Secant v. Cactus

Rapport sur les contributions

ID de Contribution: **1**   Type: **Non spécifié**

Basics of border apolarity

mardi 9 mai 2023 09:30 (1 heure)

The aim of my talk is to introduce the border apolarity idea toegether with the tools necessary for its proof. I will recall the setting of border apolarity as it was done my joint paper with Jarek Buczynski. There we have formulated a version of apolarity lemma for a toric variety embedded via very ample line bundle and have proved it in the characteristic zero case. The main tool is to use the multigraded Hilbert scheme of ideals in the Cox ring of the variety X with fixed Hilbert function. In the context of calculating border rank the most interesting is the component containing ideals of the subsets of r points in general position in X. Finally, when there is a group action on X, and the point (tensor, polynomial) is a fixed point of this action, we get an even more useful version of the apolarity lemma. I will give some examples of how one can use the border apolarity theorem to calculate the border rank of a tensor or polynomial.

Orateur: BUCZYNSKA, Weronika (University of Warsaw)

ID de Contribution: **2**   Type: **Non spécifié**

Cactus rank and varieties

mardi 9 mai 2023 11:00 (1 heure)

The cactus variety of a projective variety X is a version of the secant variety, where we take into account the linear spans of all finite subschemes of bounded length, not only the smooth ones or smoothable ones. I will discuss the definitions and basic properties of cactus rank and cactus varieties, with a particular focus on why they are relevant as an obstruction to study secant varieties. I will also explain the what is the Hilbert scheme of points and what we know about its components. Finally I will relate the components of the cactus variety (typically, one of these components would be the secant variety) to components of Hilbert scheme.

Orateur: BUCZYNSKI, Jarek (IMPAN Warsaw)

ID de Contribution: **3**   Type: **Non spécifié**

Ideal enumeration for border apolarity

mardi 9 mai 2023 14:30 (1 heure)

The first step in lower bounding the border rank of a tensor or polynomial with border apolarity is to enumerate all ideals contained in the annihilator with Hilbert series equal to the Hilbert series of an ideal of general points. The second step requires determining whether any such ideal may be deformed to an ideal of points. Typically, one simplifies these questions by asking if there are any such ideals which are additionally fixed under a given solvable group of symmetries of the tensor or polynomial.

In this talk I discuss the challenges involved in the ideal enumeration step. At a high level, the ideals are enumerated multigraded component by component, but concrete questions arise. How should partially constructed ideals be represented? How are the symmetries of the tensor or polynomial handled? How do we proceed when the answer contains positive dimensional families? Furthermore, I anticipate the successful application of both steps of border apolarity will as much as possible interleave checks for deformability of partially built ideals into the early steps of enumeration. I hope this discussion will make clear the context in which tests for deformability will need to be applied.

Orateur: CONNER, Austin (Harvard Universty)

ID de Contribution: **4**   Type: **Non spécifié**

Algorithms for rank and cactus decomposition of polynomials 1

mercredi 10 mai 2023 09:30 (1 heure)

In this talk and the next one we will revise the algorithm for polynomial decomposition originally proposed by Brachat-Comon-Mourrain-Tsidgaridas and we will show how we can improve it. Then we will see how certain modifications to the algorithm can lead to a cactus decomposition.

Orateur: TAUFER, Daniel (KU Leuven)

ID de Contribution: **5** Type: **Non spécifié**

Algorithms for rank and cactus decomposition of polynomials 2

mercredi 10 mai 2023 11:00 (1 heure)

Orateur: BERNARDI, Alessandra (Universita di Trento)

ID de Contribution: **6**   Type: **Non spécifié**

Counterexamples for the slice technique for cactus rank and border cactus rank

mardi 9 mai 2023 16:00 (1 heure)

The slice technique is a tool which let use to translate the question about rank (or border rank) of a tensor in to the analogue question about the subspace spanned by tensors of a smaller order. The technique works in the case of a rank and border rank, but not for cactus and border cactus rank. Gesmundo, Oneto and Ventura gave an example of a family of forms such that their simultaneous cactus rank cannot be read as the cactus rank of tensor living in a bigger space. With a help of Multigraded Cactus Apolarity Lemma we provide a simpler one. We also show the minimal example of a tensor p in $C^N \otimes Sym^d(C^n)$ with a different border cactus rank than the border cactus rank of $p(C^N *)$.

Orateur: RUPNIEWSKI, Filip (Universität Bern)

ID de Contribution: **7**   Type: **Non spécifié**

Rank algorithms, Hilbert functions and non-saturated ideals

mercredi 10 mai 2023 14:30 (1 heure)

Some of the classical tensor decomposition algorithms are based on the ability of solving particular zero-dimensional polynomial system, defining the set of points of the decomposition. Generalized eigenvalue methods can be used for this task, and their complexity is controlled by the regularity of certain associated ideals, which are often non-saturated. We determine these regularity values in a restricted range, drawing connections to classical problems in commutative algebra, such as the Minimal Resolution Conjecture and the Ideal Generation Conjecture. This is based on joint work with Leonie Kayser and Simon Telen.

Orateur: GESMUNDO, Fulvio (Saarland Universität)

ID de Contribution: **8**   Type: **Non spécifié**

Border rank bounds for GL_n-invariant tensors arising from spaces of matrices of constant rank

mercredi 10 mai 2023 16:00 (1 heure)

One measure of the complexity of a tensor is its border rank.

Finding the border rank of a tensor, or even bounding it, is a difficult problem that is currently an area of active research, as several problems in theoretical computer science come down to determining the border ranks of certain tensors.

For a class of $GL(V)$-invariant tensors lying in a $GL(V)$-invariant space $V \otimes U \otimes W$, where U and W are $GL(V)$-modules, we can take advantage of $GL(V)$-invariance to find border rank bounds for these tensors.

I discuss a special case where these tensors correspond to spaces of matrices of constant rank.

Orateur: WU, Derek (Texas A&M University)

ID de Contribution: **9**   Type: **Non spécifié**

Border apolarity 2

jeudi 11 mai 2023 09:30 (1 heure)

Orateur: BUCZYNSKA, Weronika (University of Warsaw)

ID de Contribution: **10**   Type: **Non spécifié**

Quot schemes and varieties of commuting matrices

jeudi 11 mai 2023 11:00 (1 heure)

Let $C_n(M_d)$ denote the affine variety of all n-tuples of commuting $d \times d$ matrices. The ADHM construction relates these varieties to Quot schemes, and in particular to Hilbert schemes. On the more applied side, varieties $C_n(M_d)$ are directly connected to the question whether a tensor has minimal border rank. Although $C_n(M_d)$ is usually reducible for $n > 2$ and $d > 3$, very few irreducible components are known. In the talk we classify irreducible components for small d and all n. Moreover, we show that $C_n(M_d)$, viewed as a scheme defined by the quadratic commutativity relations, has generically nonreduced components whenever $d \geq 8$ and $n \geq 4$, while it is generically reduced for $d \leq 7$. Our results give the corresponding results for Quot schemes of points. In particular, the Quot scheme parametrizing degree 8 quotients of a free module of rank 4 over polynomial ring in 4 variables has a generically nonreduced component.

This is joint work with Joachim Jelisiejew.

Orateur: SIVIC, Klemen (University of Ljubljana)

ID de Contribution: **11**   Type: **Non spécifié**

Open Problems

jeudi 11 mai 2023 14:30 (1 heure)

ID de Contribution: **12** Type: **Non spécifié**

On the minimal cactus rank

jeudi 11 mai 2023 16:00 (1 heure)

I will present the study of minimal cactus rank with respect to Veronese variety, Segre variety, and Segre-Veronese variety using an approach complementary to the one taken by Blaeser and Lysikov, and Jelisiejew, Pal, and Landsberg. I will analyze the case of 14th cactus variety in more detail.

Orateur: GALAZKA, Macej (University of Warsaw)

ID de Contribution: **13**   Type: **Non spécifié**

Irreducibility of multigraded Hilbert schemes of points in general position in the product of projective spaces

vendredi 12 mai 2023 09:30 (1 heure)

I will present some necessary conditions for a point of a multigraded Hilbert scheme corresponding to r points in general position in a smooth projective complex toric variaty to be in the Slip component. These criteria can be used to classify irreducible multigraded Hilbert schemes corresponding to points in general position in the product of projective spaces.

Orateur: MANDZIUK, Tomasz (University of Warsaw)

ID de Contribution: **14**   Type: **Non spécifié**

Ranks of powers of quadrics

vendredi 12 mai 2023 11:00 (1 heure)

Determining the rank of the powers of quadratic forms is a classical problem. Many examples of special decompositions appear in the literature. We analyze this problem from a modern point of view and we give an estimate of the value of the rank. Moreover, we determine its smoothable rank and its border rank.

Orateur: FLAVI, Cosimo (Universita di Firenze)

ID de Contribution: **15**   Type: **Non spécifié**

Cactus rank and varieties 2

vendredi 12 mai 2023 16:00 (1 heure)

Orateur: BUCZYNSKI, Jarek (IMPAN Warsaw)

ID de Contribution: **16**   Type: **Non spécifié**

Tensors of minimal border rank

vendredi 12 mai 2023 14:30 (1 heure)

Orateur: LANDSBERG, J.M.