

A well-balanced entropy scheme for a shallow water type system describing two-phase debris flows

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In the context of modeling two-phase debris flows involving grains and fluid, some shallow water systems arise with internal variables.

Our work focus on such a shallow water system with two internal variables and a topography b which adds a nonconservative term. \\

For numerical purposes, it is desirable to deal with a system where the mathematical entropy (the physical energy of the system) is convex with respect to the chosen conservative variables. Then at the numerical level, we can look for a scheme satisfying a semi-discrete entropy inequality. It also preserves the steady state at rest, so-called “well-balanced”.

Our system is written as

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\begin{equation}
\partial_t h + \nabla_x(hv) = 0,
\label{eq:h}
\end{equation}
\begin{equation}
\partial_t \left( h \langle hv \rangle + \nabla_x \left( h \langle v \otimes v \rangle + g_c \nabla_x \left( r \frac{h^2}{2} \right) + g_c h \nabla_x (b + \tilde{b}) \right) = T,
\label{eq:hv}
\end{equation}
\begin{equation}
\partial_t \rho + v \cdot \nabla_x \rho = \Phi_1,
\label{eq:rho}
\end{equation}
\begin{equation}
\partial_t r + v \cdot \nabla_x r = \Phi_2,
\label{eq:r}
\end{equation}
with the energy
\begin{equation}
E = h \frac{|v|^2}{2} + g_c (b + \tilde{b}) + g_{cr} \frac{h^2}{2}.
\label{entropy}
\end{equation}

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The physical unknowns of the system are the total mass h , the velocity v , the density of the mixture layer ρ and a variable r depending on the proportion of fluid between the layers.

Sources terms Φ_1 , Φ_2 and T contains multivalued friction and dilatancy effects. \\

Writing the system with conservative variables for which the energy is convex, we derive a well-balanced scheme satisfying a semi-discrete entropy inequality.

A numerical test case of injection of some mixture and fluid into a box will be discussed to illustrate the importance of the dilatancy effect.

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