Frontiers in Mathematical Physics

Rapport sur les contributions

https://indico.math.cnrs.fr/e/FMP2023

ID de Contribution: **1** Type: **Non spécifié**

On Infinite Classical Coulomb Configurations and Their Potential

lundi 26 juin 2023 09:00 (45 minutes)

I will discuss the classical infinite Coulomb gas and its extension to Riesz interaction potentials $|x|^{-s}$. I will particularly focus on the problem of defining infinite equilibrium states and their potential. This has caused a lot of confusion in the literature, even in the simplest case of periodic lattices. The general case is in fact largely open. Based on the review paper Coulomb and Riesz gases: The known and the unknown, J. Math. Phys., 2022.

Orateur: LEWIN, Mathieu (Université Paris Dauphine)

ID de Contribution: **2** Type: **Non spécifié**

Non-Renormalization of the Anomalies and Universality in Transport Coefficients

lundi 26 juin 2023 09:55 (45 minutes)

Anomalies are the breaking of classical symmetries by quantum effects, and their non-renormalization properties play a crucial role in a wide range of phenomena. I present some rigorous theorems on the (non-perturbative) anomaly non-renormalization in QFT models, based on Renormalization Group, cluster or tree expansion and determinant bounds, proving the exact cancellation of the terms coming by the lattice cut-offs. I discuss in particular a lattice fermion-vector model in $d = 3 + 1$, the Sommerfield model in $d = 1 + 1$ and the anomaly cancellation in a chiral lattice $d = 3+1$ model. Analogous results on universality in transport coefficients in graphene, Hall insulators and Weyl semimetals in presence of a many body interactions will be also briefly presented.

Orateur: MASTROPIETRO, Vieri (Università di Milano)

ID de Contribution: **3** Type: **Non spécifié**

Signal Communication and Modular Theory

lundi 26 juin 2023 11:00 (45 minutes)

I propose a conceptual frame to interpret the prolate differential operator, which appears in Communication Theory, as an entropy operator; indeed, I write its expectation values as a sum of terms, each subject to an entropy reading by an embedding suggested by Quantum Field Theory. This adds meaning to the classical work by Slepian et al. on the problem of simultaneously concentrating a function and its Fourier transform, in particular to the "lucky accident" that the truncated Fourier transform commutes with the prolate operator. The key is the notion of entropy of a vector of a complex Hilbert space with respect to a real linear subspace, recently introduced by the author by means of the Tomita-Takesaki modular theory of von Neumann algebras, and studied in collaboration with Ciolli, Morsella, and Ruzzi. I consider a generalization of the prolate operator to the higher dimensional case and show that it admits a natural extension commuting with the truncated Fourier transform; this partly generalizes the one-dimensional result by Connes to the effect that there exists a natural selfadjoint extension to the full line commuting with the truncated Fourier transform.

Orateur: LONGO, Roberto (Università di Roma Tor Vergata)

ID de Contribution: **4** Type: **Non spécifié**

Ladder Costs for Random Walks in Lévy Random Media

lundi 26 juin 2023 14:00 (35 minutes)

We consider a random walk moving on a Lévy random medium, namely a one-dimensional renewal point process with i.i.d. inter-distances in the domain of attraction of a stable law. The focus is on the characterization of the law of the first-ladder height and length of the process. The study relies on the construction of a broader class of processes, denoted as Random Walks in Random Scenery on Bonds (RWRSB), consisting of a scenery which associates two random variables with each bond of the integer lattice Z (corresponding to the two possible crossing directions of that bond) and a random walk S on Z with i.i.d increments that collects the scenery values of the bond it traverses. Under suitable assumptions, we characterize the tail distribution of the sum of the scenery values collected up to the first-passage time in the positive semi-axis. This setting will be applied to obtain results for the laws of the first-ladder length and height of random walks in Lévy media, with an additional application in the context of the (generalized) Lévy-Lorentz gas. The main tools of investigation are a generalized Spitzer-Baxter identity and a suitable representation of the RWRSB in terms of local times of the random walk S. All these results are easily generalized to the entire sequence of ladder variables. (Joint work with Alessandra Bianchi and Giampaolo Cristadoro)

Orateur: POZZOLI, Gaia (University of Milano-Bicocca)

ID de Contribution: **5** Type: **Non spécifié**

Quantum Trajectories: Purification and Invariant Measures

lundi 26 juin 2023 14:45 (35 minutes)

Quantum trajectories model quantum systems repeatedly (indirectly) measured. The resulting evolution is a Markov process. In this talk I will discuss their purification property. Proved in 2005 by Kummerer and Maassen, it shows that, in absence of so called dark subspaces, the system state has a tendency to get closer and closer to a pure state along a quantum trajectories. I will revisit this result. I will explain how it is related to the stability of filters that led me and some collaborators to a proof of uniqueness of invariant measure. I will briefly mention the relationship to error correcting codes. I will then list a few new equivalent criteria for the absence of dark subspaces related to minima of polynomials, singular values and Lyapunov exponents. I will conclude with some interesting conjectures dealing with the likeliness of purification and zeros of some random functions.

Orateur: BENOIST, Tristan (CNRS - Institut Mathématiques de Toulouse)

ID de Contribution: **6** Type: **Non spécifié**

Sine-Gordon fields with non vanishing mass on Minkowski spacetime and equilibrium states

lundi 26 juin 2023 15:50 (35 minutes)

During this talk we shall discuss the construction of the massive Sine-Gordon field in the ultraviolet finite regime when the background is a two-dimensional Minkowski spacetime. The correlation functions of the model in the adiabatic limit will be obtained combining recently developed methods of perturbative algebraic quantum field theory with techniques developed in the realm of constructive quantum field theory over Euclidean spacetimes. More precisely, perturbation theory is used to represent interacting fields as power series in the coupling constant over the free theory. Adapting techniques like conditioning and inverse conditioning to spacetimes with Lorentzian signature, we shall see that these power series converge if the interaction Lagrangian has generic compact support. The latter observation implies also convergence in the strong operator topology in the GNS representations of states in which the system is analyzed. Finally, adapting the cluster expansion technique to the Lorentzian case, we shall see that the adiabatic limit of the correlation functions of the interacting equilibrium state at finite temperature (KMS state) is finite. The talk is based on a joint work with D. Bahns and K. Rejzner [arxiv.org:2103.09328]

Orateur: PINAMONTI, Nicola (Università di Genova)

ID de Contribution: **7** Type: **Non spécifié**

Some Thoughts on Approach to Equilibrium

lundi 26 juin 2023 16:35 (35 minutes)

Inspired by the successes of algebraic quantum statistical mechanics in dealing with some fundamental nonequilibrium questions, we investigate the relation between "approach to equilibrium" (sometimes called the Zeroth Law) and the Second Law. Short of being able to provide a new example of non-trivial and physically pertinent system approaching equilibrium, we bring some partial answers to a question raised by David Ruelle in 1967. This is a joint work with Vojkan Jaksic and Clément Tauber.

Orateur: PILLET, Claude-Alain (Université de Toulon)

ID de Contribution: **8** Type: **Non spécifié**

Adiabatic Time Evolution and Quasi-Static Processes in Translation-Invariant Quantum Systems

mardi 27 juin 2023 09:00 (45 minutes)

I will talk about slowly varying and non-autonomous quantum dynamics of a translation invariant spin or fermion system on the lattice. This system is assumed to be initially in thermal equilibrium, and we consider realizations of quasi-static processes in the adiabatic limit. By combining the Gibbs variational principle with the notion of quantum weak Gibbs states, I will present some general structural results regarding such realizations. In particular, such a quasi-static process is incompatible with the property of approach to equilibrium, presented in Claude-Alain's talk. This talk is based on a joint work with V. Jaksic and C.-A. Pillet.

Orateur: TAUBER, Clément (Université de Strasbourg)

ID de Contribution: **9** Type: **Non spécifié**

Adiabatic Evolution of Low-Temperature Many-Body Systems

mercredi 28 juin 2023 09:00 (45 minutes)

I will discuss the dynamics of short-ranged, weakly interacting fermionic lattice models, exposed to extensive perturbations slowly varying in time. We shall focus on the evolution of the expectation of local observables, starting from a positive temperature equilibrium state. At zero temperature, in the last years there has been important progress in the derivation of a many-body adiabatic theorem for gapped systems, uniformly in the size of the system. A corollary of this result is the validity of linear response. A limitation of the method is that it does not extend to positive temperatures, no matter how small. In this talk, I will discuss a representation via a convergent expansion for the evolution of the expectation of local observables, which implies the validity of a many-body adiabatic theorem for gapped systems at small positive temperature and the validity of linear response. "Small"means that the temperature has to vanish with the adiabatic parameter, uniformly in the size of the system. In particular, our setting covers the case in which the temperature is sent to zero after the thermodynamic limit. Our strategy is based on a rigorous version of the Wick rotation, that allows to represent the Duhamel expansion for the real-time dynamics in terms of Euclidean correlation functions, for which precise space-time decay estimates are proved using fermionic cluster expansion. Joint work with Rafael L. Greenblatt, Markus Lange and Giovanna Marcelli.

Orateur: PORTA, Marcello (SISSA, Trieste)

ID de Contribution: **10** Type: **Non spécifié**

The Adiabatic Wigner-Weisskopf Model

mardi 27 juin 2023 09:55 (45 minutes)

We consider a slowly varying time dependent d-level atom interacting with a photon field. Restricted to the single excitation atom-field sector, the model is a time-dependent generalization of the Wigner-Weisskopf model describing spontaneous emission of an atomic excitation into the radiation field. We analyze the dynamics of the atom and of the radiation field in the adiabatic and small coupling approximations, in various regimes. In particular, starting with an excited atomic state, we provide a description of both the radiative decay of the atom and of the buildup of the photon excitation in the field, and we discuss some properties of the effective evolution of the atom. This is joint work with Marco Merkli.

Orateur: JOYE, Alain (Université Grenoble Alpes)

ID de Contribution: **11** Type: **Non spécifié**

Quasi-classical Limit and Ultraviolet Renormalization in the Nelson Model

mercredi 28 juin 2023 09:55 (45 minutes)

We review the quasi-classical limit of the Nelson model, describing nucleons interacting with a scalar bosonic field, i.e., when the field degrees of freedom becomes classical while the nucleons retain their quantum nature. It is well known that such a model admits a simple energy renormalization of the ultraviolet divergence via the so-called dressing transformation. We then investigate the interplay between such a renormalization and the quasi-classical limit in both the stationary and dynamical pictures.

Orateur: CORREGGI, Michele (Politecnico di Milano)

ID de Contribution: **12** Type: **Non spécifié**

Quantum trajectory of the one atom maser model

mardi 27 juin 2023 11:00 (45 minutes)

The evolution of a quantum system undergoing repeated indirect measurements naturally leads to a Markov chain on the set of states, and which is called a quantum trajectory. When the system under consideration is finite dimensional, and under some natural assumption related to the non-existence of so-called dark subspaces, the state of the systems tends to become pure along the trajectory, a result which goes back to Kummerer and Maassen (’2006). This purification result is then a key step to prove uniqueness of the invariant measure (Benoist et al. 2019). In this talk I will present some results concerning purification and invariant measure for the quantum trajectory associated to the (infinite dimensional) one atom maser model. This talk is based on a joint work with T. Benoist and C. Pellegrini

Orateur: BRUNEAU, Laurent (CY Cergy Paris Université)

ID de Contribution: **13** Type: **Non spécifié**

The Euclidean Φ_2^4 Theory as the Limit of an Interacting Bose Gas

mercredi 28 juin 2023 11:00 (45 minutes)

Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. On the other hand, the nonlinear Schrödinger equation can be viewed a classical limit of many-body quantum theory. We are interested in the problem of the derivation of Gibbs measures as mean-field limits of Gibbs states in many-body quantum mechanics.

The particular case we consider is when the dimension $d = 2$ and when the interaction potential is the delta function, which corresponds to the Euclidean Φ_2^4 theory. The limit that we consider corresponds to taking the density to be large and the range of the interaction to be small in a controlled way. Our proof is based on two main ingredients:

An infinite-dimensional stationary phase argument, based on a functional integral representation. A Nelson-type estimate for a nonlocal field theory in two dimensions.

This is joint work with J. Fröhlich, A. Knowles, and B. Schlein.

Orateur: SOHINGER, Vedran (University of Warwick)

ID de Contribution: **14** Type: **Non spécifié**

Entropic Fluctuations in Quantum Two-time Measurement Framework

mardi 27 juin 2023 14:00 (35 minutes)

Non-equilibrium statistical mechanics has seen some impressive developments in the last three decades, thank to the pioneering works of Evans, Cohen, Morris and Searles on the violation of the second law, soon followed by the ground-breaking formulation of the Fluctuation Theorem by Gallavotti and Cohen for entropy fluctuation in the early nineties.The extension of these results to the quantum setting has turned out to be surprisingly challenging and it is still an undergoing effort. Kurchan's seminal work (2000) showed the measurement role has to be taken in account, leading to the introduction of the so called two-time measurement statistics (also known as full counting statistics). However introducing this frameworks leads to surprising phenomena with no classical counterpart. In this talk, I will present some work in progress, where we attempt to introduce a quantum equivalent of Gallavotti-Cohen (steady) entropic functional and compare it with the Evans-Searls (transient) entropic functional. We show that, due to the invasive measurement role, the situation differs considerably to its classical counterpart. We are able to obtain general results using functional and spectral analysis and operator algebras tools. Under more restrictive hypothesis, we can extend our analysis to the experimentally accessible indirect measurement framework (through an ancilla), using resonance analysis.

Joint work with T. Benoist, L. Bruneau, V. Jaksic, C.A.Pillet.

Orateur: PANATI, Annalisa (Université de Toulon)

ID de Contribution: **15** Type: **Non spécifié**

Return Times and Waiting Times as Entropy Estimators: History and Law of Large Numbers

mardi 27 juin 2023 14:45 (35 minutes)

Poincaré's theorem and Kac's lemma on recurrence are basic results that one typically encounters quite early on when studying dynamical systems. Perhaps less well known is a 1993 result of Ornstein and Weiss on the time R_n it takes for the n first symbols in a sequence sampled from an ergodic measure $\mathbb{P}$ on a one-sided shift to reappear down this same sequence: they build on ideas of Wyner and Ziv to equate, $\mathbb{P}$-almost surely, the growth rate in n of these return times to the entropy of the measure $\mathbb{P}$. In other words, there is a strong law of large numbers for $n^{-1} \ln R_n$ with limit $h(\mathbb{P})$. In this sense, observations of recurrence provide a universal entropy estimator. I will discuss analogous results for pairs of measures and their application to the theory of entropy production. These new results, obtained in collaboration with G. Cristadoro, M. Degli Esposti and V. Jaksic, hold under decoupling conditions on the measures involved.

Orateur: RAQUÉPAS, Renaud (Courant Institute, NYU)

ID de Contribution: **16** Type: **Non spécifié**

Return Times and Waiting Times as Entropy Estimators: Large Deviations

mardi 27 juin 2023 15:50 (35 minutes)

Once the sequence $(n^{-1} \ln R_n)$ of return times introduced in Renaud's talk has been shown to satisfy a law of large numbers, a natural question is to study its large deviations. Quite surprisingly, very limited results were available. In a recent paper with Renaud Raquépas, we proved that the return times satisfy the full large deviation principle, again under some quite mild decoupling assumptions. I will present this result and outline the proof. As we will show, we find a natural expression for the large-deviation rate function. Moreover, if the definition of R_n allows for some "overlaps", we will see that the rate function is nonconvex in general.

Orateur: CUNEO, Noé (Université Paris Cité)

ID de Contribution: **17** Type: **Non spécifié**

The Merhav-Ziv Cross Entropy Estimator: Beyond Stationary Markov Measures

mardi 27 juin 2023 16:35 (35 minutes)

Introduced in 1993 by Merhav and Ziv, the Merhav-Ziv estimator Q_n is an analogue of the well-known Lempel-Ziv estimator, which estimates the Cross Entropy of two unknown probability measures $\mathbb{P}$ and $\mathbb{Q}$. The algorithm takes as an input two strings y_1^n and x_1^n and does the following: it starts by considering the largest word y_1^m which appears inside x_1^n, then looks at the largest second word $y_{m+1}^{m'}$ which appears inside x_1^n and continues as such until the entire string y_1^n has been parsed into subwords. Q_n is then the number of parsed words created by this procedure. In their paper, Merhav and Ziv show the $\mathbb{P} \times \mathbb{Q}$ a.s convergence of $n^{-1}\log(n)Q_n$ to the cross entropy of $\mathbb{P}$ relative to $\mathbb{Q}$ under the seemingly restrictive assumption that both the probability measures are stationary Markov measures. Surprisingly, no rigorous generalisation of this result, covering more general measures, can be found. I will present the most recent generalisation of the result under fairly general decoupling assumption and talk about the next steps in getting the most general result we can hope for.

Orateur: GRONDIN, Raphaël (McGill University)