

Geometry, Topology and Discrete Symmetries Revealed by Deep Neural Networks

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A natural question at the intersection of universality efforts and manifold learning is the following: What kinds of architecture are universal approximators of maps between manifolds that are topologically interesting? A (low-dimensional) manifold hypothesis has been underlying the study of inverse problems ensuring Lipschitz stability, implying a like-wise hypothesis for data. This is used, for example, in inference through flows. By exploiting the topological parallels between locally bilipschitz maps, covering spaces, and local homeomorphisms, we find that a novel network of the form $p \circ E$, where E is an injective flow and p a coordinate projection, is a universal approximator of local diffeomorphisms between compact smooth (sub)manifolds embedded in Euclidean spaces. We show that the network allows for the computation of multi-valued inversion and that our analysis holds in the interesting case when the target map between manifolds changes topology and its degree is a priori not known. We also show that the network can be used, for example, in supervised problems for recovering the group action of a group invariant map if the group is finite, and in unsupervised problems by informing the choice of topologically expressive starting spaces in the generative case.

Orateur: Prof. DE HOOP, Maarten (Rice)