

# A well-balanced entropy scheme for a shallow water type system describing two-phase debris flows

Monday, July 3, 2023 4:50 PM (50 minutes)

In the context of modeling two-phase debris flows involving grains and fluid, some shallow water systems arise with internal variables.

Our work focus on such a shallow water system with two internal variables and a topography  $b$  which adds a nonconservative term. \\

For numerical purposes, it is desirable to deal with a system where the mathematical entropy (the physical energy of the system) is convex with respect to the chosen conservative variables. Then at the numerical level, we can look for a scheme satisfying a semi-discrete entropy inequality. It also preserves the steady state at rest, so-called “well-balanced”.

Our system is written as

```

\begin{equation}
\partial_t h + \nabla_x(hv) = 0,
\end{equation}
\begin{equation}
\partial_t \left( \frac{h^2}{2} \right) + \nabla_x \left( h v \otimes v \right) + g_c \nabla_x \left( \frac{h^2}{2} \right) + g_c h \nabla_x (b + \tilde{b}) = T,
\end{equation}
\begin{equation}
\partial_t \rho + v \cdot \nabla_x \rho = \Phi_1,
\end{equation}
\begin{equation}
\partial_t r + v \cdot \nabla_x r = \Phi_2,
\end{equation}
with the energy
\begin{equation}
E = h \frac{|v|^2}{2} + g_c h (b + \tilde{b}) + g_c r \frac{h^2}{2}.
\end{equation}

```

The physical unknowns of the system are the total mass  $h$ , the velocity  $v$ , the density of the mixture layer  $\rho$  and a variable  $r$  depending on the proportion of fluid between the layers.

Sources terms  $\Phi_1$ ,  $\Phi_2$  and  $T$  contains multivalued friction and dilatancy effects. \\

Writing the system with conservative variables for which the energy is convex, we derive a well-balanced scheme satisfying a semi-discrete entropy inequality.

A numerical test case of injection of some mixture and fluid into a box will be discussed to illustrate the importance of the dilatancy effect.

**Primary authors:** DRACH, Elias (Université Gustave Eiffel); BOUCHUT, Francois (CNRS & Université Gustave Eiffel)

**Presenter:** DRACH, Elias (Université Gustave Eiffel)

**Session Classification:** Lundi après-midi

**Track Classification:** Présentation orale