

Maximal representations of uniform complex hyperbolic lattices

Let Γ be a uniform complex hyperbolic lattice, that is, a discrete subgroup of the group of biholomorphisms $PU(n, 1)$ of the ball B^n acting cocompactly on B^n . If ρ is a representation (a group homomorphism) of Γ in a semisimple Lie group of Hermitian type G , the Toledo invariant of ρ is a measure of the « complex size » of ρ . It is bounded by a quantity depending only on the real rank of G and the volume of the quotient $\Gamma \backslash B^n$. Maximal representations are those for which this bound is attained.

We show that if ρ is a maximal representation in a classical Lie group of Hermitian type G and if $n \geq 2$, then necessarily $G = SU(p, q)$ with $p \geq nq$, and there exist a ρ -equivariant holomorphic or antiholomorphic totally geodesic homothetic embedding of the ball B^n to the symmetric space associated to G . This implies that the representation ρ essentially extends to a homomorphism from the ambient Lie group $PU(n, 1)$ to G . The proof mixes the Higgs bundle theory associated to representations of Kähler groups and the dynamics and geometry of the tautological foliation on the projectivized tangent bundle of complex hyperbolic manifolds. This is a joint work with Vincent Koziarz.

Orateur: MAUBON, Julien