UNIVERSITY of York

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Local measurement theory for quantum fields

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Curved spacetimes, field theory and beyond. April 12 2024

Comm. Math. Phys. **378** (2020) 851 arXiv:1810.06512 - with R Verch; summary arXiv:1904.06944
Phys. Rev. D **103** (2021) 025017 arXiv:2003.04660 with H Bostelmann and M Ruep
Ann. H. Poincaré **24** (2023) 1137–1184 arXiv:2203.09529 with I Jubb and M Ruep
Measurement in quantum field theory, arXiv:2304.13356 with R Verch

What's the problem?

Measurement in quantum theory

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"A major scandal in the foundations of quantum physics" (Earman & Valente)

- The literature on QFT is almost silent about measurement!
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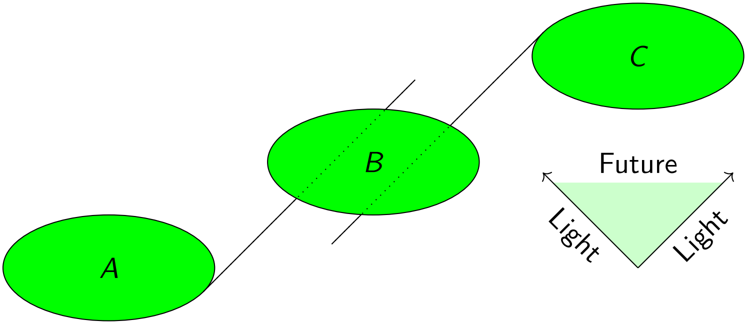
- Such literature as there is reveals more problems than solutions!

Impossible measurements Sorkin 1993



A and C are not causally connected, though A can influence B and B can influence C.

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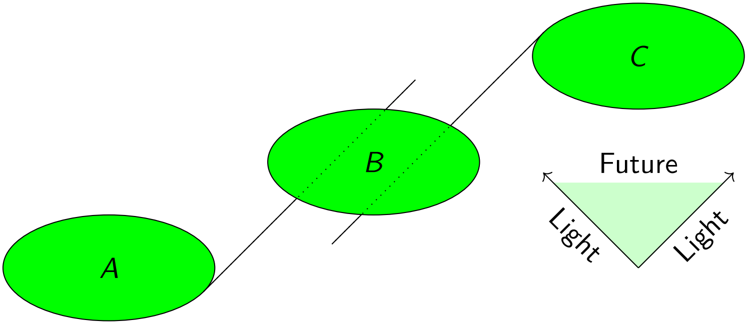
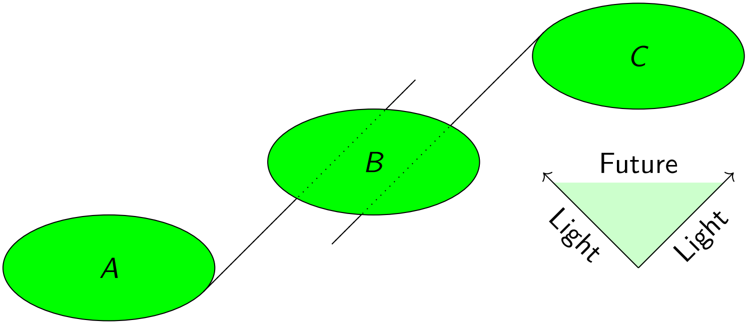


Claim: nonselective measurement of a typical observable B allows C to determine whether A has conducted a measurement – superluminal communication.

Presumably B represents an impossible measurement (spacetime extension is crucial).

Impossible measurements Sorkin 1993



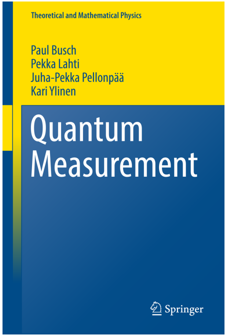
"[I]t becomes a priori unclear, for quantum field theory, which observables can be measured consistently with causality and which can't.

This would seem to deprive [QFT] of any definite measurement theory, leaving the issue of what can actually be measured to (at best) a case-by-case analysis"

What's the cure?

Operational approach CJF & Verch, 2018

Instead of constructing rules for QFT *de novo*, apply a systematic approach by modelling the measurement process, combining Quantum Measurement Theory with modern QFT in curved spacetimes



Describes measurement chain in QM

Little attention to QFT

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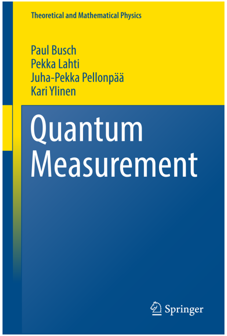

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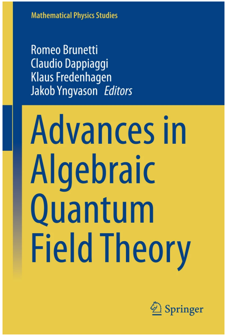
Conceptual framework for QFT

Little attention to measurement



Algebraic QFT – brief reminder (See arXiv:1904.04051 for a pedagogical intro)

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit, together with subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$. ($\mathcal{A}(\boldsymbol{M}; M) = \mathcal{A}(\boldsymbol{M})$)

Typical elements of $\mathcal{A}(\boldsymbol{M}; N)$ include smeared fields

$$\Phi(f) \in \mathcal{A}(\boldsymbol{M}; N) \qquad \text{if } f \equiv 0 \text{ outside } N$$

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Terms and conditions apply

- $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$ Isotony
- $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$ Timeslice
- ...

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Self-adjoint elements of $\mathcal{A}(\boldsymbol{M}; N)$ are interpreted as observables localisable in N.
An observable may be localisable in many distinct regions.

A state is a linear map $\omega : \mathcal{A}(\boldsymbol{M}) \to \mathbb{C}$ so that $\omega(\mathbf{1}) = 1$ and $\omega(A^*A) \geq 0\ \forall A \in \mathcal{A}(\boldsymbol{M})$.
Interpretation: $\omega(A)$ is the expectation value for measurements of A in state ω.

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NB No specific Lagrangian has been assumed.

Outline of the idea

Describe the system and probe by QFTs $\mathcal{A}$, $\mathcal{B}$ on spacetime $\boldsymbol{M}$ (globally hyperbolic).
$\mathcal{A}(\boldsymbol{M})$ = alg. of system observables on $\boldsymbol{M}$; $\mathcal{A}(\boldsymbol{M}; N)$ = subalgebra localisable in N.
Compare:

- the uncoupled combination $\mathcal{U}$ of $\mathcal{A}$ and $\mathcal{B}$

$$\mathcal{U}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M}; N) \otimes \mathcal{B}(\boldsymbol{M}; N)$$

- a coupled combination $\mathcal{C}$ with bounded coupling region K in spacetime.

Only assumption: $\mathcal{C}$ and $\mathcal{U}$ coincide 'outside' K.

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- a coupled combination $\mathcal{C}$ with bounded coupling region K in spacetime.

Only assumption: $\mathcal{C}$ and $\mathcal{U}$ coincide 'outside' K. Combining this assumption with spacetime geometry & standard AQFT rules, there are isomorphisms

$$\tau^{\pm} : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$$

reflecting the identifications between the two theories at early $(-)$ and late $(+)$ times.

The scattering map $\Theta = (\tau^{-})^{-1} \circ \tau^{+}$ is an automorphism of $\mathcal{U}(\boldsymbol{M})$. Details

Measurement scheme: prepare early, measure late

$\tau^{\pm}$ translate statements in 'uncoupled language' to the physical coupled system.

	Uncoupled	Coupled
Prepare system & probe independently at early times	$\omega \otimes \sigma$	$\omega_\sigma = (\omega \otimes \sigma) \circ (\tau^-)^{-1}$

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Description purely at system level: Seek induced observable $A \in \mathcal{A}(M)$ so that

$$\omega(A) = \underset{\sim}{\omega}_\sigma(\widetilde{B}) \qquad \text{(matching expectation values).}$$

Notation: $A = \varepsilon_\sigma(B)$.

Induced system observables

$\varepsilon_\sigma(B)$ is the system observable you learn about by measuring B on the probe.

- Explicit formula for $\varepsilon_\sigma(B)$ can be given in terms of Θ, σ and B,

$$\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B)), \qquad \text{where} \qquad \eta_\sigma(A \otimes C) = \sigma(C)A.$$

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  Can be computed in specific models.
- The induced observables are localisable in any suitable neighbourhood of K
- Probe observables localisable spacelike to K induce trivial observables.

A specific probe model

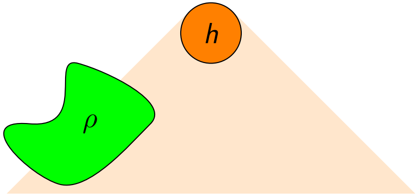
Two free scalar fields: Φ (system) and Ψ (probe) are coupled via an interaction term

$$\mathcal{L}_{\text{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

As formal power series in $h \in C_0^\infty(M^+)$,

$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma(e^{i\Psi(h^-)})e^{i\Phi(f^-)}$$

(f^- and $h^- - h$ vanish outside $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$).



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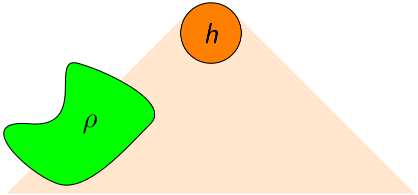

$$\begin{pmatrix} f^- \\ h^- \end{pmatrix} = \begin{pmatrix} 0 \\ h \end{pmatrix} - \begin{pmatrix} 0 & \rho \\ \rho & 0 \end{pmatrix} E^- \begin{pmatrix} 0 \\ h \end{pmatrix}$$

where E^- is the retarded Green function for the coupled system.

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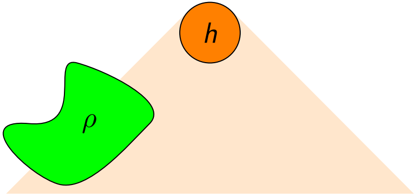

$$\begin{aligned}
\varepsilon_\sigma(\mathbf{1}) &= \mathbf{1} \\
\varepsilon_\sigma(\Psi(h)) &= \Phi(f^-) + \sigma(\Psi(h^-))\mathbf{1}, \\
\varepsilon_\sigma(\Psi(h)^2) &= \Phi(f^-)^2 + \sigma(\Psi(h^-))\Phi(f^-) + \sigma(\Psi(h^-)^2)\mathbf{1} \qquad \text{etc}
\end{aligned}$$

Asymptotic measurement schemes CJF, Jubb & Ruep 2022

Continuing with the coupled fields, replace ρ by $\lambda\rho$ and h by h/λ, taking $\lambda \to 0$.
(Walk softly and carry a big stick.)

$$\begin{pmatrix} f^- \\ h^- \end{pmatrix} = \begin{pmatrix} 0 \\ h \end{pmatrix} - \lambda \begin{pmatrix} 0 & \rho \\ \rho & 0 \end{pmatrix} E_\lambda^- \begin{pmatrix} 0 \\ h/\lambda \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ h \end{pmatrix} - \begin{pmatrix} 0 & \rho \\ \rho & 0 \end{pmatrix} E_0^- \begin{pmatrix} 0 \\ h \end{pmatrix}$$

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With a little ingenuity one can now design h and ρ to achieve a desired f^- in the limit.
Then use

$$e^{i\Phi(f_\lambda^-)} = \varepsilon_\sigma\left(\frac{e^{i\Psi(h/\lambda)}}{\sigma(e^{i\Psi(h_\lambda^-)})}\right)$$

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Further ingenuity extends this to arbitrary elements of the algebra of observables, both in $*$-algebra and Weyl algebra formulations.

Correlations of spacelike separated effects

Consider two probes $\mathcal{P}_A$ and $\mathcal{P}_B$ with spacelike separated coupling regions K_A and K_B.

Consider two effects E_A and E_B (yes/no observables) of the respective probe theories, localisable in spacelike separated regions.

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The observable recording success in both tests is the effect

$$E_A \otimes E_B \in \mathcal{P}_A(M) \otimes \mathcal{P}_B(M)$$

in the combined probe theory $\mathcal{P}_A \otimes \mathcal{P}_B$.

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Assuming the causal factorisation property $\Theta_{AB} = \hat{\Theta}_A \circ \hat{\Theta}_B$, one may compute

$$\varepsilon^{AB}_{\sigma_A \otimes \sigma_B}(E_A \otimes E_B) = \varepsilon^{A}_{\sigma_A}(E_A)\varepsilon^{B}_{\sigma_B}(E_B)$$

Consequently,

$$\omega(\varepsilon^{A}_{\sigma_A}(E_A)\varepsilon^{B}_{\sigma_B}(E_B))$$

is the joint success probability for the observables $\varepsilon^{A}_{\sigma_A}(E_A)$ and $\varepsilon^{B}_{\sigma_B}(E_B)$.

Remarks on Bell inequalities

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Example: CHSH inequality

$$\langle A_1(B_1 + B_2) + A_2(B_1 - B_2)\rangle \leq 2$$

for observables A_i spacelike separated from B_i, and $|A_i| \leq 1$, $|B_i| \leq 1$.

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Invoke:

- the existence of spacelike separated observables in QFT witnessing arbitrarily closely to maximal violation of Bell inequalities in the Minkowski vacuum state Summers & Werner
- asymptotic measurement schemes

to conclude that the measurement framework can exhibit close to maximal violation.

An algebra of observables for de Sitter Chandrasekaran, Longo, Penington & Witten 2023

Aim to find an algebra of observables for QFT 'gravitationally dressed' to the worldline of a observer following a geodesic in a static patch of de Sitter.

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- The observer is given by a simple QM clock for the worldline proper time
- Physical observables are defined as those joint observables of the clock & QFT that are invariant under the static flow on dS
- The resulting vN algebra is of type II_1 rather than the usual type III_1 of QFT $\implies$ there is a finite trace that can be used to define entropy.

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However, the motivation for the particular clock system used is unclear, and there is no real understanding of how the 'observer' actually observes the QFT.

Measurement schemes and QRF CJF, Janssen, Loveridge, Rejzner & Waldron, 2024

Our approach: start from the description of measurement theory in QFT

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- To determine the measurement scheme used, invoke a quantum reference frame covariant w.r.t. the isometries
- Physical observables are the invariant joint observables of the QRF and QFT
- Significant generalisation of CPLW
  - the clock is one of many systems that could be used
  - as in CPLW, the physical algebra is a compressed crossed product algebra
  - there is a semifinite trace that is finite if the QRF has good thermal properties

What about states?

State update rules CJF + Verch; CJF + Bostelmann & Ruep

Operational ideology

- The role of a state is to compute probabilities for measurement outcomes

$$\mathrm{Prob}(B;\omega) = \omega(B)$$

for effect B (yes/no measurement)

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(conditional probability for B, subsequent to a successful measurement of A).

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Operational ideology

- The role of … tcomes

It is not necessary to assume that the state actually changes.

The update rule conveniently does the book-keeping needed to compute the conditional probability, given additional knowledge from the A-measurement.

- The role … abilities for subsequent outcomes conditioned on the measurement result

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Properties of the update rule

Explicit formulae

$$\omega_A(C) = \frac{(\omega \otimes \sigma_A)(\Theta_A(C \otimes A))}{(\omega \otimes \sigma_A)(\Theta_A(\mathbf{1} \otimes A))} \qquad \omega_A^{\text{n.s.}}(C) = (\omega \otimes \sigma_A)(\Theta_A(C \otimes \mathbf{1}))$$

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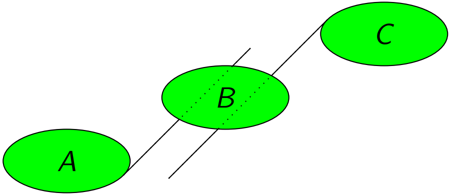
Unspooky 'action' at a distance $\omega_A(B) = \omega(B)$ iff B is uncorrelated with $\varepsilon_\sigma(A)$ in ω.

NB Correlations include those due to entanglement.

Impossible measurements resolved

Impossible measurements? Bostelmann, CJF & Ruep

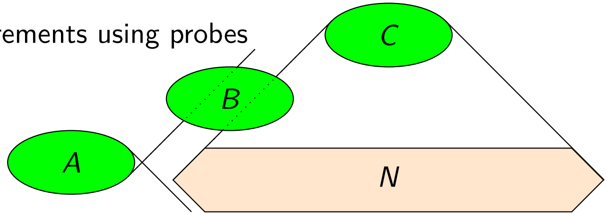


- Alice chooses whether to make a nonselective measurement
- Bob certainly makes a nonselective measurement
- Can Charlie determine whether Alice performed the measurement?

$$\omega_{AB}^{\text{n.s.}}(C) \overset{?}{\neq} \omega_{B}^{\text{n.s.}}(C)$$

Impossible measurements? Bostelmann, CJF & Ruep

Model A and B measurements using probes



Detailed investigation of locality properties and the geometric situation gives:

$$\hat{\Theta}_B C \otimes \mathbf{1} \otimes \mathbf{1} \in \mathcal{U}(\boldsymbol{M}; N) \quad \text{for a region } N \subset K_A^{\perp} \cap M_B^{-}$$

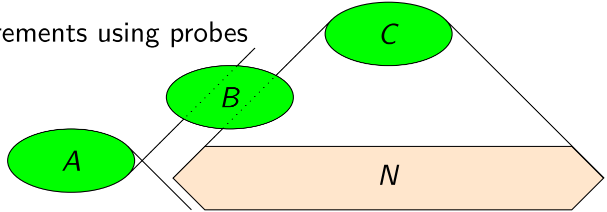
Theorem Charlie cannot determine whether Alice has measured:

$$\omega_{AB}^{\text{n.s.}}(C) = \omega_B^{\text{n.s.}}(C)$$

Proof by blissful ignorance.

Impossible measurements? Bostelmann, CJF & Ruep

Model A and B measurements using probes



The analysis shows that the measurement scheme is free of Sorkin-type pathologies.

Key assumption – the probes and couplings are described by physics respecting locality.

Impossible measurements can only be performed using impossible apparatus.

Impossible measurements – morals of the tale

- In our framework there are no impossible measurement pathologies and (at least in models) all local observables can be measured asymptotically.
- The problematic aspect of Sorkin's example is his update rule, assumed to be administered by a typical 'unitary kick' localisable in Bob's region.
  By contrast, we use state update rules derived from QFT.
- The same problem can occur in classical field theories Much & Verch
- An operator can be localisable without representing an operation that can be implemented using local physical interactions.
  Classifying those that can be is an interesting open problem.

A better [but less catchy] name might have been impossible updates.

Summary

- QFT has a consistent system of measurement schemes and update rules
- Fully consistent with relativity and curved spacetimes
- Allows for multiple observers, protects ignorance in all the right places
- Excludes 'impossible measurements' – all problematic aspects resolved!
- Is comprehensive as well as consistent.
- Clarifies the interpretation of AQFT: local algebra elements should be interpreted primarily as observables rather than operations.
- Based on QFT itself – derived from minimal, general assumptions.

UNIVERSITY of York

Engineering and Physical Sciences Research Council

Multiple causally orderable probes

Probes with coupling regions $K_1, \ldots, K_N$ are causally ordered if each K_{r+1} lies outside the causal past of K_r. There may be many compatible causal orderings.

Theorem Assume causal factorisation between probes. Then

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- (a) if effects $A_1, \ldots, A_{N+1}$ are measured by causally ordered probes,

$$\mathrm{Prob}(A_{N+1}|A_1 \& A_2 \& \cdots \& A_N; \omega) = \mathrm{Prob}(A_{N+1}; ((\omega_{A_1})_{A_2})_{\cdots A_N})$$

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- (b) if probes are coupled in causally ordered regions

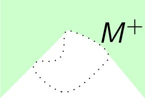
$$K_{A_1}, \ldots, K_{A_M}, K_B, K_{C_1}, \ldots, K_{C_N}$$

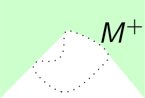
  and effects $A_1, \ldots, A_M$, $C_1, \ldots, C_N$ are measured without selection, then

$$\mathrm{Prob}(B; \omega) = ((\omega_{A_1}^{\text{n.s.}})_{A_2}^{\text{n.s.}})_{\cdots A_N}^{\cdots \text{n.s.}}(B)$$

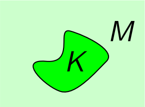
  which depends on the past measurements, but not on the future ones.

(Valid for all compatible causal orderings.)

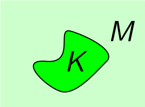




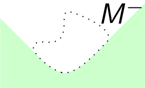
$(\mathcal{A} \otimes \mathcal{B})(M^+)$ $\mathcal{C}(M^+)$

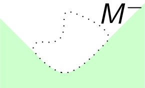


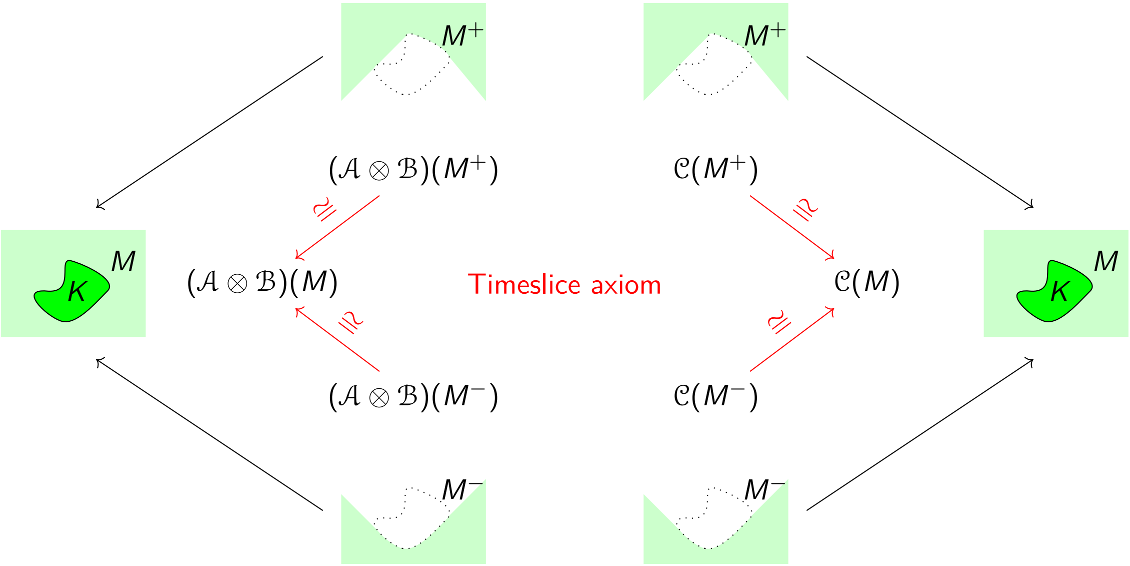
$(\mathcal{A} \otimes \mathcal{B})(M)$ $\mathcal{C}(M)$

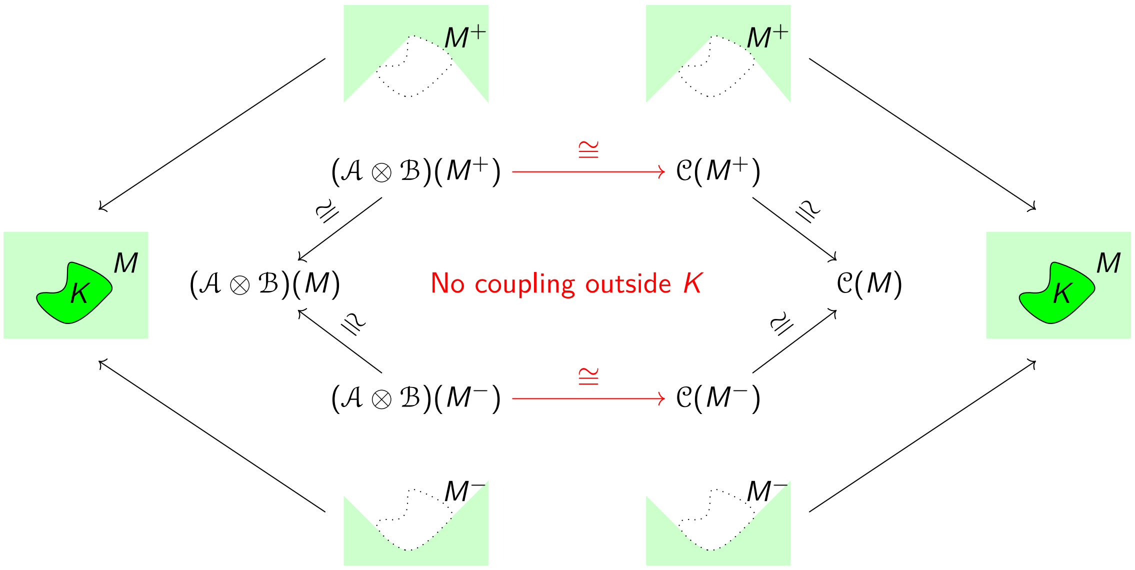


$(\mathcal{A} \otimes \mathcal{B})(M^-)$ $\mathcal{C}(M^-)$






M+
M+
(A ⊗ B)(M+)
C(M+)
≅
≅
M
K
(A ⊗ B)(M)
Timeslice axiom
C(M)
M
K
≅
≅
(A ⊗ B)(M−)
C(M−)
M−
M−


M^+
M^+
$(\mathcal{A}\otimes\mathcal{B})(M^+)$
$\cong$
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M
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$(\mathcal{A}\otimes\mathcal{B})(M)$
No coupling outside K
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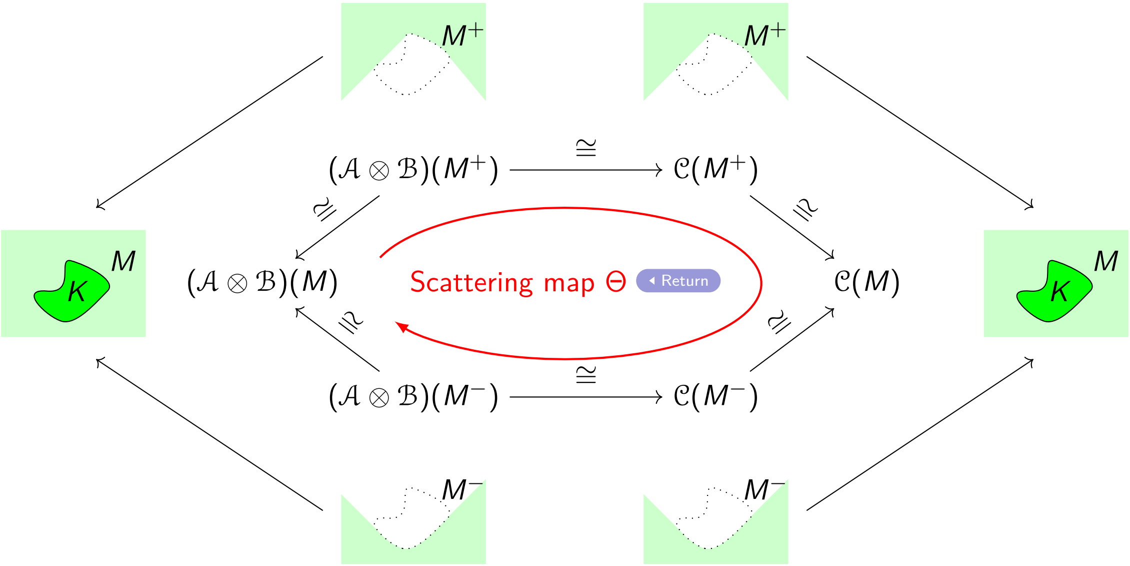
$\cong$
$(\mathcal{A}\otimes\mathcal{B})(M^-)$
$\cong$
$\mathcal{C}(M^-)$
M^-
M^-


M^+
M^+
$(\mathcal{A} \otimes \mathcal{B})(M^+)$
$\cong$
$\mathcal{C}(M^+)$
$\cong$
$\cong$
M
K
$(\mathcal{A} \otimes \mathcal{B})(M)$
Scattering map Θ
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$\cong$
$\cong$
$(\mathcal{A} \otimes \mathcal{B})(M^-)$
$\cong$
$\mathcal{C}(M^-)$
M^-
M^-