

Mini-cours : Birational geometry of moduli spaces

Overview: The ideal situation in moduli theory is to have a geometric/modular compactification for a given moduli space, and to have a good understanding of its structure so that one can compute various invariants (e.g. Betti numbers). Unfortunately, this is rarely the case (e.g. for varieties of general type, there is a geometric compactification, the so called KSBA compactification, but we have little understanding of the boundary objects, and even less is known about the global structure of the KSBA compactification). In recent years, starting with the VGIT theory of Thaddeus and Dolgachev-Hu, a different perspective appeared: It might be better to study the birational geometry of the moduli space itself. Namely, in certain cases, there might be two or more natural compactifications for a moduli space (one might be of interest as a geometric compactification, while another one might have a simple description). Then the interest is to understand (explicitly) the birational geometry relating these various models. More precisely, the goal is to decompose the birational maps relating various models of a moduli space into simple birational maps (simple flips and divisorial contractions), and then compute the invariants of interest via “wall-crossing” formulas.

Plan of the lectures: I will start by reviewing the basic story of moduli spaces and methods of constructing them (see [K] and [L2]). I will then briefly discuss the VGIT theory (see [L2]), and the variation of log canonical models for the moduli space of curves (aka Hassett-Keel program, see [HH]). I will then focus (probably half of my lecture time) on the emerging story of the variation models for moduli spaces of polarized K3 surfaces (aka Hassett-Keel-Looijenga program, see [LOG1, LOG2]).

REFERENCES:

General Moduli Theory

- [K] Kollár “Moduli of varieties of general type”, Handbook of moduli, vol II
- [L1] Laza “GIT and moduli with a twist”, Handbook of moduli, vol II
- [L2] Laza “Perspectives on the construction and compactification of moduli spaces”, in Compactifying moduli spaces, (Birkhauser, 2016)

Hassett-Keel Program

- [HH] Hassett-Hyeon “Log canonical models for the moduli space of curves: the first divisorial contraction”. Trans. Amer. Math. Soc. 361 (2009)
- [FS] Fedorchuk-Smyth “Alternate compactifications of moduli spaces of curves”, Handbook of Moduli, vol I

Hassett-Keel-Looijenga Program

- [Lo] Looijenga “Compactifications defined by arrangements II”, Duke Math. J. 119 (2003)
- [LOG1] Laza-O’Grady “Birational geometry of the moduli space of quartic K3 surfaces”, arXiv:1607.01324 (2016)
- [LOG2] Laza-O’Grady “GIT versus Baily-Borel compactification for K3’s which are quartic surfaces or double covers of quadrics”, arXiv:1612.07432 (2016)

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