Neutrino emission in hot Neutron Stars: Modified Urca

Seminaire Institut Denis Poisson

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In collaboration with Micaela Oertel & Marco Mancini

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Outline

1. Introduction to Neutron Star Physics
    - Neutron stars and their extreme conditions
    - The equation of state mystery
2. The role of neutrinos in Neutron Star Physics
3. Derivation of Modified Urca neutrino emission
4. Conclusion

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Neutron stars and their extreme conditions

- Compactness
  $C = GM/(Rc^2) \sim 0.2 - 0.3$
  - → test of gravity theories [Kramer et al. 2021] for the double pulsar binary PSR J0737-3039
  - → precise measurement of post-Keplerian parameters allow for a *mass* – *mass* diagram
- Structure in layers
  - → Atmosphere: light elements
  - → Crust: lattice of nuclei (free neutrons in the inner crust)
  - → Core: homogeneous matter
- Extreme densities, up to 10^{15} g/cm^3
  - → Conditions of density outside the reach of Earth laboratories
  - → Neutron stars (NS) observations = extra-terrestrial laboratory for ultra dense matter

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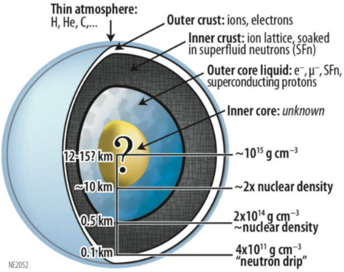

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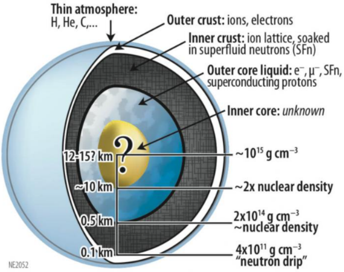

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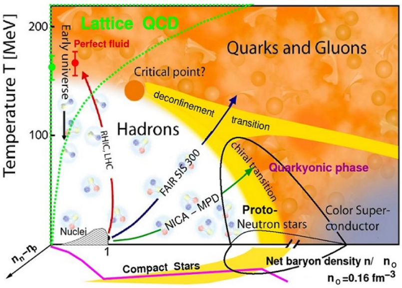
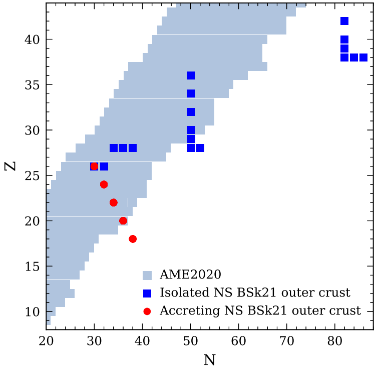


Figure 1: Credits, Larry McLerran

The equation of state mystery

Description of Neutron Star's interior

- Core composition
  - ◇ nucleonic matter
  - ◇ hyperonic matter
  - ◇ phase transition to deconfined quarks
- Equation of state
  - ◇ various theoretical frameworks:
    - phenomenological: Relativistic Mean Field, Cogny or Skyrme forces
    - microscopic calculations: χ-EFT
  - ◇ constrains from nuclear physics
- Constraints from observations:
  - ◇ $\mathrm{M_{obs}}$ vs. $\mathrm{M_{eos}}$
  - ◇ NICER telescope measurement of M and R
  - ◇ Gravitational wave detection of Λ (GW170817)



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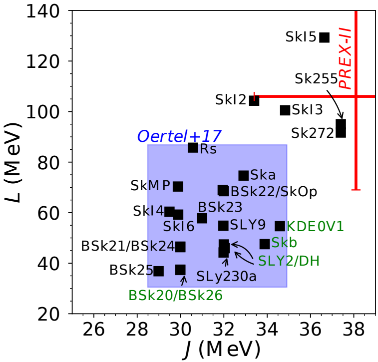

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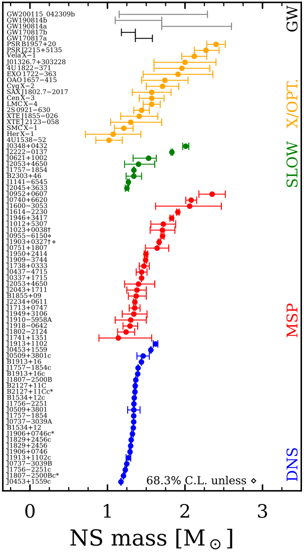

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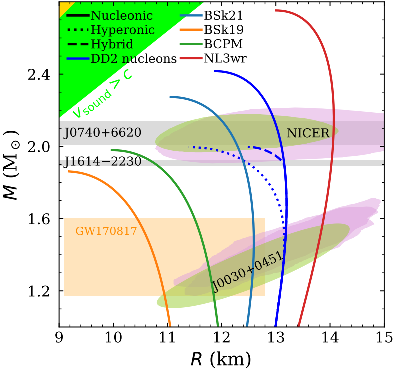

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    - Core-collapse supernovae
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Core-collapse supernovae

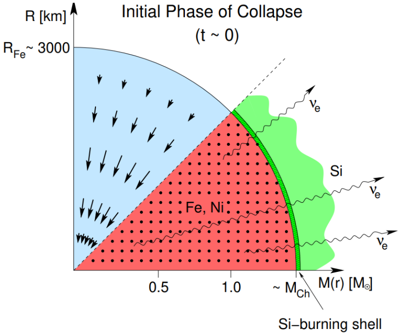
The stages of the Neutron Star birth:

- Iron core collapse leads to neutronization of matter

$$ {}^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}X + \nu_e \ . $$

- Decrease in the electron pressure leads to further collapse → density increase → neutrinosphere → stops the neutronization under 10^{-3} fm^{-3}.
- Nuclei dissolve to nucleons, rising the role of repulsive force and leading to a core bounce.
- Resulting hydrodynamic shock propagates outward until it reaches the neutrinosphere.

- The remaining core is a Proto-NS.
- Neutrino heating is key in the explosion mechanism.



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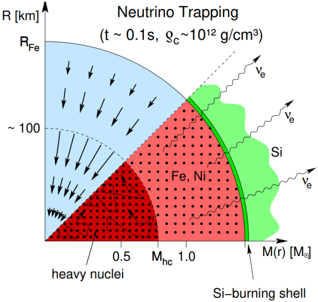

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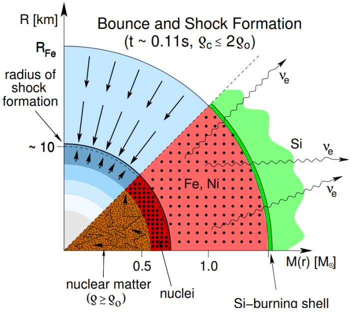

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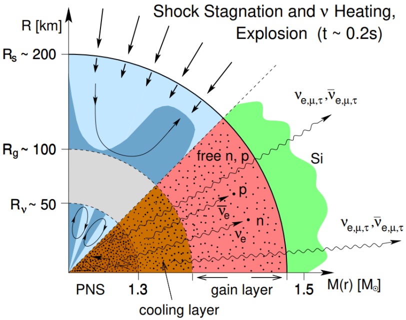

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Cooling and neutrino transport

Neutrino emissive processes in Neutron Stars

Efficient processes in the core:

$$B_1 \leftrightarrow B_2 + l^{\pm} + \nu_l \ ,$$
$$B_2 + l^{\pm} \leftrightarrow B_1 + \bar{\nu}_l \ ,$$

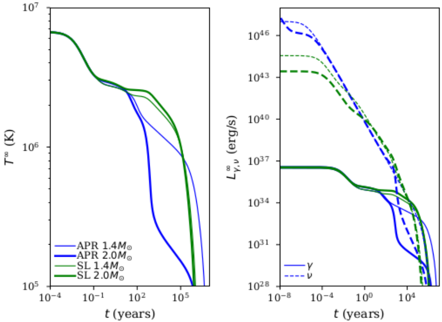
Slow cooling processes in the core

$$\mathbf{B} + B_1 \leftrightarrow \mathbf{B} + B_2 + l^{\pm} + \nu_l$$
$$\mathbf{B} + B_2 + l^{\pm} \leftrightarrow \mathbf{B} + B_1 + \bar{\nu}_l$$
$$B + B \leftrightarrow B + B + \nu\bar{\nu}$$
$$l^{\pm} + C \to l^{\pm} + C + \nu\bar{\nu}$$

Neutrino emissivity and mean free path

$$\frac{\partial}{\partial_t}\mathcal{F}_{\nu} = j(E_{\nu})\big(1-\mathcal{F}_{\nu}\big) - \frac{1}{\lambda(E_{\nu})}\mathcal{F}_{\nu} \ ,$$
$$\frac{\partial}{\partial_t}\mathcal{F}_{\bar{\nu}} = \bar{j}(E_{\nu})\big(1-\mathcal{F}_{\bar{\nu}}\big) - \frac{1}{\bar{\lambda}(E_{\nu})}\mathcal{F}_{\bar{\nu}} \ ,$$

Direct Urca, Modified Urca and Bremsstrahlung are the most important processes to study.



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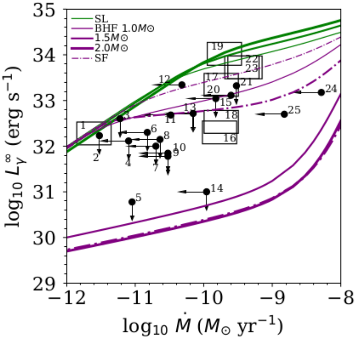

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Neutrino emissivity and mean free paths in QTFT

Modified Urca neutrino emission for *npe* matter

$$N + n \leftrightarrow N + p + e^- + \bar{\nu}\ ,$$
$$N + p + e^- \leftrightarrow N + n + \nu\ ,$$
$$N + n + e^+ \leftrightarrow N + p + \bar{\nu}\ ,$$
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The neutrino emissivity for a modified electron capture gives:

$$j^{[e^-\nu]} = -\frac{G_F^2 V_{ud}^2}{8}\int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\,\frac{\mathcal{F}_{e^-}(Q^{[e^-\nu]})\left(1+\mathcal{F}_B(Q_0^{[e^-\nu]})\right)}{E_e E_\nu} \times L_{\alpha\beta}(Q^{[e^-\nu]})\,\mathrm{Im}\,\Pi^{\alpha\beta}(Q^{[e^-\nu]})$$

with Q four-momentum of the weak boson.

The hardship is with the hadronic polarization function.

Calculation in Thermal Quantum Field Theory:

- write the Green's function of neutrinos G_ν and charged lepton G (forward and reverse),
- write neutrino's self-energy Σ_ν (forward and reverse) introducing the premises of Π,
- write the kinetics equation for the Green's function of neutrinos introducing traces and in turn lepton tensor $L_{\alpha\beta}$ and hdronic polarization function $H_{\alpha\beta}$,
- ensure that particles are on shell introduces imaginary part of Π.

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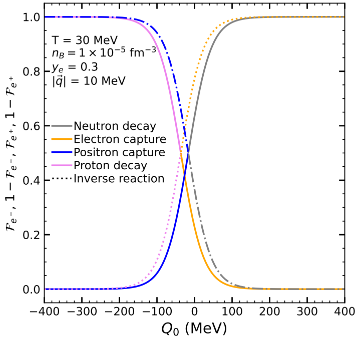
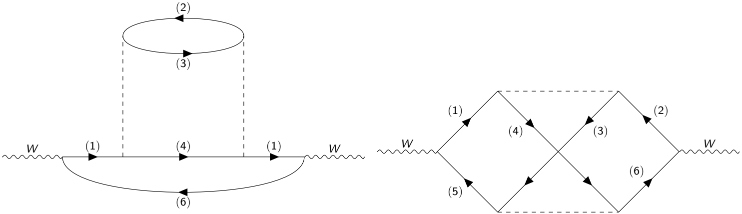


The hadronic polarization function

$$\Pi^{\alpha\beta}(Q) = \left(\prod_{j=1}^{4} \int \frac{\mathrm{d}^4 p_j}{(2\pi)^4} \right) \sum_X I_X X^{\alpha\beta}_{\mathrm{spin}}(Q) \delta^4(p_1 + p_2 - p_3 - p_4)(2\pi)^4 \ ,$$



$$I_{\mathrm{D1}} = S_p(p_1)^2 \left[S_p(p_2)S_p(p_3)S_p(p_4) + S_n(p_2)\Big(4S_p(p_3)S_n(p_4) + S_n(p_3)S_p(p_4) \Big) \right] S_n(p_6) \ ,$$

$$X^{\alpha\beta}_{y;\mathrm{spin}}(\vec{k},\vec{k}') = \left(\frac{f_{\pi NN}}{m_\pi} \right)^4 \begin{pmatrix} F(\vec{k},\vec{k}')C_V^2 & H_1^i(\vec{k},\vec{k}')C_A C_V \\ H_2^j(\vec{k},\vec{k}')C_A C_V & G^{ij}(\vec{k},\vec{k}')C_A^2 \end{pmatrix}$$

Approximations:

- non-relativistic nucleons,
- One-Pion-Exchange interaction,
- neglecting the nucleons self-energy full derivation.

However, no approximations for:

- the matrix element,
- the denominator of the intermediary line particles.

Numerical approach to the calculation.

The hadronic polarization function

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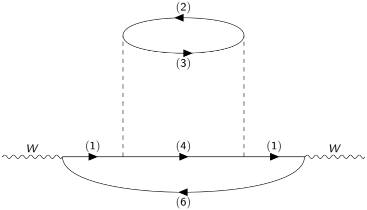

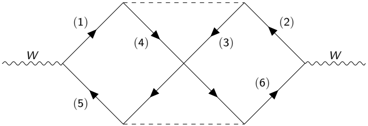

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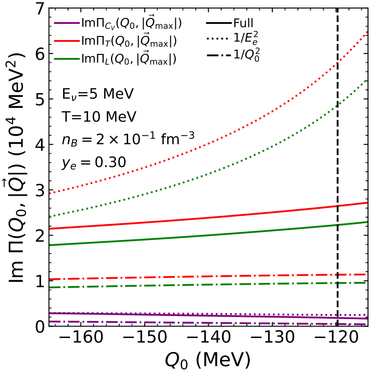
Hadronic polarization function:

$$\mathrm{Im}\,\Pi^{\alpha\beta} \propto \left(\widetilde{\mathscr{E}}_{\vec{p}_1;x} - \widetilde{\mathscr{E}}_{\vec{p}_6;w} - Q_0\right)^{-2}$$

Drastic approximations in the literature

- $\mathrm{Im}\,\Pi^{\alpha\beta} \propto E_e^{-2}$ [Friman & Maxwell (1979)]
- $\mathrm{Im}\,\Pi^{\alpha\beta} \propto Q_0^{-2}$ [Bacca et al. (2012)]



We provide results for finite temperature neutron star matter outside of commonly used approximation.

Based on the Monte-Carlo importance sampling numerical approach.

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Temperature and density regimes of MUrca vs DUrca

Estimation of the ratio between MUrca and DUrca neutrino emissivity:

- Treating zero temperature NS: [Yakovlev et al.(2001)]

$$\frac{I^{\mathrm{Mu}}}{I^{\mathrm{Du}}} = 10^{-6} \ ,$$

- Treatment in the high temperature regime: approximation for the nucleon distribution

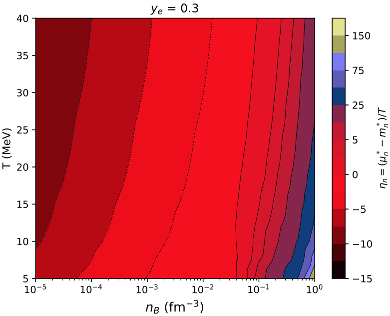
$$n_i \rightarrow e^{-\beta \epsilon_i} e^{\eta_i} \ , \qquad \frac{I^{\mathrm{Mu}}}{I^{\mathrm{Du}}} \propto e^{\eta_i} \ .$$

Reveals **different regimes of temperature and density**:

- around $\eta = 0 \rightarrow \frac{I^{\mathrm{Mu}}}{I^{\mathrm{Du}}} \sim 1$,
- for $\eta \leq 0$, then $\frac{I^{\mathrm{Mu}}}{I^{\mathrm{Du}}} \ll 1$.

Our calculations apply to proto-NS, mergers, core-collapse supernovae.

$$\eta_i = \frac{\mu_i^* - m_i^*}{T} \ .$$



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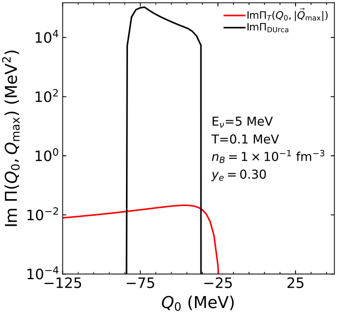

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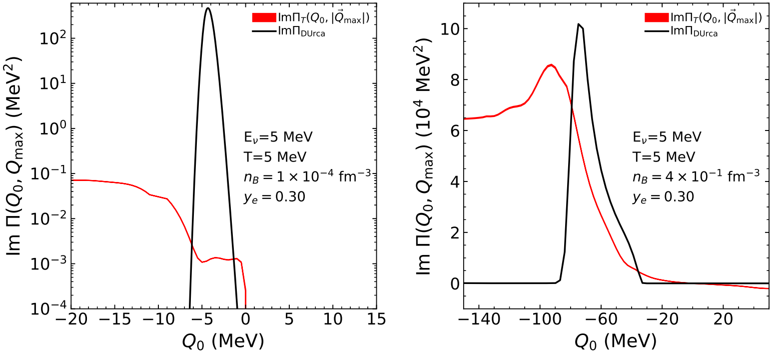

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MUrca neutrino emission at finite temperature:

- Neutrinos play an important role in Neutron Star's Physics, at various stages in the star's life.
- We have provided results for MUrca neutrino emission at finite temperature without drastic approximations usually taken.
- A simple approximation for the distribution of nucleons revealed temperature and density regimes in which MUrca is not necessarily suppressed with respect to DUrca; results were confirmed numerically.
- *We are calculating results including the lepton part of the process.*
- *Our results depend on a parameter* Γ *accounting for nucleon's self-energy. This quantity must be accurately calculated. We also intend to extend the derivation outside the non-relativistic approach.*

Questions ?