CHALMERS

Control of Auto-Resonant Beat-Wave Excitation of Plasma Waves

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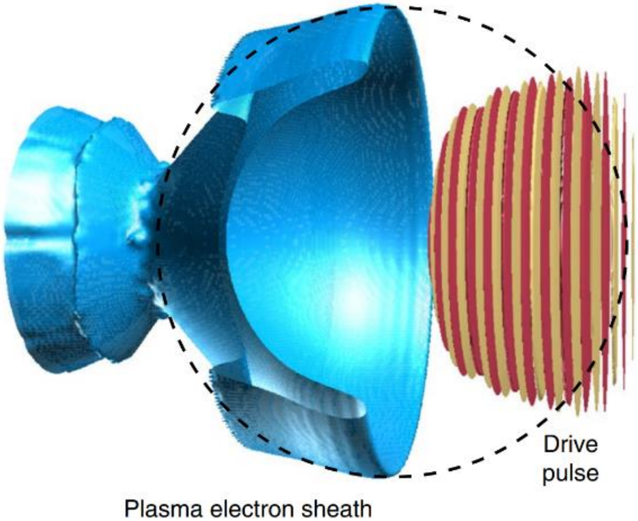
OUTLINE

- Plasma wave excitation using laser beams
- **P**lasma **B**eat **W**ave **A**ccelerator (PBWA)
  - 1D standard PBWA
  - 1D Auto-Resonant PBWA
- Kinetic study of Auto-resonant PBWA
- Summary

Plasma wave excitation using laser beams

- **L**aser **W**ake**F**ield **A**ccelerator

$$cT_{dura} \sim \lambda_{plasma} \sim c \cdot fs$$



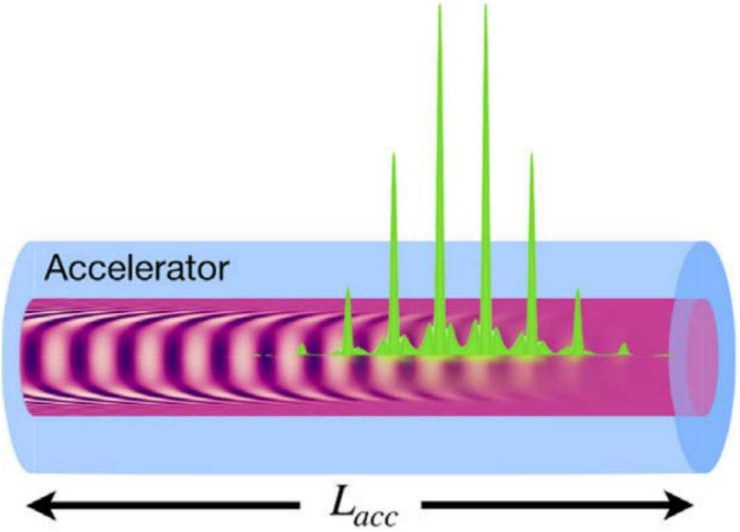
Z. Nie, *et al*., Nat. Photonics 12, 489 (2018)

Plasma wave excitation using laser beams

- **L**aser **W**ake**F**ield **A**ccelerator

$$N \times cT_{dura} \sim N \times \lambda_{plasma} \sim N \times c \cdot fs$$

- **S**cheme of **M**ultiple **L**aser **P**ulses



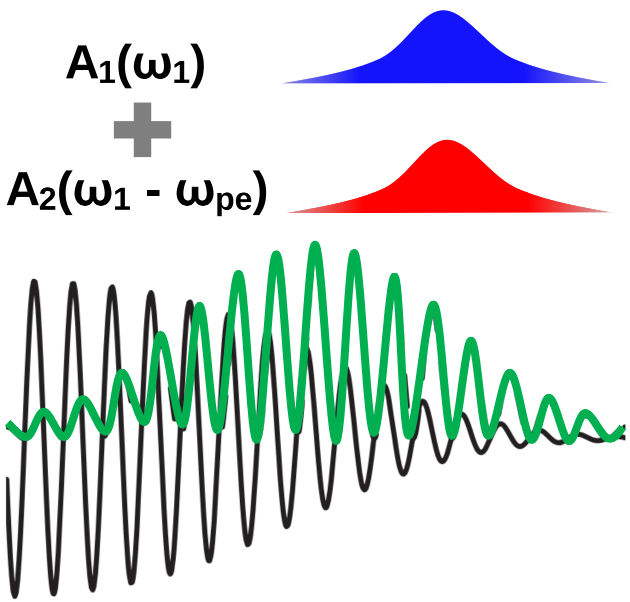
O. Jakobsson, *et al.*, Phys. Rev. Lett. 127, 184801 (2021)

Plasma wave excitation using laser beams

- **L**aser **W**ake**F**ield **A**ccelerator

- **S**cheme of **M**ultiple **L**aser **P**ulses

$$cT_{dura} \geq / \gg n\lambda_{plasma} \sim c \cdot ps$$



- **P**lasma **B**eat **W**ave **A**ccelerator

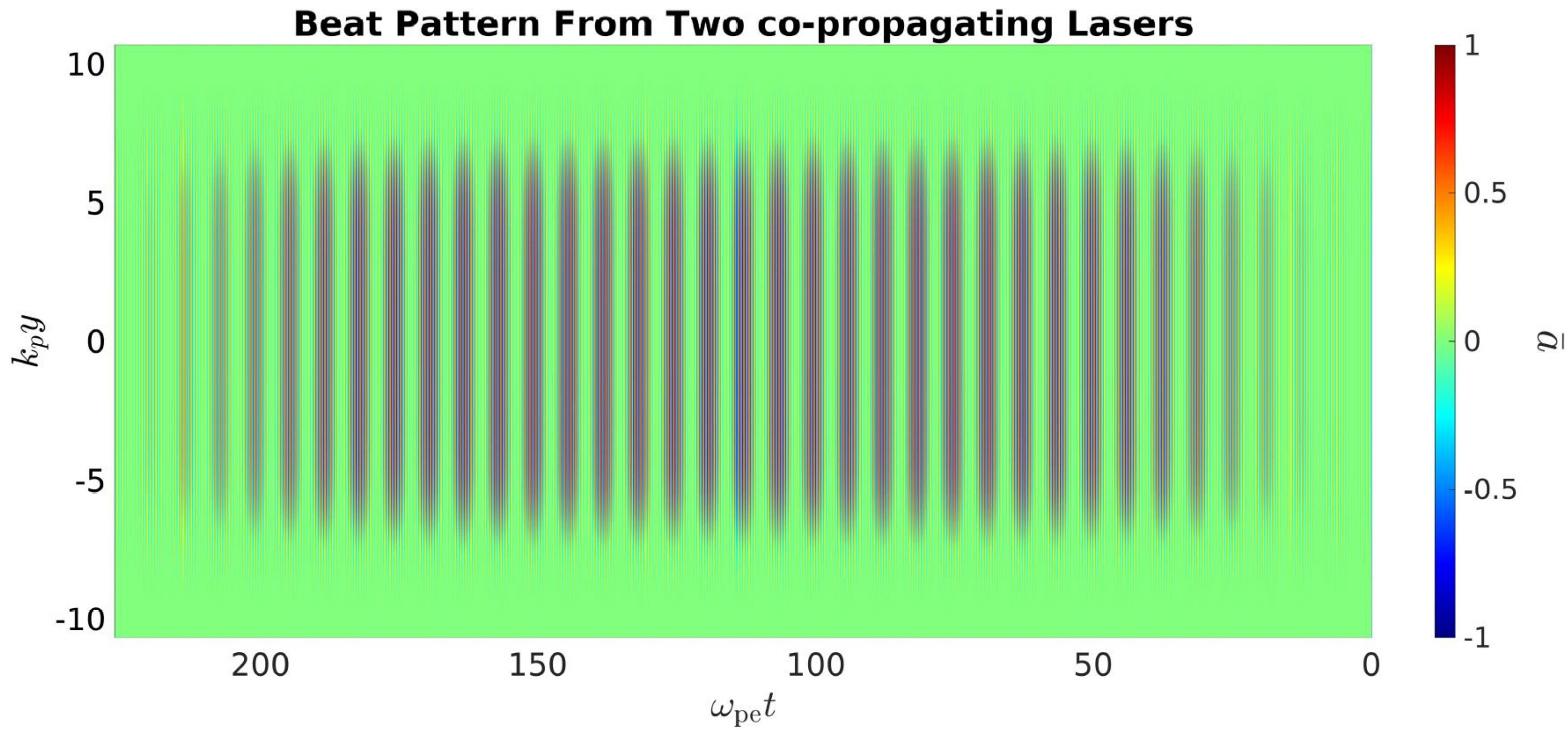
- ☐ Exciting a plasma wave with low transverse spread; (E. Ponomareva, *et al.*, Phys. Rev. Accel. Beams **26**, 061301 (2023))
- ☐ Favoring electron acceleration under near-critical density; (E Barraza-Valdez, *et al.*,Photonics, *9*(7), 476 (2022))
- ☐ Modification of the plasma wave phase velocity; (A. Pukhov, *et al.*, Plasma, *6*(1), 29-35 (2023))
- ☐ Autoresonant excitation of plasma wave.

Plasma **B**eat **W**ave **A**ccelerator

Laser beam1 : $a_1 cos(\phi_1)$, $\phi_1 = \int \omega_1 dt$ $\quad a = eA/m_e c$

Laser beam2 : $a_2 cos(\phi_2)$, $\phi_2 = \int \omega_2 dt$ $\quad \Delta\omega = \omega_1 - \omega_2 \approx \omega_{pe}$ $\quad \omega_{pe} = (n_e e^2/\epsilon_0 m_e)^{1/2}$

$$\overline{a} = a_1 cos(\phi_1) + a_2 cos(\phi_2)/(a_1 + a_2)$$



- ➢ $\omega_{pe} T_{dura} < 100\pi$ to avoid the ion motion [1], valid for the singly ionized He plasma.

[1] P. Mora *et al.*, Phys. Rev. Lett. **61**, 1611 (1988).

Plasma **B**eat **W**ave **A**ccelerator

Laser beam1 : $a_1 cos(\phi_1), \phi_1 = \int \omega_1 dt$ $a = eA/m_e c$

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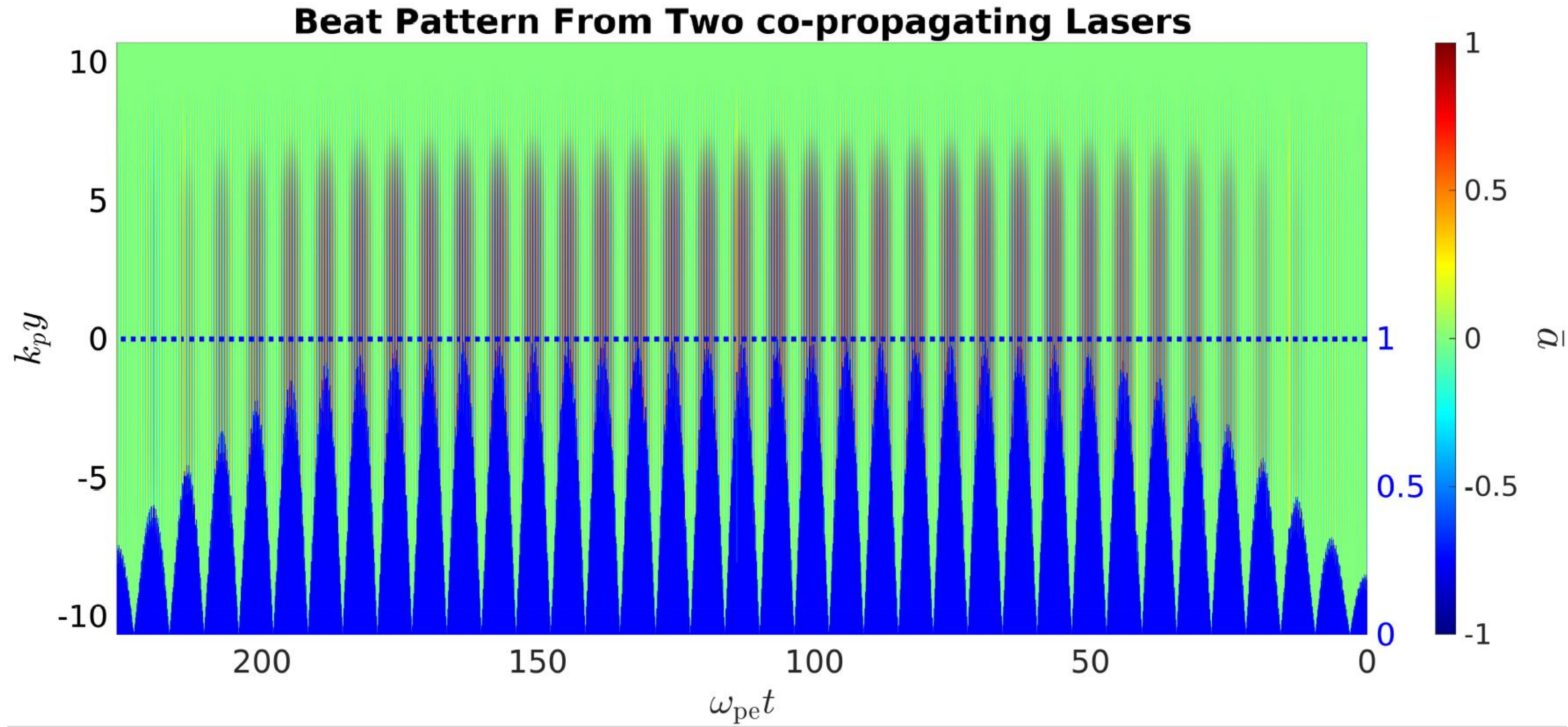

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Standard PBWA

Fluid Model (FM) with No density dependence [1]

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1+\frac{a_1^2}{2}+\frac{a_2^2}{2}+a_1 a_2\cos(\phi_1-\phi_2)}{(1+\phi)^2}\right]$$

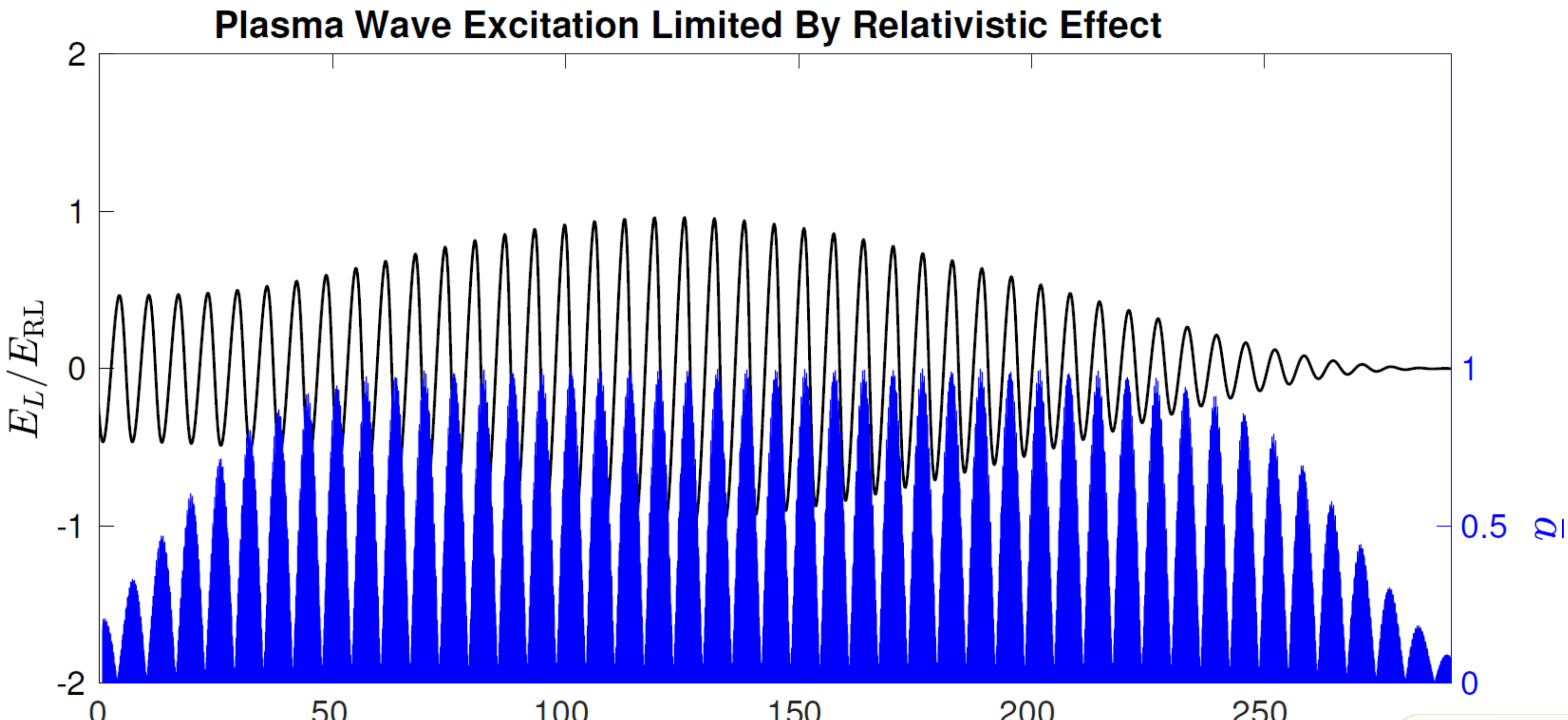
$\xi = \omega_{pe}(t - x/v_g)$

$a = eA/m_e c$

$\Delta\omega = \frac{\partial(\varphi_1-\varphi_2)}{\partial\xi} = \omega_1 - \omega_2 = \omega_{pe}$

$\omega_{pe} = (n_e e^2/\epsilon_0 m_e)^{1/2}$

Basic parameters: $a_1 = a_2 = 0.12,\ \omega_{pe}T_{dura} = 80\pi$



Relativistic effect leads to wavelength shift ⟹ Rosenbluth-Liu (RL) limit [2]:

$E_{RL} = E_0(16a_1a_2)^{\frac{1}{3}}$

$E_0 = m_e\omega_{pe}c/e$

[1] R. R. Lindberg. *et. al.*, Phys. Rev. Lett. 93.055001 (2003); [2] Rosenbluth, *et al.*, Phys. Rev. Lett. **29**, 701 (1972).

Phase information **in Standard PBWA**

Fluid Model (FM) with No density dependence [1]

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1+\frac{a_1^2}{2}+\frac{a_2^2}{2}+a_1a_2\cos(\phi_1-\phi_2)}{(1+\phi)^2}\right]$$

$\xi = \omega_{pe}(t - x/v_g)$

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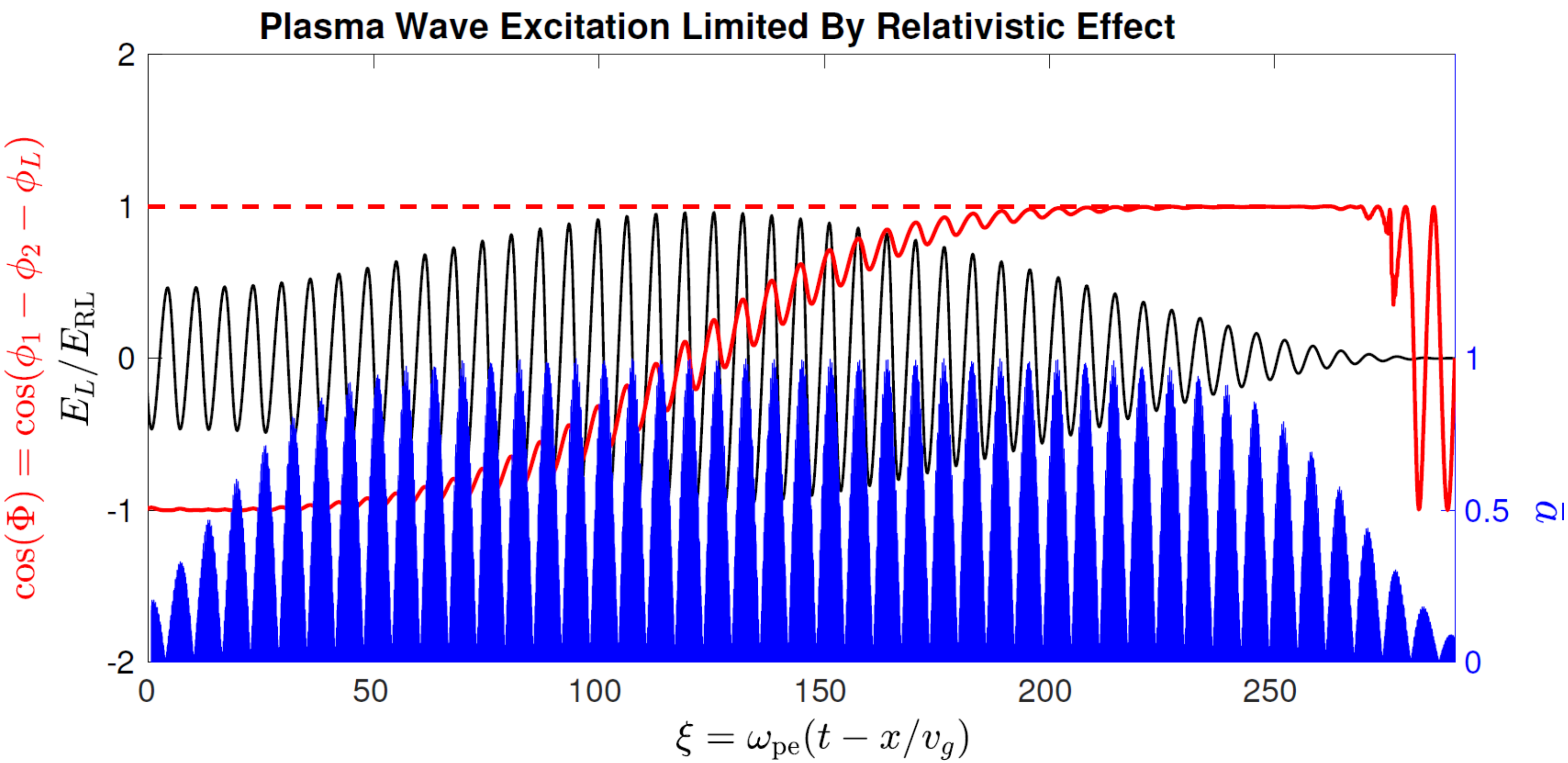


Phase-locking, $\cos\Phi = \cos(\phi_1 - \phi_2 - \mathrm{Arg}(E_L)) \approx 1$, promotes the excitation

[1] R. R. Lindberg. *et. al.*, Phys. Rev. Lett. 93.055001 (2003).

AutoResonant (AR) PBWA

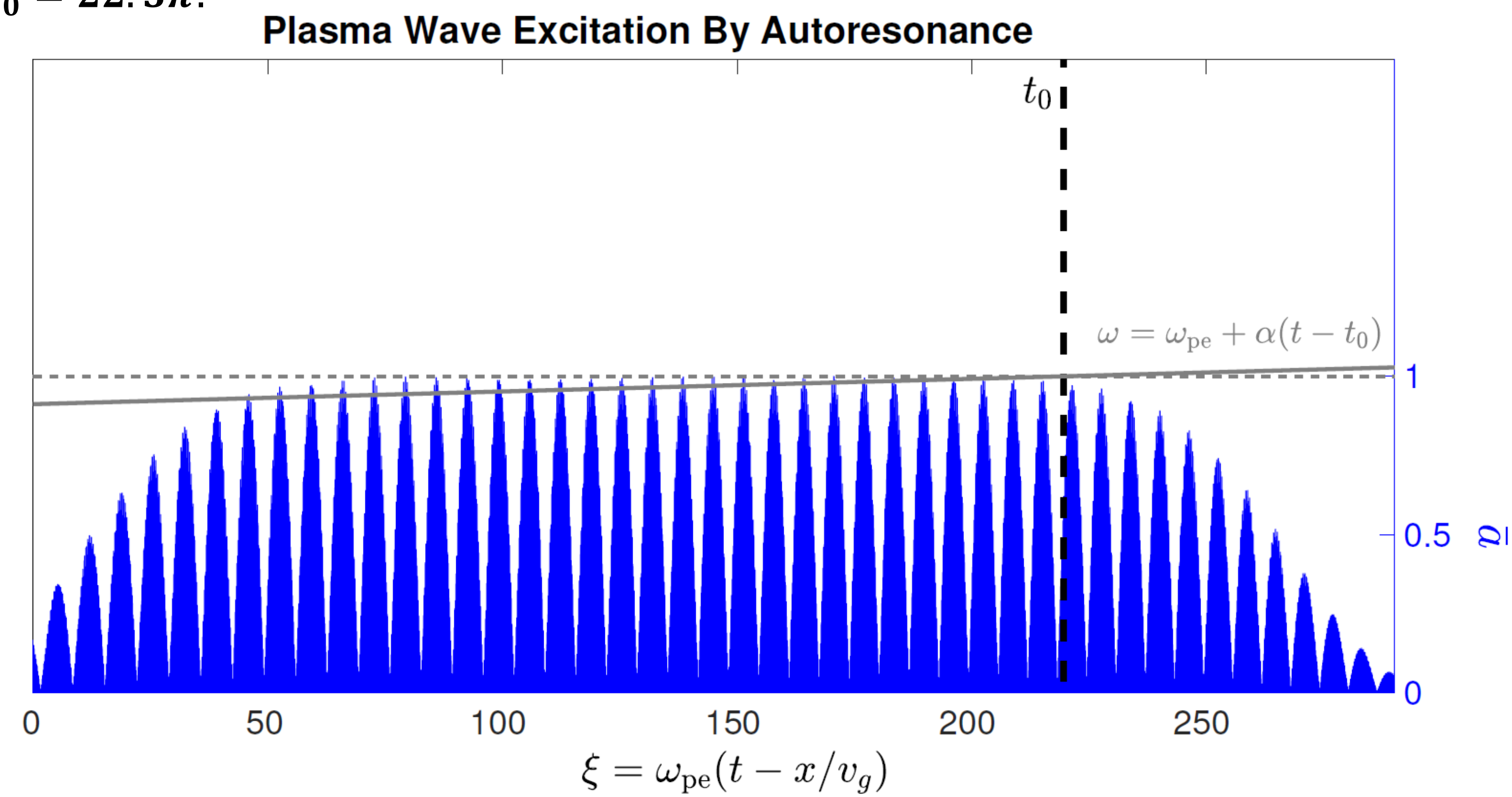
Fluid Model (FM) with No density dependence [1]

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1+\frac{a_1^2}{2}+\frac{a_2^2}{2}+a_1 a_2 \cos(\phi_1-\phi_2)}{(1+\phi)^2}\right]$$

$$\xi = \omega_{pe}(t - x/v_g)$$

$$\Delta\omega = \frac{\partial(\varphi_1-\varphi_2)}{\partial\xi} = \omega_1 - \omega_2 = \omega_{pe} + \alpha(t - t_0)$$

Basic parameters: $a_1 = a_2 = 0.12$, $\omega_{pe}T_{dura} = 80\pi$, linear chirp rate $\alpha = -0.0004$, $\omega_{pe}t_0 = 22.5\pi$.



[1] R. R. Lindberg. *et. al.*, Phys. Rev. Lett. 93.055001 (2003).

Plasma wave excitation in **AR PBWA**

Fluid Model (FM) with No density dependence [1]

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1+\frac{a_1^2}{2}+\frac{a_2^2}{2}+a_1 a_2\cos(\phi_1-\phi_2)}{(1+\phi)^2}\right]$$

$$\xi = \omega_{pe}(t - x/v_g)$$

$$\Delta\omega = \frac{\partial(\varphi_1-\varphi_2)}{\partial\xi} = \omega_1-\omega_2 = \omega_{pe}+\alpha(t-t_0)$$

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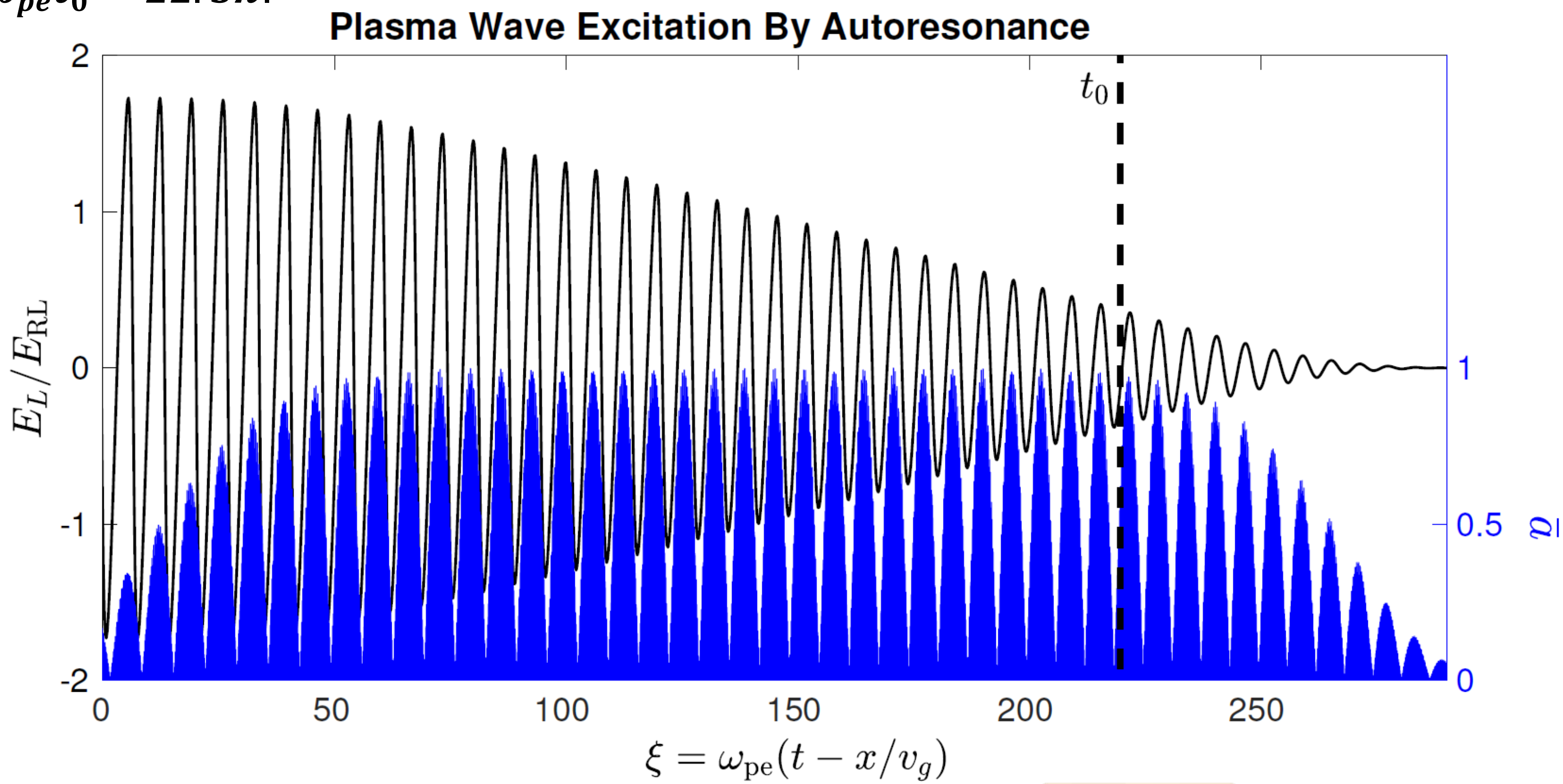

Proper chirp choice leads to an enhancement [1,2], e.g., $E_L > E_{RL}$

Phase information **in AR PBWA**

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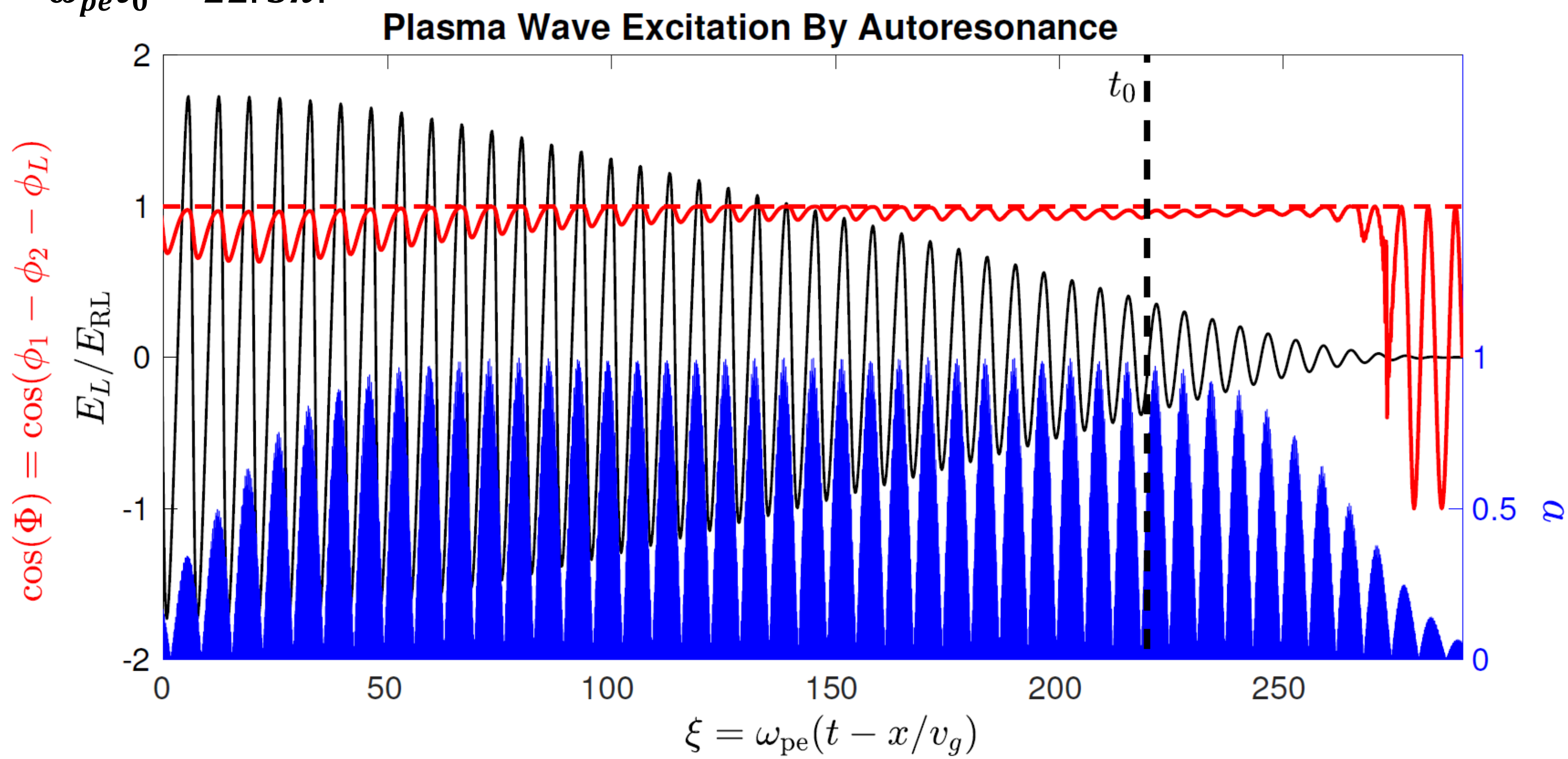

Proper chirp choice leads to the broaden phase-locking distance [2].

[1] R. R. Lindberg. *et. al.*, Phys. Rev. Lett. 93.055001 (2003); [2] R. R. Lindberg. *et. al.*, POP 13, 123103 (2006)

What effects are missing in fluid model?

- No explicit density dependent **F**luid **M**odel (**FM**):

$$\frac{d^2}{d\xi^2}\phi = \frac{1}{2}\left[\frac{1+\frac{a_1^2}{2}+\frac{a_2^2}{2}+a_1 a_2\cos(\phi_1-\phi_2)}{(1+\phi)^2}\right]$$

$$\xi = \omega_{pe}\left(t - x/v_g\right)$$

$$\Delta\omega = \frac{\partial(\varphi_1-\varphi_2)}{\partial\xi} = \omega_1 - \omega_2 = \omega_{pe} + \alpha(t - t_0)$$

- No explicit density dependence;
- Particle acceleration, wave-breaking limitation, are not included;
- The up/down shift of the laser frequencies due to the density perturbation is not included;
- The Stokes / Anti-Stokes scattering instability of the laser beams are not included.

Comparison between **FM** and **PIC** results

Kinetic simulations are performed with SMILEI PIC code[1] (Convenient and Friendly) with the same basic parameters as the fluid one

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	α	$k_p dx$
0.0004	0.12	80π	22.5π	-0.0004	0.008

[1] J. Derouillat. *et. al*., Phys. Commun. 222, 351-373 (2018)

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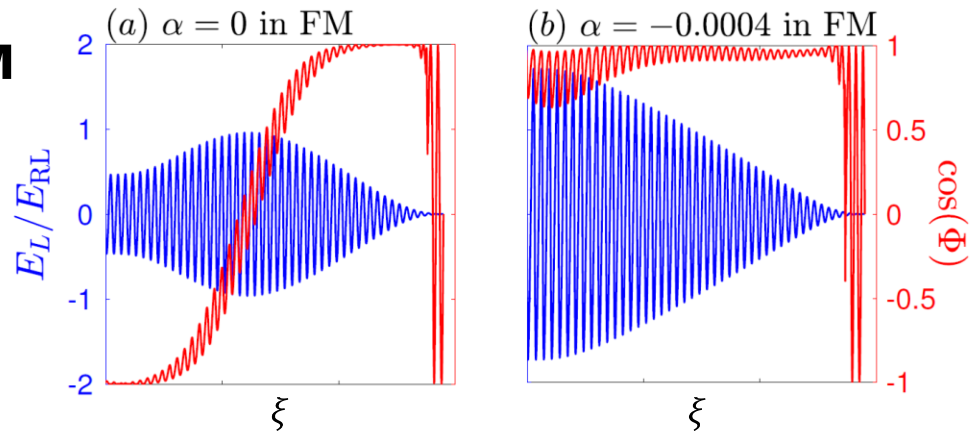
FM



- Blue lines: E_L/E_{RL} ;

  Red lines: $\cos[\varphi_1 - \varphi_2 - \mathrm{Arg}(E_L)]$;

- $\mathrm{Cos}[\varphi_1 - \varphi_2 - \mathrm{Arg}(E_L)] \approx 1$, gives the optimal condition for the plasma wave excitation;

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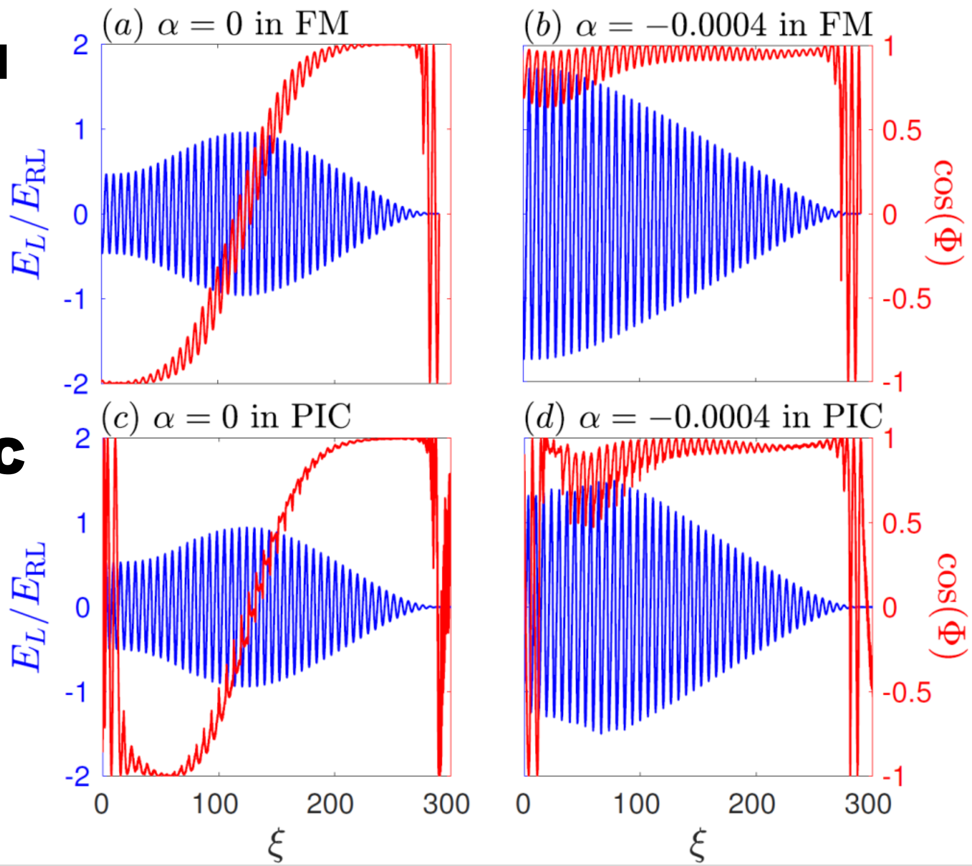
FM

PIC



- Blue lines: E_L/E_{RL} ;
  Red lines: $\cos[\varphi_1 - \varphi_2 - \mathrm{Arg}(E_L)]$;
- $\mathrm{Cos}[\varphi_1 - \varphi_2 - \mathrm{Arg}(E_L)] \approx 1$, gives the optimal condition for the plasma wave excitation;
- AR PBWA is reproduced in PIC.

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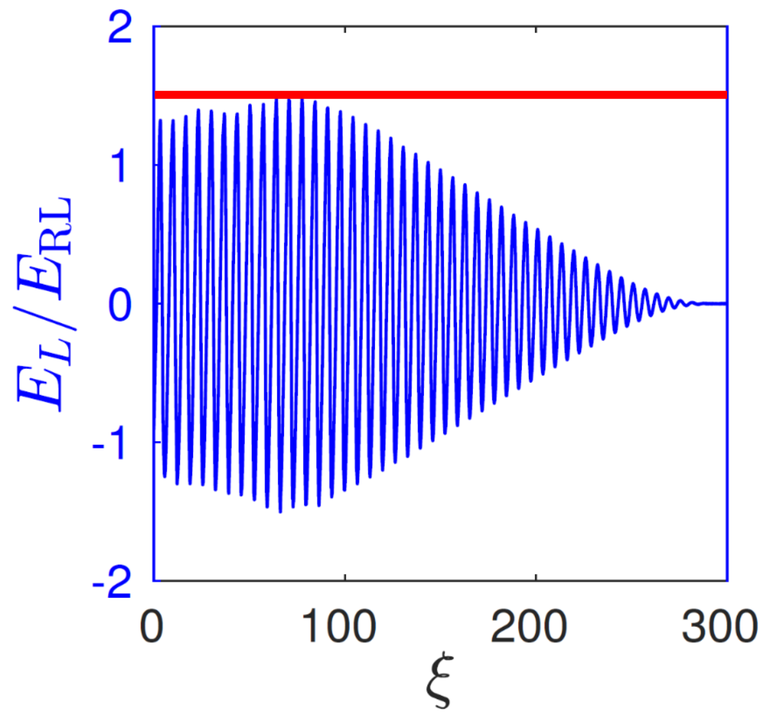
Density dependence is observed in **PIC**

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004,	**0.12**	**80π**	**22.5π**	**0.008**

$\boldsymbol{\alpha = -0.0004}$



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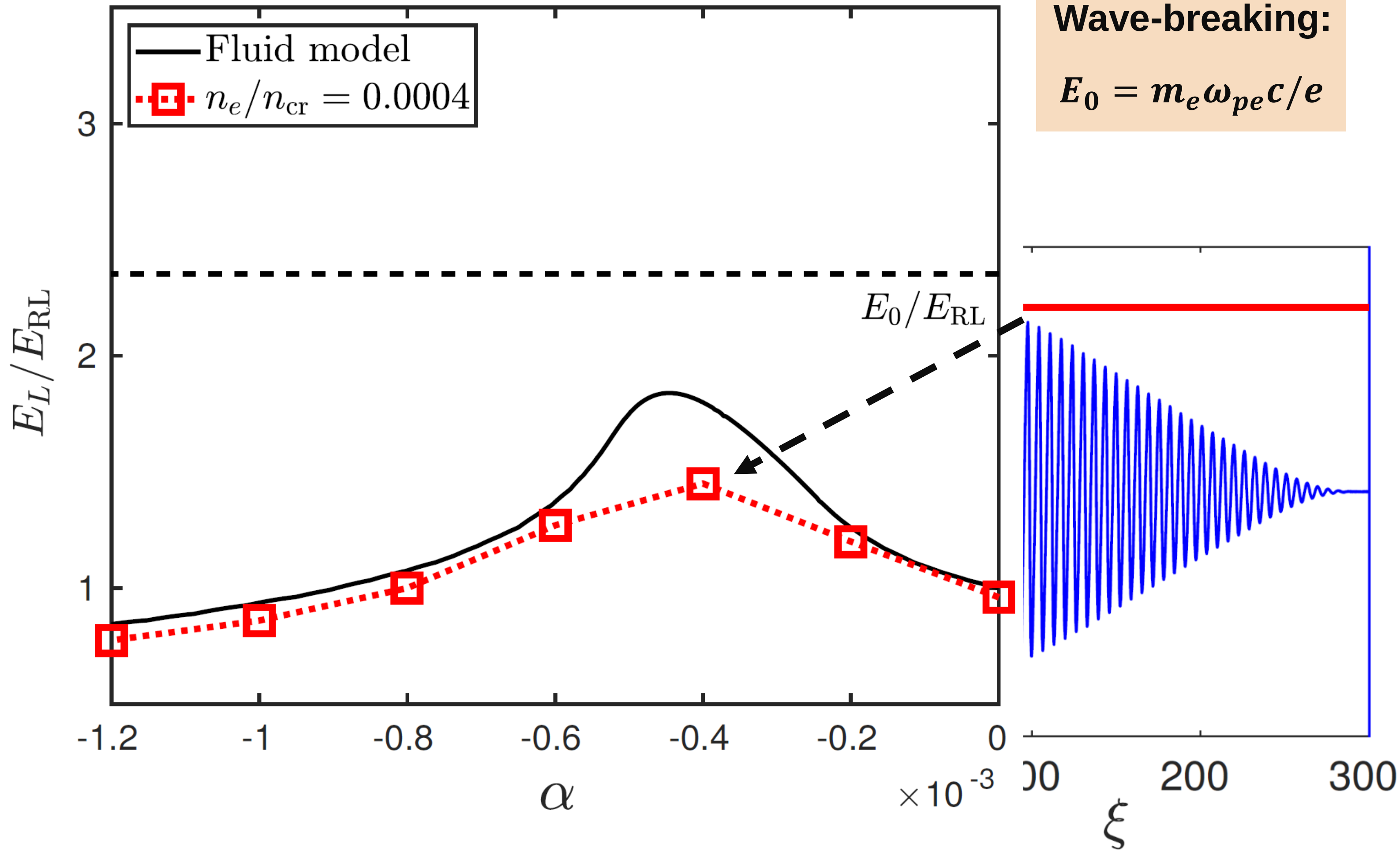
0.0004,	0.12	80π	22.5π	0.008

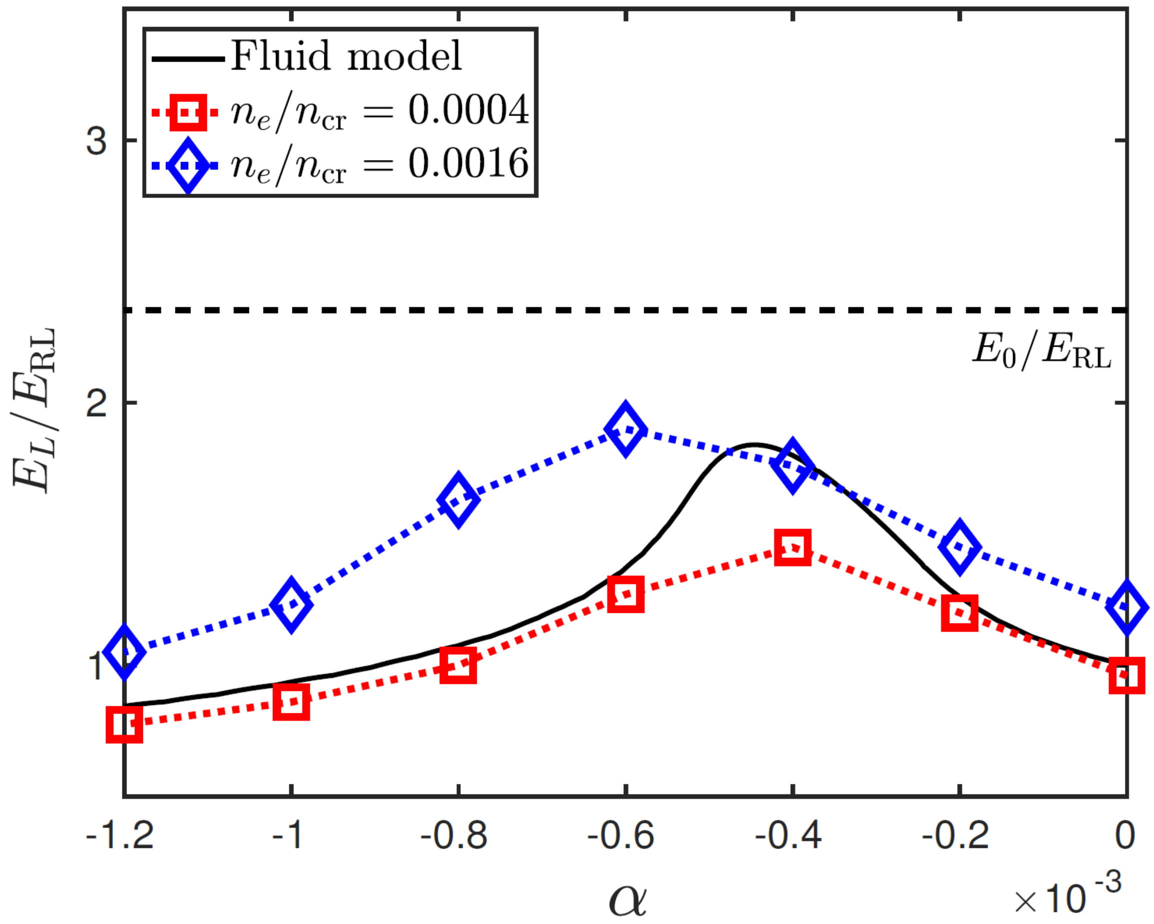
Wave-breaking:

$$E_0 = m_e \omega_{pe} c/e$$



Density dependence is observed in **PIC**

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004, 0.0016	0.12	80π	22.5π	0.008



Wave-breaking:

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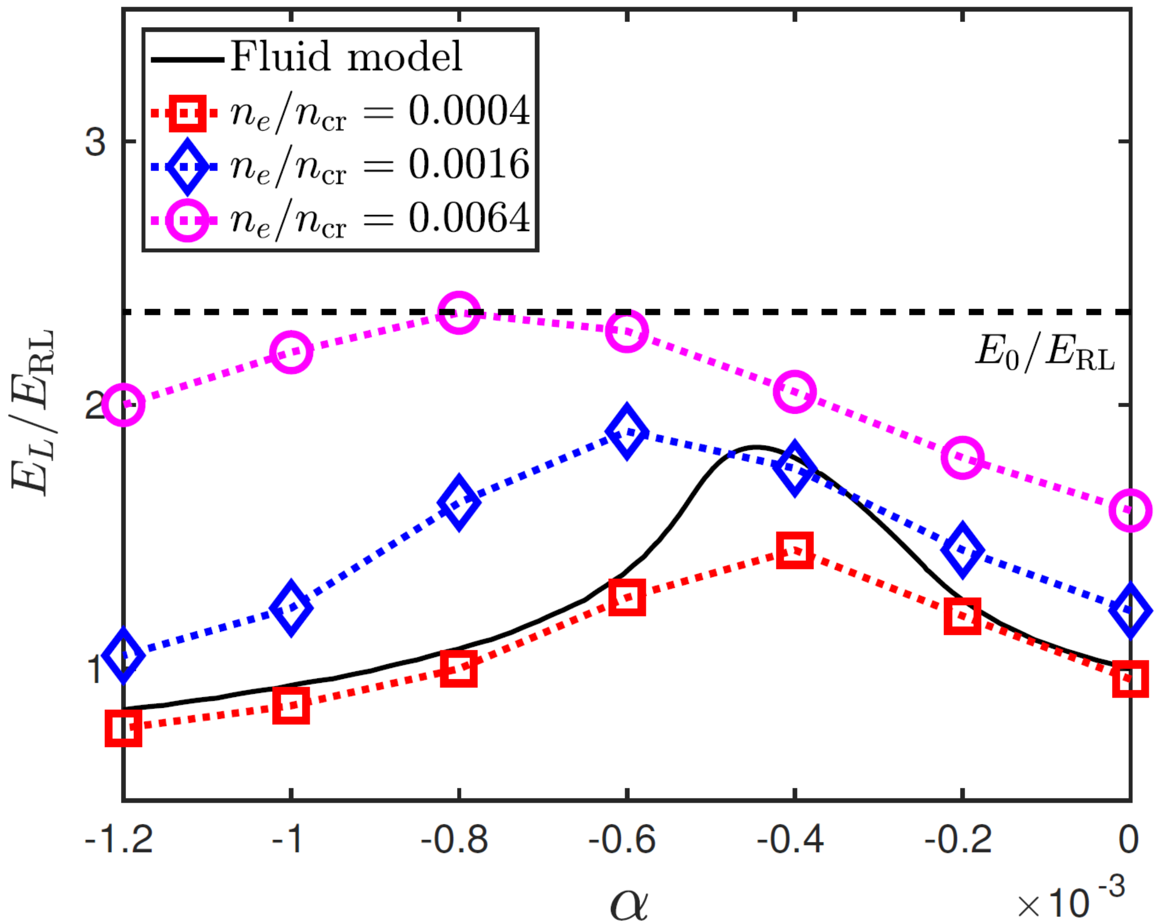
Density dependence is observed in **PIC**

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004, 0.0016, 0.0064	0.12	80π	22.5π	0.008

Wave-breaking:

$E_0 = m_e \omega_{pe} c/e$

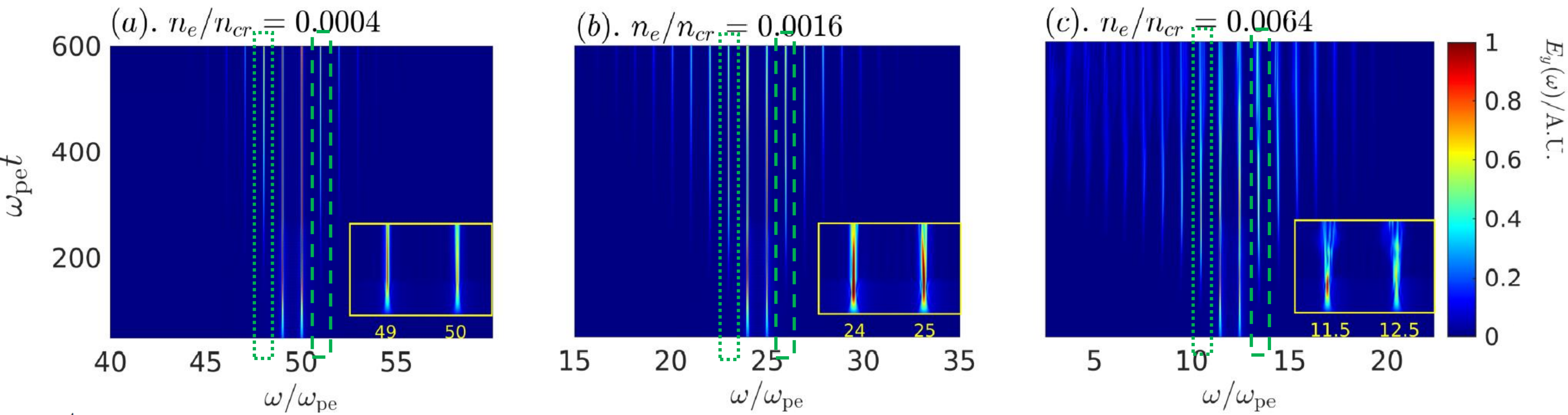


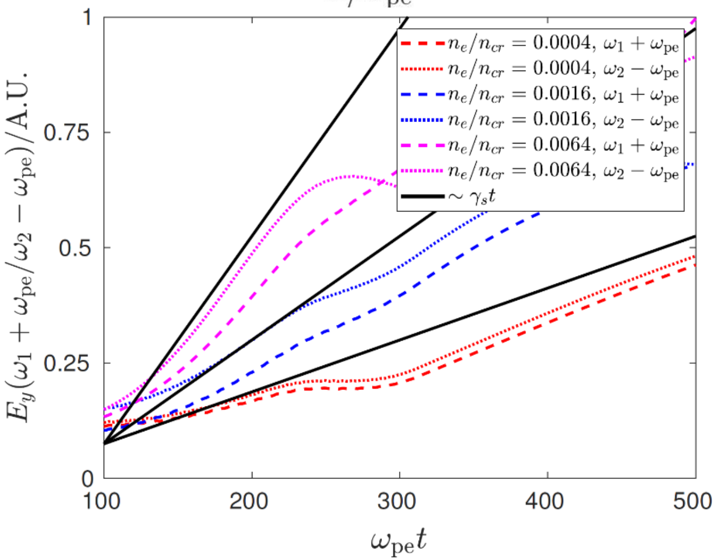
- For the lowest plasma density, e.g., n_e/n_{cr} = **0.0004**, good agreement is obtained;
- **Higher** density, **higher** maximum amplitude, but **faster** decline.

Laser beams evolution

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	α	$k_p dx$
0.0004, 0.0016, 0.0064	0.12	80π	0	0.008

Stokes/ anti-Stokes scattering:





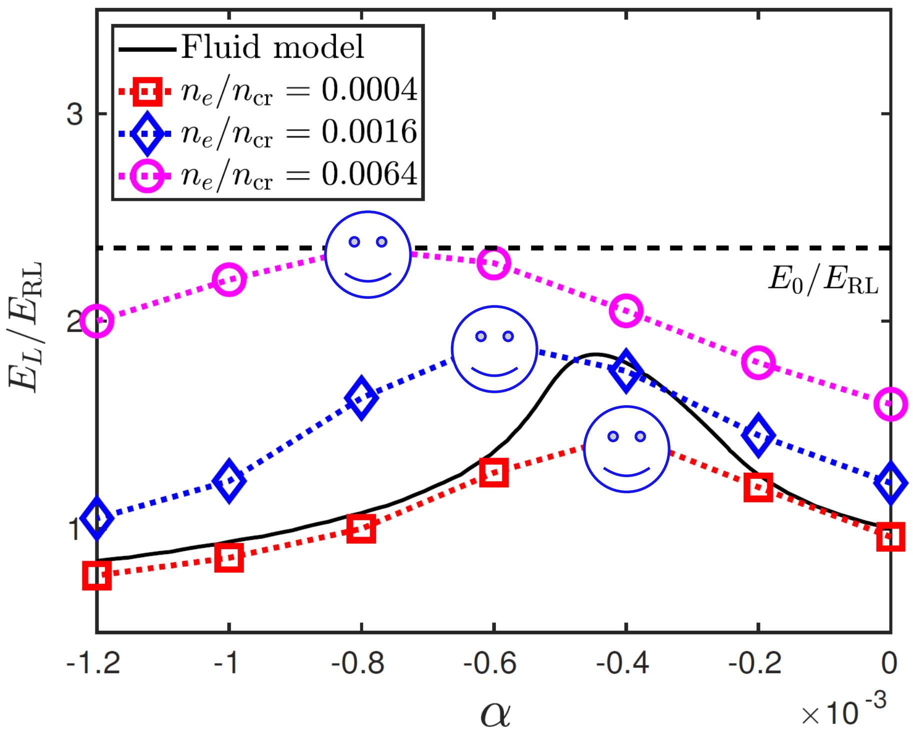
The growth rate is analytically obtained:

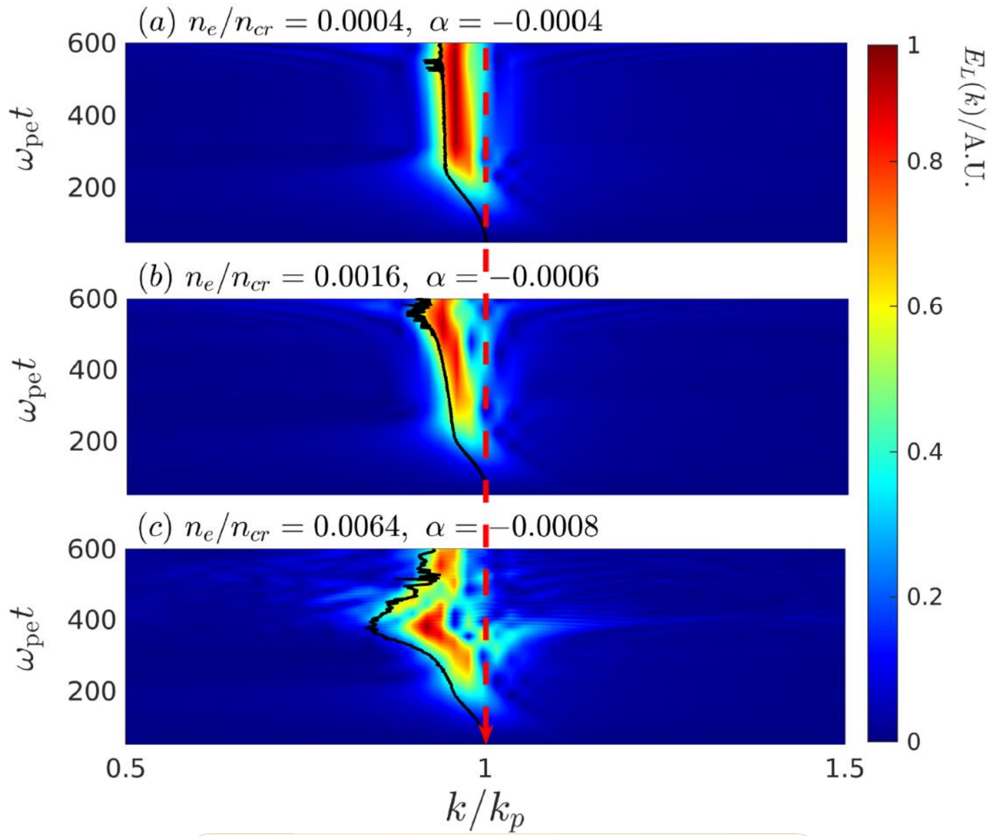
$$\frac{\gamma_s}{\omega_{pe}} = \frac{\sqrt{n_e/n_{cr}}}{4} a_{1,2}\,(16a_1a_2)^{\frac{1}{3}}$$

- Coupling between the neighboring mode may enhance the transient excitation;
- Frequency shift due to $\delta n_e/n_e$, $\sim \boldsymbol{n_e/n_{cr}}$.

Coherence of the plasma wave

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004, 0.0016, 0.0064	0.12	80π	22.5π	0.008





Maximum nonlinear wavenumber:

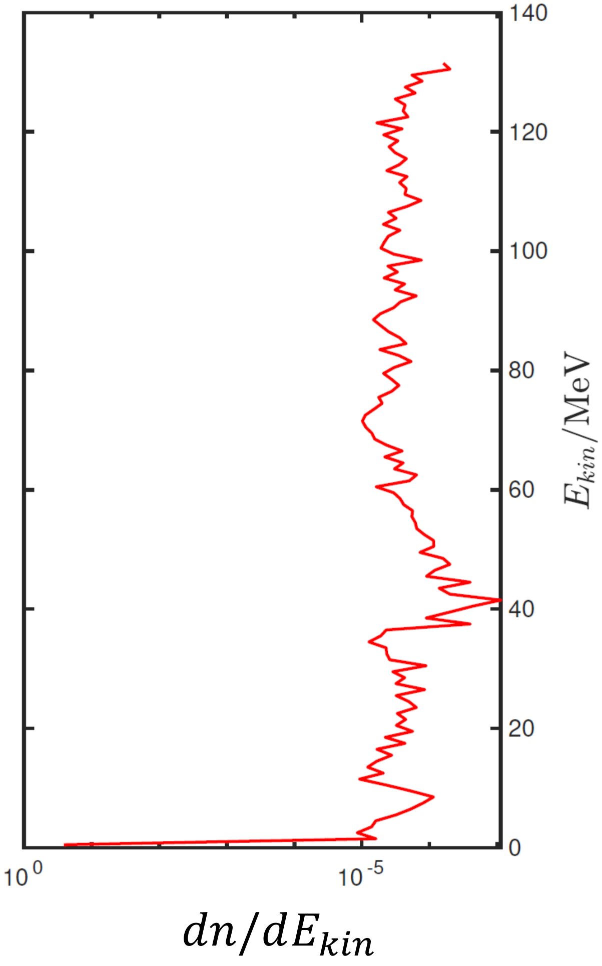
$$k_{np} = k_p/[1 + 3/16(E_{L-max}/E_0)^2]$$

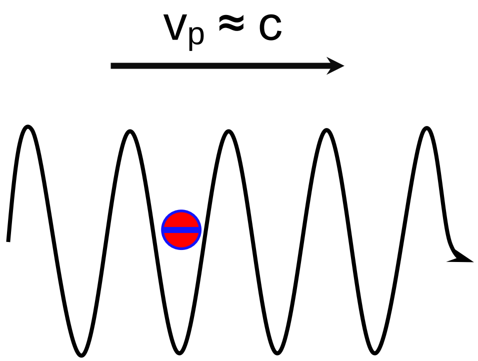
- **Higher** density, **higher** maximum amplitude, but **reduced** coherence.

Recording the acceleration process

A good accelerator has high accelerating field, and should be stable

Distribution Function





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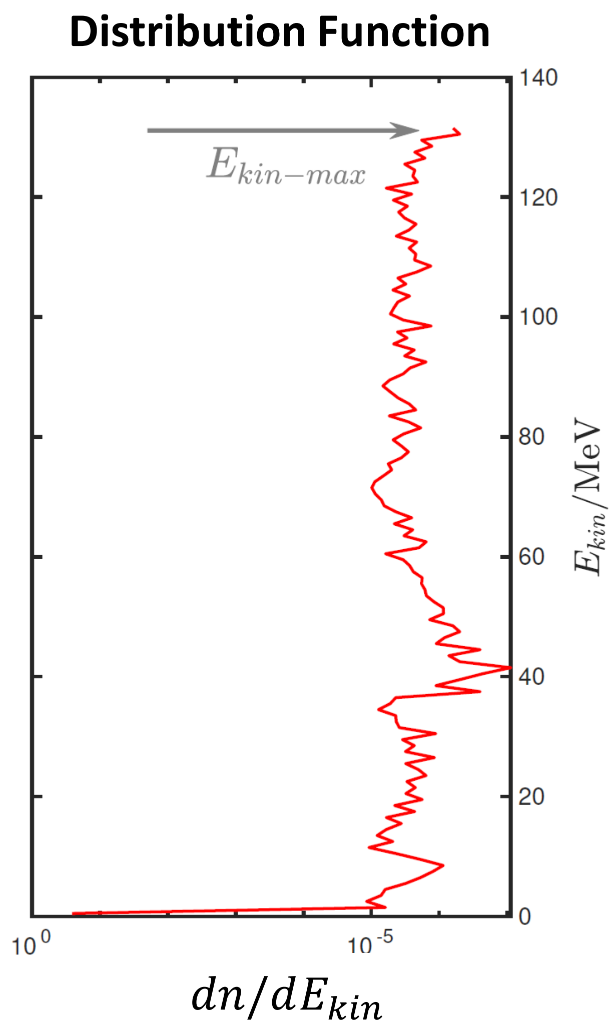
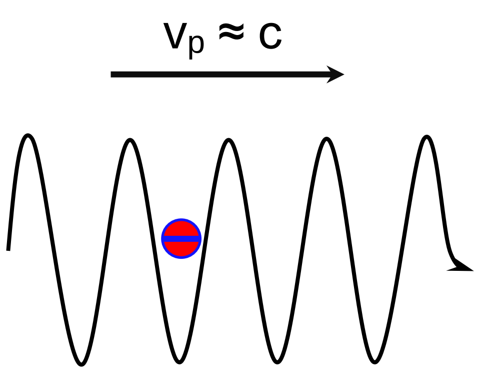
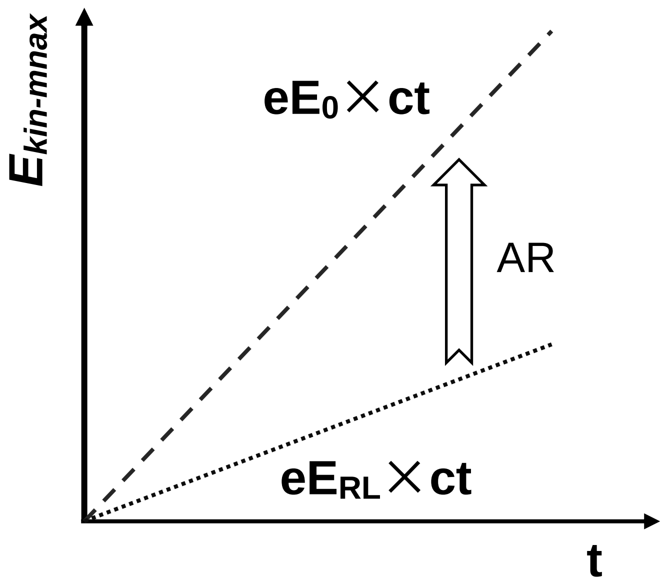




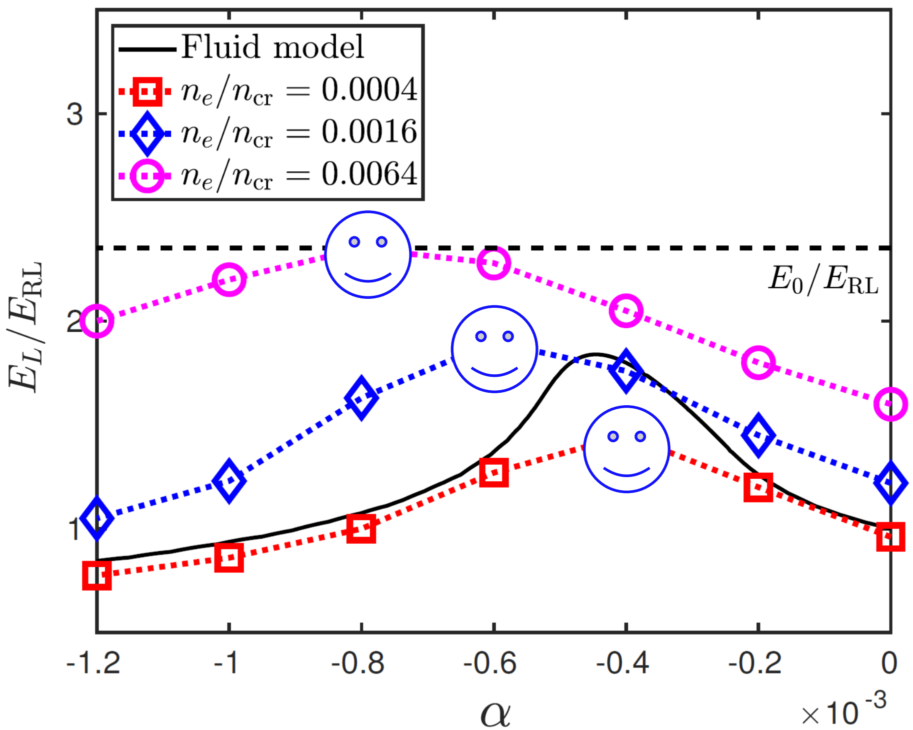
Kinetic energy $E_{kin\text{-}max} = eE_L \times ct$

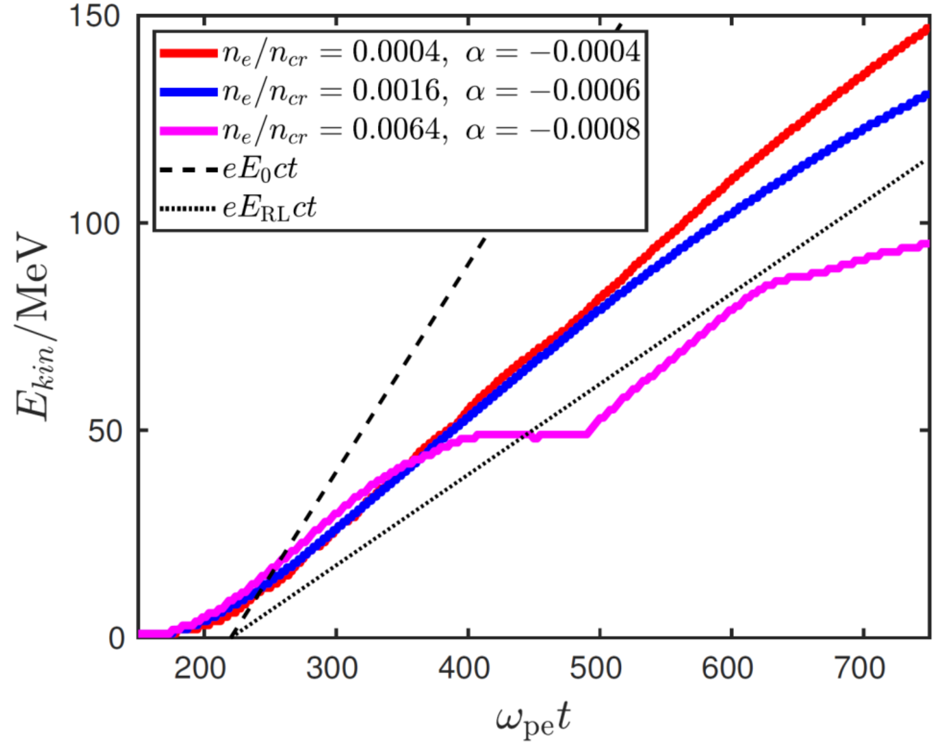


Coherence influences the acceleration process

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004, 0.0016, 0.0064	0.12	80π	22.5π	0.008

NOTING: $eE_0ct = e\frac{m_ec\omega_p}{e}ct = m_ec^2\omega_{pe}t = \omega_{pe}t \times 0.5\text{MeV}, \quad E_{RL} = E_0(16a_1a_2)^{\frac{1}{3}}$



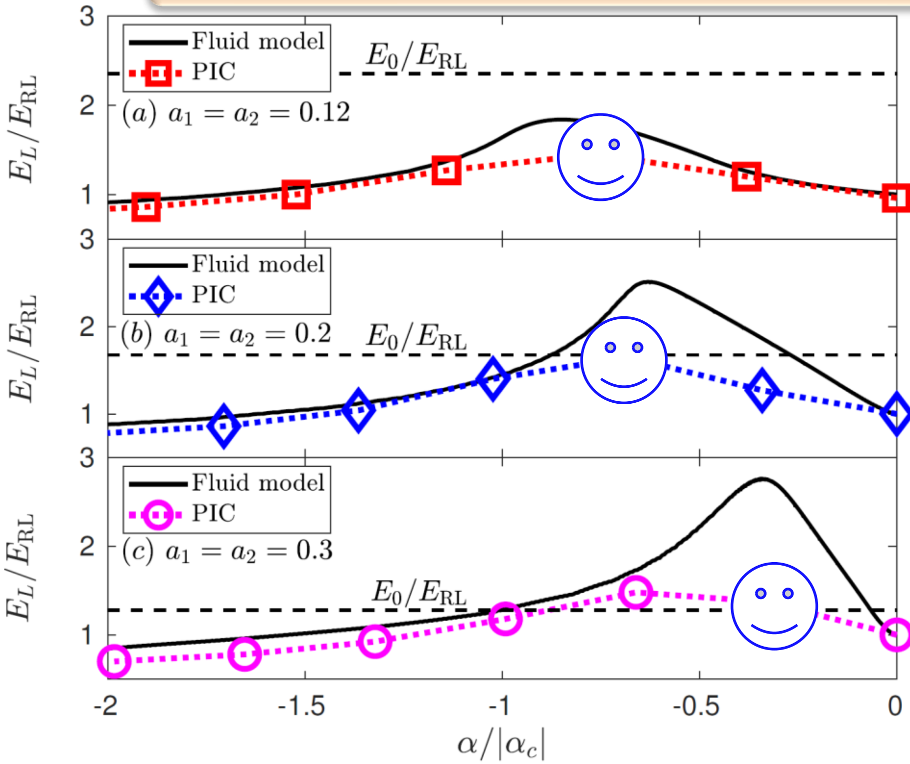


- **Higher** density, **higher** maximum amplitude, but **reduced** coherence leads to **lower** acceleration efficiency.

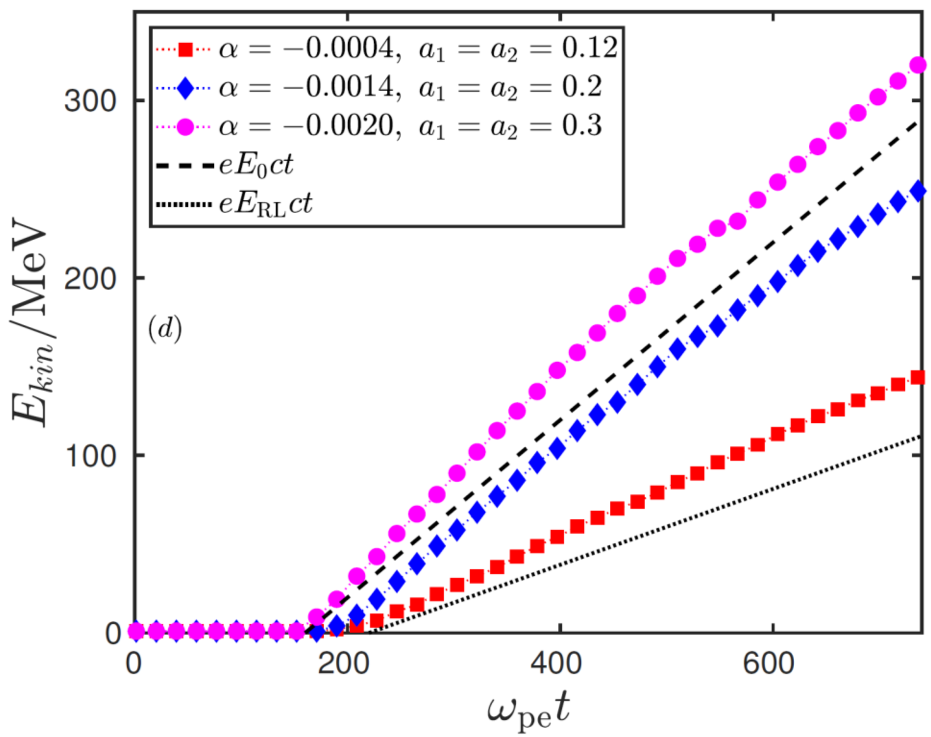
Optimal laser intensity to reach wave-breaking

n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	$k_p dx$
0.0004	0.12, 0.2, 0.3	80π	22.5π	0.008

NOTING: $eE_0ct = e\frac{m_e c\omega_p}{e}ct = m_e c^2\omega_{pe}t = \omega_{pe}t \times 0.5\text{MeV}, \quad E_{RL} = E_0(16a_1a_2)^{\frac{1}{3}}$



$\alpha_c = 0.15(a_1a_2)^{4/3}$



- The **optimal** parameters:

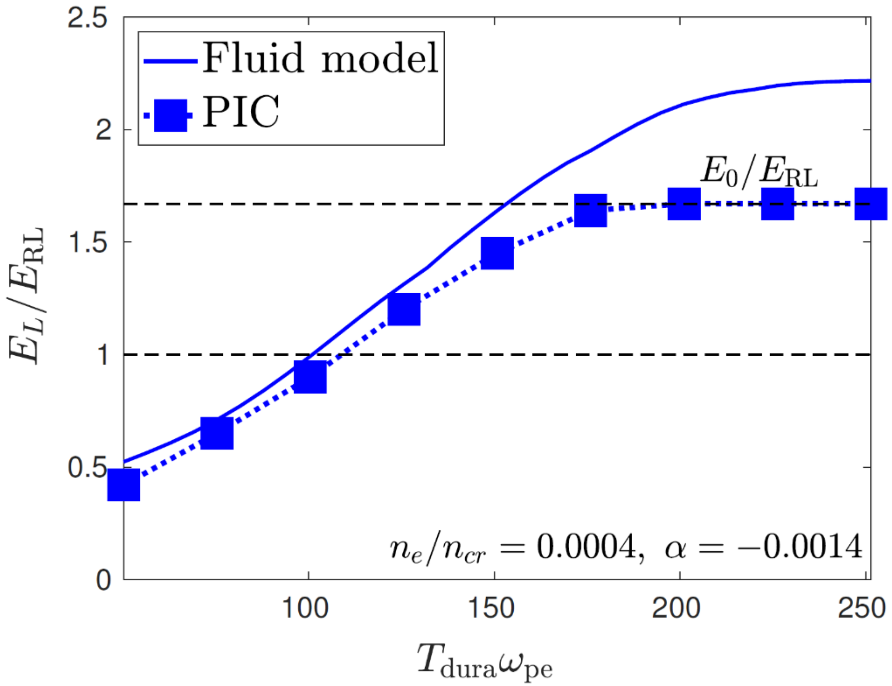
density $n_e/n_{cr} \approx$ **0.0004, intensity** $a_1 = a_2 \approx 0.2$, **chirp rate** $\alpha \approx -0.0014$

Control of Auto-Resonant **PBWA**

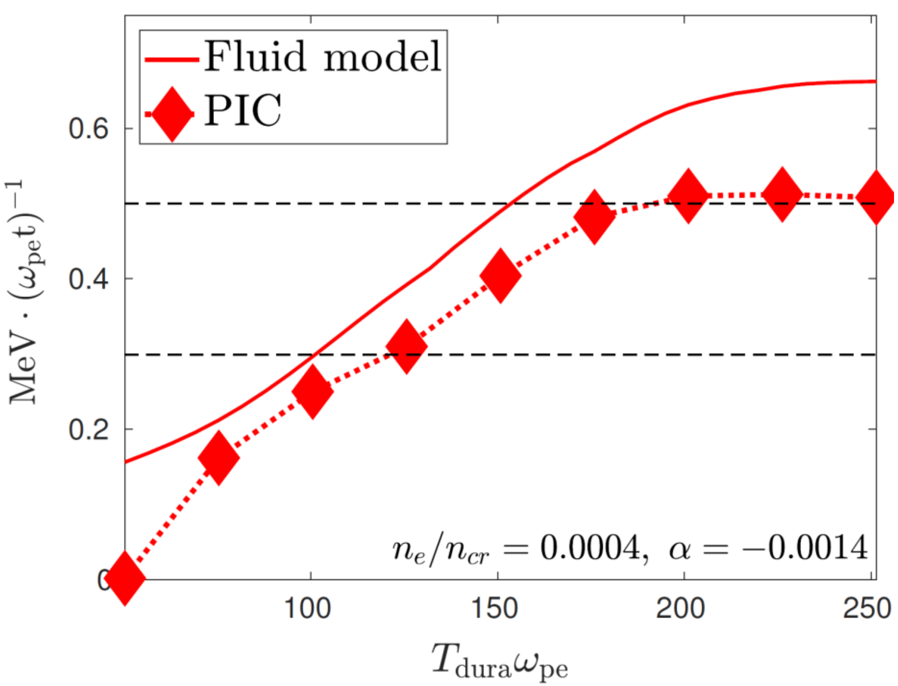
n_e/n_{cr}	$a_1 = a_2$	$\omega_{pe}T_{dura}$	$\omega_{pe}t_0$	α
0.0004	0.2	$16\pi\sim80\pi$	22.5π	-0.0014

NOTING: $eE_0ct = e\frac{m_ec\omega_p}{e}ct = m_ec^2\omega_{pe}t = \omega_{pe}t \times 0.5\text{MeV}, \quad E_{RL} = E_0(16a_1a_2)^{\frac{1}{3}}$

Accelerating electric field



Energy gain rate



SUMMARY of the1D work[1]

- The **Auto-Resonant beat-wave generation** has been studied through **fluid model** and **kinetic simulations (SMILEI)**. An appropriate choice of the chirp rate can allow the excitation of plasma waves beyond the RL-limit;

- Through **kinetic simulations (SMILEI),** the optimal set of parameters is established for enhancing plasma waves up to the point of wave breaking. **Control of Auto-Resonant plasma wave excitation and energy gain can be realized;**

- Considering the **800nm** laser, $\boldsymbol{I_1 = I_2 \approx 8.5 \times 10^{16} W/cm^2}$, $\boldsymbol{n_e \approx 7 \times 10^{17} cm^{-3}}$, laser duration $\boldsymbol{T_{dura} \approx 4ps}$, frequency bandwidth $\Delta\boldsymbol{\omega/\omega_0} \approx$ **0.5%**, **RL limit ~ 47GV/m**, **wave breaking ~ 80 GV/m**.

Thanks for your attention

Analysis of auto-resonant PBWA in 2D is ongoing and indicate that:

- Our main conclusions stand the test of higher dimensionality
- But additional rich physics shows up. Stay tuned!

[1] MF. Luo. *et. al*., XXX (under review);