10:20. Bruno Volzone: New results on nonlinear Aggregation-Diffusion equations with Riesz kernels

One of the archetypical aggregation-diffusion models is the so-called classical parabolic-elliptic Patlak-Keller-Segel (PKS for short) model. This model was classically introduced as the simplest description for chemotactic bacteria movement in which linear diffusion tendency to spread fights the attraction due to the logarithmic kernel interaction in two dimensions. For this model there is a well-defined critical mass. In fact, here a clear dichotomy arises: if the total mass of the system is less than the critical mass, then the long time asymptotics are described by a self-similar solution, while for a mass larger than the critical one, there is finite time blow-up. In this talk we will first give an overview about some results obtained in the papers [1]-[2] concerning the characterization of the stationary states for a nonlinear variant of the PKS model, of the form

$$\partial_t \rho = \Delta \rho^m + \nabla \cdot (\rho \nabla (W * \rho)), \tag{1}$$

being $W \in C^1(\mathbb{R}^d \setminus \{0\})$ a Riesz kernel aggregation, namely $W(x) = c_{d,s}|x|^{2s-d}$ for $s \in (0, d/2)$, in the assumptions of dominated diffusion, i.e. for $m > 2 - (2s)/d$. In particular, all stationary states of the model are shown to be radially symmetric decreasing and uniquely identified with global minimizers of the associated free energy functionals. In the second part of the talk we will discuss the recent results established in the joint paper [3], in which an addition of a quadratic diffusion term in equation (1) produces a more precise competition with the aggregation term for small s, as they have the same scaling if $s = 0$. We characterize the asymptotic behavior of the stationary states behavior as s goes to zero. Finally, we establish the existence of gradient flow solutions to the evolution problem by applying the JKO scheme.

[1] J. A. Carrillo, F. Hoffmann, E. Mainini, B. Volzone. *Ground States in the Diffusion-Dominated Regime*, Calc. Var. Partial Differ. Equ. 57, No. 5, Paper No. 127, 28 p. (2018).

[2] H. Chan, M. González, Y. Huang, E. Mainini, B. Volzone. *Uniqueness of entire ground states for the fractional plasma problem.*, Calc. Var. Partial Differ. Equ. 59, No. 6, Paper No. 195, 41 p. (2020).

[3] Y. Huang, E. Mainini, J. L. Vázquez, B. Volzone. *Nonlinear aggregation-diffusion equations with Riesz potentials*, arXiv:2205.13520 [math.AP] (2022)

11:00 - 11:30. Artur Stephan: On time-splitting methods for gradient flows with two dissipation mechanisms

A gradient system $(X, \mathcal{E}, \mathcal{R})$ consists of a state space X (a separable, reflexive Banach space), an energy functional $\mathcal{E} : \mathrm{dom}(\mathcal{E}) \subset X \to \mathbb{R} \cup \{\infty\}$ and a dissipation potential $\mathcal{R} : X \to [0, \infty[$, which is convex, lower semicontinuous and satisfies $\mathcal{R}(0) = 0$. The associated gradient-flow equation is then given by

$$0 \in \partial\mathcal{R}(u'(t)) + \mathrm{D}\mathcal{E}(u(t)) \quad \text{or equivalently} \quad u'(t) \in \partial\mathcal{R}^*(-\mathrm{D}\mathcal{E}(u(t))).$$

In my talk we are interested in the case where the dual dissipation potential $\mathcal{R}^*$ is given by the sum $\mathcal{R}^* = \mathcal{R}_1^* + \mathcal{R}_2^*$ for two dissipation potentials $\mathcal{R}_j : X_j \to [0, \infty[$, $X_j \subset X$. This splitting provides also a decomposition of the right-hand side of the combined gradient-flow equation

$$u'(t) \in \partial(\mathcal{R}_1^* + \mathcal{R}_2^*)(-\mathrm{D}\mathcal{E}(u(t))) = \partial\mathcal{R}_1^*(-\mathrm{D}\mathcal{E}(u(t))) + \partial\mathcal{R}_2^*(-\mathrm{D}\mathcal{E}(u(t))),$$

and enables to construct solutions via a split-step method. For this let $\tau = T/N$ define a uniform partition of the interval $[0, T]$. Starting from an initial datum $u_0 \in \mathrm{dom}(\mathcal{E})$, we define a piecewise constant time-discrete solution $U_\tau : [0, T] \to \mathrm{dom}(\mathcal{E}) \subset X_1$ by setting $U_\tau(0) = u_0$ and by performing the *Alternating Minimizing Movement* of the form

$$\begin{aligned}
&U_\tau(t) = U_k^1 \text{ for } t \in](k{-}1)\tau, (k{-}1/2)\tau], \quad U_\tau(t) = U_k^2 \text{ for } t \in](k{-}1/2)\tau, k\tau] \\
&\text{where } U_k^1 \in \mathrm{Argmin}_{U \in X_1}\{\tfrac{\tau}{2}\mathcal{R}_1(\tfrac{2}{\tau}(U - U_{k-1}^2)) + \mathcal{E}((k{-}1/2)\tau, U)\} \\
&\text{and } U_k^1 \in \mathrm{Argmin}_{U \in X_2}\{\tfrac{\tau}{2}\mathcal{R}_2(\tfrac{2}{\tau}(U - U_k^1)) + \mathcal{E}(k\tau, U)\}.
\end{aligned}$$

In my talk I will show how the curves U_τ indeed converge to the solution of the combined gradient-flow equation (involving $\mathcal{R}^* = \mathcal{R}_1^* + \mathcal{R}_2^*$) in the limit $N \to \infty$. The analysis relies on methods from calculus of variations, and uses the energy-dissipation principle for gradient flows.

This is joint work with Alexander Mielke (Berlin) and Riccarda Rossi (Brescia).

11:40 - 12:10. Juliette Venel: A free boundary model describing corrosion process

In this talk we propose to consider some mechanisms involved in corrosion processes which occur in underground nuclear waste repositories. More precisely we would like to describe the evolution of the oxide layer appearing at the surface of carbon steel canisters in contact with a claystone formation. First we will explain that the proposed model takes the form of a diffusion equation with a moving boundary and nonlinear boundary conditions. Then we will introduce a JKO minimizing scheme and we will obtain the existence of semi-discrete solutions. Finally, using a priori estimates, we will take the limit to prove the existence of global weak solutions to this problem.

16:00 - 16:30. Havva Yoldas: Hypocoercivity for run and tumble equations

We present some results on the long-time behaviour of run and tumble equations which is used as a model for bacteria movement under the effect of a chemoattractant. Even though the run and tumble equation is a hypocoercive kinetic equation, it differs in many ways from the kinetic equations arising in gas dynamics. Therefore, the classical L^2/H^1 hypocoercivity techniques, e.g. due to Dolbeault, Mouhot, Schmeiser, TRAN, 2015, are not applicable to this equation. I will summarize recent results obtained in a collaboration with Jo Evans (Warwick) on the long-time behaviour of the run and rumble equations for different tumbling kernels using Harris-type theorems.

16:40 - 17:10. Matthias Erbar: Variational characterisation of a fuzzy Boltzmann equation method

This talk will be concerned with a fuzzy Boltzmann equation where particles interact via delocalised collisions as compared to the classical Boltzmann equation. We will discuss a variational characterisation of this equation by casting it into the framework of GENERIC (General Equations for Non-Equilibrium Reversible-Irreversible Coupling) systems and exploiting a suitable geometry on the space of probability measures. This talk is based on joint work with Zihui He.

Titles and abstracts for Tuesday 12

9:10 - 9:40. Antonio Esposito: Graph approximation of nonlocal interaction equations

In this talk I will discuss the connection between nonlocal dynamics on graphs and the corresponding local counterparts in the underlying Euclidean space. Equations on graphs have been recently introduced in applications to data science, social dynamics, and synchronization. Starting from the nonlocal interaction equation on graphs, we can obtain a class of nonlocal interaction equations with the presence of tensor-mobility encoding the information on the localised graph. This talk is based on joint works with G. Heinze (Augsburg), F. Patacchini (Paris), A. Schlichting (Muenster), and D. Slepčev (Pittsburgh).

9:50 - 10:20. Emanuela Radici: Optimal control problems for nonlocal interaction equations

We discuss the existence of solutions for optimal control problems associated to transport equations. The behaviour of a population of individuals will be influenced by the presence of a population of control agents whose role is to lead the dynamics of the individuals towards a specific goal. The dynamics of the population of individuals is described by a suitable nonlocal transport equation, while the role of the population of agents is designed by the optimal control problem. This model has been first studied in a paper by Bongini and Butazzo in 2017 for a class of continuous nonlocal potentials, while in our case we consider the case of mildly singular potentials in a gradient flow formulation of the target transport equation.

11:00 - 11:30. Valeria Iorio: Second order two-species systems with nonlocal interactions and large damping

In this talk we present a second order system with two species coupled through nonlocal interactions, with an additional damping parameter. We take initial data in a space of probability measures, and in case of collisions a sticky particles condition is adopted. We first consider the case of smooth nonlocal interaction potentials, not subject to any symmetry condition, and prove existence and uniqueness of a measure solution. Then a large-time large-damping result is obtained, proving the convergence towards the corresponding first order system. Finally, we consider the case of Newtonian potentials for the self-interaction terms, with additional confining external potentials and we prove existence of solutions and a large time collapse result, providing the convergence towards Dirac delta solutions. We complement the results with numerical simulations. This is a joint work with M. Di Francesco and S. Fagioli.

11:40 - 12:10. Lorenzo Portinale: Stochastic Homogenisation of Discrete Transport Problems

We discuss discrete-to-continuum limits of optimal transport problems, with particular attention to recent contributions in the periodic and stochastic setting. We introduce a natural discretization of a broad class of dynamical transport problems and, in this talk, focus in particular on their convergence in the framework of stationary (and possibly ergodic) graphs with random transport problems. The content of the talk is based on a collaboration with Peter Gladbach and Jan Maas.

16:00 - 16:30. Ivan Gentil: Gradient flows to prove functional inequalities

In this talk, we would like to revisit the gradient flow methods to prove functional inequalities and apply it to get an optimal Sobolev inequality for the spherical Caffarelli-Kohn-Nirenberg models. This is a joint work with Louis Dupaigne and Simon Zugmeyer.

16:40 - 17:10. Matteo Bonforte: Stability in Gagliardo-Nirenberg-Sobolev inequalities: nonlinear flows, regularity and the entropy method

We discuss stability results in Gagliardo-Nirenberg-Sobolev inequalities, a joint project with J. Dolbeault, B. Nazaret and N. Simonov. We have developed a new quantitative and constructive "flow method", based on entropy methods and sharp regularity estimates for solutions to the fast diffusion equation (FDE). This allows to study refined versions of the Gagliardo-Nirenberg-Sobolev inequalities that are nothing but explicit stability estimates. Using the quantitative regularity estimates, we go beyond the variational results and provide fully constructive estimates, to the price of a small restriction of the functional space which is inherent to the method.

17:20 - 17:50. Lisa Kreusser: Γ-convergence of an Ambrosio-Torterelli approximation scheme for image segmentation

Given an image, the aim of minimizing the Mumford-Shah functional is to find a decomposition of the image domain into sub-domains and a piecewise smooth approximation u of the image such that u varies smoothly within each sub-domain. Since the Mumford-Shah functional is highly non-smooth, regularizations such as the Ambrosio-Tortorelli approximation can be considered which is one of the most computationally efficient approximations of the Mumford-Shah functional for image segmentation. Our main result is the Γ-convergence of the Ambrosio-Tortorelli approximation of the Mumford-Shah functional for piecewise smooth approximations. This requires the introduction of an appropriate function space. As a consequence of our Γ-convergence result, we can infer the convergence of minimizers of the respective functionals.

Titles and abstracts for Wednesday 13

9:10 - 9:40. Philippe Laurençot: Bounded weak solutions to the thin film Muskat problem

A countably infinite family of Liapunov functionals is constructed for the thin film Muskat problem, which is a second-order degenerate parabolic system featuring cross-diffusion. As a consequence, bounded weak solutions are obtained. This is a joint work with Bogdan-Vasile Matioc from Regensburg.

9:50 - 10:20. Stefano Lisini: On a fractional thin-film equation with non-linear mobility

We illustrate a formal gradient flow structure of a family of fractional thin film equations with non-linear mobility. The equations are of the form $\partial_t u = \nabla \cdot (u^n \nabla p)$, where $p = (-\Delta)^s u$. We show the existence of solutions and entropy estimates for this family of equations.

11:00 - 11:30. Markus Schmidtchen: A degenerate Cross-Diffusion system as the inviscid limit of a nonlocal tissue growth model

In recent years, there has been a spike of interest in multi-phase tissue growth models. Depending on the type of tissue, the velocity is linked to the pressure through Stoke's law, Brinkman's law or Darcy's law. While each of these velocity-pressure relations has been studied in the literature, little emphasis has been placed on the fine relationship between them. In this paper, we want to address this dearth in the literature, providing a rigorous argument that bridges the gap between a viscoelastic tumour model (of Brinkman type) and an inviscid tumour model (of Darcy type).

11:40 - 12:10. Marco Di Francesco: Nonlocal particle approximation via Morse potential for the one-dimensional Porous Medium Equation

We consider a deterministic particle scheme approximating a nonlocal interaction equation with repulsive Morse potential, where the latter is subject to a dilation scaling. We first prove convergence of the scheme to the nonlocal equation when the Morse potential has a fixed variance and the number of particles goes to infinity. This result is proven for initial data in the space of probability measures and uses a measure-to-L^∞ smoothing effect of the scheme. Then, we prove that by letting the variance of the Morse potential go to zero as the number of particles goes to infinity (under a suitable scaling constraint) one obtains weak solutions to the Porous Medium Equation. Here we prescribe additionally that the initial condition is in L^∞. The latter convergence result holds strongly in L^2, which improves previous approximation results, and uses the L^p contractivity of the scheme. This is a joint work with Valeria Iorio (L'Aquila) and Markus Schmidtchen (TU Dresden).

16:00 - 16:30. Noemi David: New Lipschitz estimates and long-time asymptotic behavior for porous medium and fast diffusion equations

Among the class of nonlinear diffusion equations, the porous medium equation has certainly attracted major interest and its regularity theory is nowadays well established and understood. In this talk I will present some recent results based on a joint work with F. Santambrogio. We obtain new estimates for the solution of both the porous medium and the fast diffusion equations by studying the evolution of suitable Lipschitz norms. Our results include instantaneous regularization for all positive times, long-time decay rates of the norms which are sharp and independent of the initial support, and new convergence results to the Barenblatt profile. Moreover, we address nonlinear diffusion equations including quadratic or bounded potentials as well. In the slow diffusion case, our strategy requires exponents close enough to 1, while in the fast diffusion case, our results cover any exponent for which the problem is well-posed and mass-preserving in the whole space.

16:40 - 17:10. Nikita Simonov: Fast diffusion equations, tails and convergence rates

Understanding the intermediate asymptotic and computing convergence rates towards equilibria are among the major problems in the study of parabolic equations. Convergence rates depend on the tail behaviour of solutions. This observation raised the following question: how can we understand the tail behaviour of solutions from the tail behaviour of the initial datum? In this talk, I will discuss the asymptotic behaviour of solutions to the fast diffusion equation. It is well known that non-negative solutions behave for large times as the Barenblatt (or fundamental) solution, which has an explicit expression. In this setting, I will introduce the Global Harnack Principle (GHP), precise global pointwise upper and lower estimates of non-negative solutions in terms of the Barenblatt profile. I will characterize the maximal (hence optimal) class of initial data such that the GHP holds by means of an integral tail condition. As a consequence, I will provide rates of convergence towards the Barenblatt profile in entropy and in stronger norms such as the uniform relative error.

Titles and abstracts for Thursday 14

9:10 - 9:40. André Schlichting: EGSYWTK:DLSS - Every gradient structure you want to know for the Derrida-Lebowitz-Speer-Spohn equation

We present a spatial discretization of the fourth-order nonlinear DLSS equation on the circle. Our choice of discretization is motivated by a novel gradient flow formulation with respect to a diffusive transport metric that generalizes martingale transport. The discrete dynamics inherits this gradient flow structure, and in addition further structural properties. We prove the convergence in the limit of vanishing mesh size. This is a joint work with Daniel Matthes and Eva-Maria Rott.

9:50 - 10:20. Anastasiia Hraivoronska: The Scahrfetter-Gummel scheme for the aggregation-diffusion equation and vanishing diffusion limit

In this talk, I will discuss the Scharfetter-Gummel finite-volume scheme for aggregation-diffusion equations. We leverage appropriate gradient structures on discrete and continuous levels for the analysis. An interesting aspect is the vanishing diffusion limit for the Scharfetter-Gummel scheme corresponding to the upwind scheme for the aggregation equation. I will discuss the challenges presented by studying the discrete-to-continuum convergence in the zero-diffusion regime and some gains from the relevant gradient structures. This talk is based on a joint work with André Schlichting and Oliver Tse.

11:00 - 11:30. Bertram Düring: A Lagrangian scheme for the solution of nonlinear diffusion equations

Many nonlinear diffusion equations can be interpreted as gradient flows whose dynamics are driven by internal energies and given external potentials, examples include the heat equation and the porous medium equation. When solving these equations numerically, schemes that respect the equations' special structure are of particular interest. In this talk we present a Lagrangian scheme for nonlinear diffusion equations. For discretization of the Lagrangian map, we use a finite subspace of linear maps in space and a variational form of the implicit Euler method in time. We present numerical experiments for the porous medium equation in two space dimensions.

11:40 - 12:10. Simone Di Marino: TBA