

**Secant v. Cactus**

# **Report of Contributions**

Contribution ID: 1

Type: **not specified**

## Basics of border apolarity

*Tuesday, May 9, 2023 9:30 AM (1 hour)*

The aim of my talk is to introduce the border apolarity idea together with the tools necessary for its proof. I will recall the setting of border apolarity as it was done in my joint paper with Jarek Buczyński. There we have formulated a version of apolarity lemma for a toric variety embedded via a very ample line bundle and have proved it in the characteristic zero case. The main tool is to use the multigraded Hilbert scheme of ideals in the Cox ring of the variety  $X$  with fixed Hilbert function. In the context of calculating border rank the most interesting is the component containing ideals of the subsets of  $r$  points in general position in  $X$ . Finally, when there is a group action on  $X$ , and the point (tensor, polynomial) is a fixed point of this action, we get an even more useful version of the apolarity lemma. I will give some examples of how one can use the border apolarity theorem to calculate the border rank of a tensor or polynomial.

**Presenter:** BUCZYŃSKA, Weronika (University of Warsaw)

Contribution ID: 2

Type: **not specified**

## Cactus rank and varieties

*Tuesday, May 9, 2023 11:00 AM (1 hour)*

The cactus variety of a projective variety  $X$  is a version of the secant variety, where we take into account the linear spans of all finite subschemes of bounded length, not only the smooth ones or smoothable ones. I will discuss the definitions and basic properties of cactus rank and cactus varieties, with a particular focus on why they are relevant as an obstruction to study secant varieties. I will also explain what is the Hilbert scheme of points and what we know about its components. Finally I will relate the components of the cactus variety (typically, one of these components would be the secant variety) to components of Hilbert scheme.

**Presenter:** BUCZYNSKI, Jarek (IMPAN Warsaw)

Contribution ID: 3

Type: **not specified**

## Ideal enumeration for border apolarity

*Tuesday, May 9, 2023 2:30 PM (1 hour)*

The first step in lower bounding the border rank of a tensor or polynomial with border apolarity is to enumerate all ideals contained in the annihilator with Hilbert series equal to the Hilbert series of an ideal of general points. The second step requires determining whether any such ideal may be deformed to an ideal of points. Typically, one simplifies these questions by asking if there are any such ideals which are additionally fixed under a given solvable group of symmetries of the tensor or polynomial.

In this talk I discuss the challenges involved in the ideal enumeration step. At a high level, the ideals are enumerated multigraded component by component, but concrete questions arise. How should partially constructed ideals be represented? How are the symmetries of the tensor or polynomial handled? How do we proceed when the answer contains positive dimensional families? Furthermore, I anticipate the successful application of both steps of border apolarity will as much as possible interleave checks for deformability of partially built ideals into the early steps of enumeration. I hope this discussion will make clear the context in which tests for deformability will need to be applied.

**Presenter:** CONNER, Austin (Harvard Universty)

Contribution ID: 4

Type: **not specified**

# Algorithms for rank and cactus decomposition of polynomials 1

*Wednesday, May 10, 2023 9:30 AM (1 hour)*

In this talk and the next one we will revise the algorithm for polynomial decomposition originally proposed by Brachat-Comon-Mourrain-Tsidgaridas and we will show how we can improve it. Then we will see how certain modifications to the algorithm can lead to a cactus decomposition.

**Presenter:** TAUFER, Daniel (KU Leuven)

Contribution ID: 5

Type: **not specified**

## **Algorithms for rank and cactus decomposition of polynomials 2**

*Wednesday, May 10, 2023 11:00 AM (1 hour)*

**Presenter:** BERNARDI, Alessandra (Universita di Trento)

Contribution ID: 6

Type: **not specified**

## Counterexamples for the slice technique for cactus rank and border cactus rank

*Tuesday, May 9, 2023 4:00 PM (1 hour)*

The slice technique is a tool which let use to translate the question about rank (or border rank) of a tensor in to the analogue question about the subspace spanned by tensors of a smaller order. The technique works in the case of a rank and border rank, but not for cactus and border cactus rank. Gesmundo, Oneto and Ventura gave an example of a family of forms such that their simultaneous cactus rank cannot be read as the cactus rank of tensor living in a bigger space. With a help of Multigraded Cactus Apolarity Lemma we provide a simpler one. We also show the minimal example of a tensor  $p$  in  $C^N \otimes Sym^d(C^m)$  with a different border cactus rank than the border cactus rank of  $p(C^N *)$ .

**Presenter:** RUPNIEWSKI, Filip (Universität Bern)

Contribution ID: 7

Type: **not specified**

## Rank algorithms, Hilbert functions and non-saturated ideals

*Wednesday, May 10, 2023 2:30 PM (1 hour)*

Some of the classical tensor decomposition algorithms are based on the ability of solving particular zero-dimensional polynomial system, defining the set of points of the decomposition. Generalized eigenvalue methods can be used for this task, and their complexity is controlled by the regularity of certain associated ideals, which are often non-saturated. We determine these regularity values in a restricted range, drawing connections to classical problems in commutative algebra, such as the Minimal Resolution Conjecture and the Ideal Generation Conjecture. This is based on joint work with Leonie Kayser and Simon Telen.

**Presenter:** GESMUNDO, Fulvio (Saarland Universität)

Contribution ID: 8

Type: **not specified**

## Border rank bounds for $GL_n$ -invariant tensors arising from spaces of matrices of constant rank

*Wednesday, May 10, 2023 4:00 PM (1 hour)*

One measure of the complexity of a tensor is its border rank.

Finding the border rank of a tensor, or even bounding it, is a difficult problem that is currently an area of active research, as several problems in theoretical computer science come down to determining the border ranks of certain tensors.

For a class of  $GL(V)$ -invariant tensors lying in a  $GL(V)$ -invariant space  $V \otimes U \otimes W$ , where  $U$  and  $W$  are  $GL(V)$ -modules, we can take advantage of  $GL(V)$ -invariance to find border rank bounds for these tensors.

I discuss a special case where these tensors correspond to spaces of matrices of constant rank.

**Presenter:** WU, Derek (Texas A&M University)

Contribution ID: 9

Type: **not specified**

## **Border apolarity 2**

*Thursday, May 11, 2023 9:30 AM (1 hour)*

**Presenter:** BUCZYNSKA, Weronika (University of Warsaw)

Contribution ID: 10

Type: **not specified**

## Quot schemes and varieties of commuting matrices

*Thursday, May 11, 2023 11:00 AM (1 hour)*

Let  $C_n(M_d)$  denote the affine variety of all  $n$ -tuples of commuting  $d \times d$  matrices. The ADHM construction relates these varieties to Quot schemes, and in particular to Hilbert schemes. On the more applied side, varieties  $C_n(M_d)$  are directly connected to the question whether a tensor has minimal border rank. Although  $C_n(M_d)$  is usually reducible for  $n > 2$  and  $d > 3$ , very few irreducible components are known. In the talk we classify irreducible components for small  $d$  and all  $n$ . Moreover, we show that  $C_n(M_d)$ , viewed as a scheme defined by the quadratic commutativity relations, has generically nonreduced components whenever  $d \geq 8$  and  $n \geq 4$ , while it is generically reduced for  $d \leq 7$ . Our results give the corresponding results for Quot schemes of points. In particular, the Quot scheme parametrizing degree 8 quotients of a free module of rank 4 over polynomial ring in 4 variables has a generically nonreduced component.

This is joint work with Joachim Jelisiejew.

**Presenter:** SIVIC, Klemen (University of Ljubljana)

Contribution ID: 11

Type: **not specified**

## Open Problems

*Thursday, May 11, 2023 2:30 PM (1 hour)*

Contribution ID: 12

Type: **not specified**

## On the minimal cactus rank

*Thursday, May 11, 2023 4:00 PM (1 hour)*

I will present the study of minimal cactus rank with respect to Veronese variety, Segre variety, and Segre-Veronese variety using an approach complementary to the one taken by Blaeser and Lysikov, and Jelisiejew, Pal, and Landsberg. I will analyze the case of 14th cactus variety in more detail.

**Presenter:** GALAZKA, Macej (University of Warsaw)

Contribution ID: 13

Type: **not specified**

## Irreducibility of multigraded Hilbert schemes of points in general position in the product of projective spaces

*Friday, May 12, 2023 9:30 AM (1 hour)*

I will present some necessary conditions for a point of a multigraded Hilbert scheme corresponding to  $r$  points in general position in a smooth projective complex toric variety to be in the Slip component. These criteria can be used to classify irreducible multigraded Hilbert schemes corresponding to points in general position in the product of projective spaces.

**Presenter:** MANDZIUK, Tomasz (University of Warsaw)

Contribution ID: 14

Type: **not specified**

## Ranks of powers of quadrics

*Friday, May 12, 2023 11:00 AM (1 hour)*

Determining the rank of the powers of quadratic forms is a classical problem. Many examples of special decompositions appear in the literature. We analyze this problem from a modern point of view and we give an estimate of the value of the rank. Moreover, we determine its smoothable rank and its border rank.

**Presenter:** FLAVI, Cosimo (Universita di Firenze)

Contribution ID: 15

Type: **not specified**

## **Cactus rank and varieties 2**

*Friday, May 12, 2023 4:00 PM (1 hour)*

**Presenter:** BUCZYNSKI, Jarek (IMPAN Warsaw)

Contribution ID: 16

Type: **not specified**

## Tensors of minimal border rank

*Friday, May 12, 2023 2:30 PM (1 hour)*

**Presenter:** LANDSBERG, J.M.