Conférence GAGC 2016 (en 2017)

Rapport sur les contributions

ID de Contribution: **5** Type: **Non spécifié**

Mini-cours : Birational geometry of moduli spaces

Overview: The ideal situation in moduli theory is to have a geometric/modular compactification for a given moduli space, and to have a good understanding of its structure so that one can compute various invariants (e.g. Betti numbers). Unfortunately, this is rarely the case (e.g. for varieties of general type, there is a geometric compactification, the so called KSBA compactification, but we have little understanding of the boundary objects, and even less is known about the global structure of the KSBA compactification). In recent years, starting with the VGIT theory of Thaddeus and Dolgachev-Hu, a different perspective appeared: It might be better to study the birational geometry of the moduli space itself. Namely, in certain cases, there might be two or more natural compactifications for a moduli space (one might be of interest as a geometric compactification, while another one might have a simple description). Then the interest is to understand (explicitly) the birational geometry relating these various models. More precisely, the goal is to decompose the birational maps relating various models of a moduli space into simple birational maps (simple flips and divisorial contractions), and then compute the invariants of interest via "wall-crossing" formulas.

Plan of the lectures: I will start by reviewing the basic story of moduli spaces and methods of constructing them (see [K] and [L2]). I will then briefly discuss the VGIT theory (see [L2]), and the variation of log canonical models for the moduli space of curves (aka Hassett-Keel program, see [HH]). I will then focus (probably half of my lecture time) on the emerging story of the variation models for moduli spaces of polarized K3 surfaces (aka Hassett-Keel-Looijenga program, see [LOG1, LOG2]).

REFERENCES:

General Moduli Theory

·[K] Kollár "Moduli of varieties of general type", Hanbook of moduli, vol II

·[L1] Laza "GIT and moduli with a twist", Handbook of moduli, vol II

·[L2] Laza "Perspectives on the construction and compactification of moduli spaces", in Compactifying moduii spaces, (Birkhauser, 2016)

Hassett-Keel Program

·[HH] Hassett-Hyeon "Log canonical models for the moduli space of curves: the first divisorial contraction". Trans. Amer. Math. Soc. 361 (2009)

·[FS] Fedorchuck-Smyth "Alternate compactifications of moduli spaces of curves", Handbook of Moduli, vol I

Hassett-Keel-Looijenga Program

·[Lo] Looijenga "Compactifications defined by arrangements II", Duke Math. J. 119 (2003)

·[LOG1] Laza-O'Grady "Birational geometry of the moduli space of quartic K3 surfaces", arXiv:1607.01324 (2016)

·[LOG2] Laza-O'Grady "GIT versus Baily-Borel compactification for K3's which are quartic surfaces or double covers of quadrics", arXiv:1612.07432 (2016)

Orateur: LAZA, Radu (Stony Brook University)

ID de Contribution: **6** Type: **Non spécifié**

Twisted Kodaira-Spencer classes and their use in the study of invariants of surfaces in $\mathbb{P}^4$

(joint work Igor Reider)

Let X be a projective surface. A twisted Kodaira-Spencer class is an element of the cohomology group $H^1(T_X(-D))$, with D "sufficiently positive". We study the connection between the existence of a non-trivial twisted class and the geometry of X. In particular, we show that, for a minimal general type surface satisfying $c_2/c_1^2 < 5/6$, the non-vanishing of $H^1(T_X(-K_X))$ imposes the existence of configurations of rational curves on the surface.

The techniques used to obtain this result are based on the interpretation of a non-trivial twisted class as an extension —a short exact sequence of locally free sheaves on X—, and on the detailed study of this sequence.

The above point of view and techniques are applied to the study of surfaces in $\mathbb{P}^4$. Indeed, a surface of non-negative Kodaira dimension contained in a hypersurface of degree ≤ 5 displays a natural non-trivial twisted class, allowing us to address the Hartshorne-Lichtenbaum problem for, and to slightly control the irregularity of these surfaces.

Orateur: NAIE, Daniel

ID de Contribution: **7** Type: **Non spécifié**

Enumeration of curves on K3 surfaces by polyhedral degenerations

Let (S, L) be a primitively polarized K3 surface, k an integer. Integral curves of geometric genus g in the linear system $|kL|$ form a family of dimension g (if non-empty).
One wants to count the number of such curves passing through g general points fixed on S.
Gromov-Witten theory provides a complete answer to this question when $k = 1$, but poses serious problems when$k > 1$. I shall present an approach based upon degenerating the surface S immersed by the system $|kL|$ in a union of planes incarnating a triangulation of the S^2 sphere.
This is a joint project with Ciro Ciliberto.

Orateur: DEDIEU, Thomas

ID de Contribution: **8** Type: **Non spécifié**

The eventual paracanonical map of an irregular variety.

I will report on joint work with. M.A. Barja (UPC, Barcelona) and L. Stoppino (Universita' dell'Insubria, Italy)

Given a map $a : X \longrightarrow A$ from a smooth projective variety to an abelian variety and a line bundle L on X, we study the "eventual" behaviour of the linear system $|L|$ under base change with the d-th multiplication map $A \longrightarrow A$. We prove a factorization theorem stating, roughly speaking, that the correponding map stabilizes for d large and divisible enough.
When X is of general type, A is the Albanese map and L is the canonical bundle, we obtain the so-called "eventual paracanonical" map, which is a new geometrical object intrinsically attached to X.

REFERENCES: arXiv: 1606.03301, arXiv: 1606.03290

Orateur: PARDINI, Rita

ID de Contribution: **9** Type: **Non spécifié**

On the unirationality of Hurwitz spaces

In this talk I will discuss about the unirationality of the Hurwitz spaces $H_{g,d}$ parametrizing d-sheeted branched simple covers of the projective line by smooth curves of genus g. I will summarize what is already known and formulate some questions and speculations on the general behaviour. I will then present a proof of the unirationality of $H_{12,8}$ and $H_{13,7}$, obtained via liaison and matrix factorizations. This is part of two joint works with Frank-Olaf Schreyer.

Les résultats dont je voudrais parler sont contenus dans les travaux "Matrix factorizations and curves in $\mathbb{P}^4$" (https://arxiv.org/abs/1611.03669) et "Unirational Hurwitz spaces and liaison" (en préparation).

Orateur: TANTURRI, Fabio

ID de Contribution: **10** Type: **Non spécifié**

Sur l'amplitude du cotangent des intersections complètes

C'est un travail commun avec Damian Brotbek. Nous prouvons que toute variété projective lisse M contient des sous-variétés avec cotangent ample en toute dimension $n \leq dim(M)/2$. Nous construisons de telles variétés comme certaines intersections complètes.

Orateur: DARONDEAU, Lionel

ID de Contribution: **11** Type: **Non spécifié**

Maximal representations of uniform complex hyperbolic lattices

Let Γ be a uniform complex hyperbolic lattice, that is, a discrete subgroup of the group of biholomorphisms $PU(n,1)$ of the ball B^n acting cocompactly on B^n. If ρ is a representation (a group homomorphism) of Γ in a semisimple Lie group of Hermitian type G, the Toledo invariant of ρ is a measure of the « complex size » of ρ. It is bounded by a quantity depending only on the real rank of G and the volume of the quotient $\Gamma\backslash B^n$. Maximal representations are those for which this bound is attained.
We show that if ρ is a maximal representation in a classical Lie group of Hermitian type G and if $n \geq 2$, then necessarily $G = SU(p,q)$ with $p \geq nq$, and there exist a ρ-equivariant holomorphic or antiholomorphic totally geodesic homothetic embedding of the ball B^n to the symmetric space associated to G. This implies that the representation ρ essentially extends to a homomorphism from the ambient Lie group $PU(n,1)$ to G.
The proof mixes the Higgs bundle theory associated to representations of Kähler groups and the dynamics and geometry of the tautological foliation on the projectivized tangent bundle of complex hyperbolic manifolds.
This is a joint work with Vincent Koziarz.

Orateur: MAUBON, Julien

ID de Contribution: **12** Type: **Non spécifié**

Perverse motives and mixed Hodge modules

Let X be a smooth complex algebraic variety. In this talk, I will explain a way to use perverse homology sheaves of families of algebraic varieties over X to extend Nori's construction of an Abelian category of motives to a relative setting.
This approach (which may also be applied to perverse sheaves over finite field) leads to a notion of motivic perverse sheaves and provides a mean to select, among mixed Hodge modules and their extensions, those coming from geometry.

I will also explain how the geometry of these specific perverse sheaves can be used to relate the theory of motives developed by Morel and Voevodsky to the derived category of mixed Hodge modules.

Orateur: IVORRA, Florian

ID de Contribution: **13** Type: **Non spécifié**

Finiteness results for real structures on rational surfaces

A real structure on a complex projective variety X is an antiregular (or antiholomorphic) involution. The data of such a structure on X is equivalent to the data of a real variety whose complexification is isomorphic to X (i.e. a real form of X). The aim of this talk is to show how the study of automorphism groups of rational surfaces can be used in order to give a partial answer to the question : does every rational surface have finitely many real forms (up to isomorphism)? On the one hand, we show that every rational surface whose automorphism group does not contain a nonabelian free group has finitely many real forms. On the other hand, we will show that there exist rational surfaces with large automorphism groups which also have finitely many real forms, like unnodal Coble surfaces studied by Cantat and Dolgachev, or KLT Calabi-Yau pairs.

Orateur: BENZERGA, Mohamed

ID de Contribution: **14** Type: **Non spécifié**

Liouville's inequality for transcendental points on projective varieties

Liouville inequality is a lower bound of the norm of an integral section of a line bundle on an algebraic point of a variety. It is an important tool in may proofs in diophantine geometry and in transcendence. On transcendental points an inequality as good as Liouville inequality cannot hold. We will describe similar inequalities which hold for "many" transcendental points and some applications

Orateur: GASBARRI, Carlo

ID de Contribution: **15** Type: **Non spécifié**

Mini-cours : Dynamical degrees of birational transformations of surfaces

The dynamical degree $\lambda(f)$ of a birational transformation f of a surface measures the exponential growth of the formula that define the iterates of f. It allows to study the complexity of the dynamic of f. For instance, over the field of complex number, the number $\log(\lambda(f))$ gives a upper bound for the topological entropy. The number $\lambda(f)$ is an algebraic integer of a special kind: a Pisot or Salem number. Its value gives informations on the transformation, for instance on the fact that it is conjugate or not to an automorphism of a projective surface. More generally, I will try to explain in this mini-course how a precise information on the dynamical degree allows to understand the geometry of the transformation, for instance its conjugacy class or its centraliser in the group of all birational transformations.

References:

-J. Diller and C. Favre, Dynamics of bimeromorphic maps of surfaces, Amer. J. Math. 123 (2001), no. 6, 1135–1169

-J. Blanc and S. Cantat, Dynamical degrees of birational transformations of projective surfaces. J. Amer. Math. Soc. 29 (2016), no. 2, 415–471.

Orateur: BLANC, Jérémy

ID de Contribution: **16** Type: **Non spécifié**

Mini-cours : Log Calabi-Yau varieties, degenerations of Calabi-Yau varieties, and mirror symmetry

I will try to provide some context and background for recent breakthroughs by Siebert and myself involving constructions of mirrors to various types of varieties. In particular, I will give a gentle introduction to logarithmic Gromov-Witten theory and explain how log GW invariants can be used to generalize constructions of Gross-Siebert and Gross-Hacking-Keel to give a very general mirror symmetry construction.

Suggested background reading is our survey/announcement paper https://arxiv.org/abs/1609.00624, and background on log geometry that should be useful for the talks can be found in Chapter 3 of my book, “Tropical geometry and mirror symmetry”.

Orateur: GROSS, Mark

ID de Contribution: **17** Type: **Non spécifié**

Mini-cours : The non-archimedean SYZ fibration

The SYZ conjecture gives a geometric description of the relation between mirror pairs of Calabi-Yau varieties. It was a fundamental insight of Kontsevich and Soibelman that the structures predicted by the SYZ conjecture can be found in the world of non-archimedean geometry (Berkovich spaces). I will explain some of the main ideas, as well as the connections with the minimal model program in birational geometry. If time permits, I will also discuss how these results have led to a proof of Veys's conjecture on poles of maximal order of p-adic zeta functions. These talks are based on joint work with Mircea Mustata and Chenyang Xu.

Reference. "Berkovich skeleta and birational geometry"in: M. Baker and S. Payne (eds.), Nonarchimedean and Tropical Geometry, Simons Symposia, pages 179-200 (2016), arXiv:1409.5229

Orateur: NICAISE, Johannes

ID de Contribution: **18** Type: **Non spécifié**

Mini-cours : Rationalité stable

Soit X une variété algébrique complexe, projective et lisse. On dispose de plusieurs notions pour déterminer si X est 'proche' à un espace projectif : la variété X est rationnelle si un ouvert de X est isomorphe à un ouvert d'un espace projectif, on dit que X est stablement rationnelle si cette propriété vaut en remplaçant X par un produit avec un espace projectif, enfin X est unirationnelle si X est rationnellement dominée par un espace projectif. Dans le problème classique de Lüroth on s'intéresse à trouver des exemples de variétés unirationnelles non rationnelles. Ce problème fut ouvert jusqu'à les années 1970, où trois séries d'exemples ont étés construites : les solides cubiques (Clemens et Griffiths), certaines solides quartiques (Iskovskikh et Manin), ainsi qu'une fibration en coniques (Artin et Mumford). Dans ce dernier exemple il s'agit d'une variété qui n'est même pas stablement rationnelle. Pour d'autres exemples la question de la stabilité rationnelle était ouverte.

Dans un travail récent C. Voisin montre qu'un solide double ramifié le long d'une quartique très générale n'est pas stablement rationnel. Inspirés par son travail, on montre que 'beaucoup' de solides quartiques ne sont pas stablement rationnelles (travail en commun avec J.-L. Colliot-Thélène). B. Totaro a ensuite établi qu'une hypersurface très générale de degré d n.est pas stablement rationnelle, si d/2 est au moins le plus petit entier supérieur à (n+2)/3.
Les mêmes méthodes ont permis d'établir que la rationalité n'est pas stable par déformation (travail en commun avec B. Hassett et Y. Tschinkel).

Dans ce mini-cours, on va présenter des méthodes pour obtenir les résultats ci-dessus : l'étude des propriétés universelles du groupe de Chow des zéro-cycles, la décomposition diagonale, ainsi que des méthodes de spécialisation.

References:

https://webusers.imj-prg.fr/~claire.voisin/Articlesweb/jdgvoisin.pdf;

http://math1.unice.fr/~beauvill/conf/Cime.pdf;

http://cims.nyu.edu/~pirutka/survey.pdf;

Orateur: PIRUTKA, Alena