Journée du GDR MEGA

Rapport sur les contributions

https://indico.math.cnrs.fr/e/9405

ID de Contribution: **1** Type: **Non spécifié**

Colloquium : Sur la théorie conforme des champs de Liouville (Amphi Schwartz)

vendredi 17 février 2023 14:00 (1 heure)

Les théories conformes des champs en dimension 2 sont apparues en physique
dans les années 80 pour décrire certaines limites d'échelles de modèles de physique statistique, comme Ising par exemple. Il s'agit de théories quantiques des champs particulières : elles contiennent de nombreuses symétries, les transformations conformes locales, et peuvent être parfois résolues par des outils de théorie de représentation. On expliquera ici comment utiliser des outils probabilistes pour définir la théorie de Liouville, une théorie conforme liée aux surfaces aléatoires, et comment calculer les corrélations de façon exacte sur toutes les surfaces. Ceci fait intervenir la méthode du « bootstrap conforme », en combinant des techniques d'analyse, de probabilité, d'espaces de Teichmüller et de représentations d'algèbre de Lie de dimension infinie.

Orateur: GUILLARMOU, Colin

ID de Contribution: **2** Type: **Non spécifié**

Interpolation between random matrices and free operators.

vendredi 17 février 2023 10:30 (1h 30m)

We shall start by introducing the problems that this method was first designed to solve, notably the question of strong convergence of a family of random matrices. Indeed if one considers a polynomial in those random matrices, one can wonder how the spectrum of the resulting matrix behaves when the dimension goes to infinity. In particular, a recurring question is whether the operator norm converges. After having defined some usual notions of Free Probability, we will talk about free stochastic calculus, which is an analogue of classical stochastic calculus where we replaced the notion of independence by the one of "freeness". We will then explain how to use this tool to compute some highly non trivial quantities in Random Matrix Theory. In particular, this let us compute so-called topological expansions, i.e. considering the trace of a functional in our random matrices, one can get a Taylor series of its expectation in terms of the dimension of the matrices. We will study the case of GUE as well as Haar unitary random matrices.

Orateur: FÉLIX, Parraud

ID de Contribution: **3** Type: **Non spécifié**

Estimation of (I_n - XDX^T)^{-1} with large dependence and concentration hypotheses.

vendredi 17 février 2023 15:30 (1 heure)

We provide in this presentation an estimation of the expectation of the matrix Q(D) = (I_p - XDX^T)^{-1} when the data matrix X = (x_1, …,x_n) \in M_{p,n} has independent columns (but not identically distributed) and D is random, bounded, not independent with X but satisfies some constraints on the dependence on each x_i: for any i, there exists a random diagonal matrix D_i independent of x_i and sufficiently close to D. The formula giving the estimation of Q is a classical generalization of known deterministic equivalents, the difficulty mainly lies in the proof of the convergence. It is proven under concentration of the measure hypotheses on X and it relies in particular on a formula giving the concentration of the product of such random vectors. In a sense, the study of Q(D) is a perfect example to expose the efficiency of the concentration of measure framework to prove random matrix theory inferences.
We will also provide a machine learning application of the estimation of the matrix Q(D) that concerns the prediction of the performances of robust regularized regression (Ridge regression with a general convex loss replacing the squared loss).

Orateur: LOUART, Cosme