Asymptotic Behavior of systems of PDE arising in physics and biology: theoretical and numerical points of view

2nd edition

June 15$^{\mathrm{th}}$-17$^{\mathrm{th}}$ 2016 – Lille

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June 15th-17th 2016 – Lille
M2 Building, 1st floor

Wednesday

9:30	Welcome
10:10	**A. Vasseur** *Global Existence of Solutions to the 3D Navier-Stokes Equations with Degenerate Viscosities*
10:55	Coffee break
11:30	**M. Ribot** *A (mainly numerical) study of a hyperbolic model for chemotaxis*
12:15	**X. Lhebrard** *Modelling and numerical approximation for the nonconservative bi-temperature MHD model*
12:50	Lunch
14:35	**M. Lemou** *A multiscale numerical approach for a class of time-space oscillatory problems*
15:20	**D. Kazerani** *Global existence for small data of the viscous Green-Naghdi equations*
15:55	Coffee break
16:30	**M. Pierre** *Convergence to equilibrium for gradient-like systems with analytic features*
17:05	**H. Mathis** *Numerical convergence rate for the diffusive limit of the p-system with damping*

Thursday

8:45	Coffee
9:30	**G. Dimarco** *Implicit-explicit linear multistep methods for stiff kinetic equations*
10:15	**F. Charles** *From particle methods to hybrid semi-Lagrangian schemes*
11:00	Coffee break
11:30	**M. Bessemoulin** *Exponential decay of a finite volume scheme to the thermal equilibrium for drift-diffusion systems*
12:15	**M. Postel** *Dimensional reduction of a multiscale model based on long time asymptotics*
12:50	Lunch
14:40	**K. Fellner** *Global existence and large-time behaviour for reaction-diffusion models*
15:25	**C. Mifsud** *Asymptotic analysis for a simplified model of model of dynamical perfect plasticity*
16:00	Coffee break
16:00	Poster session
19:30	Gala dinner

Friday

8:45	Coffee
9:30	**S. Nicaise** *Stability results of dissipative systems via the frequency domain approach*
10:15	**E. Franck** *Uniform asymptotic preserving scheme for hyperbolic systems in 2D*
10:50	Coffee break
11:25	**J. ten Thije Boonkkamp** *Complete flux schemes for conservation laws of advection-diffusion-reaction type*
12:10	**Y. Ma** *Non linear stability of Minkowski space-time with massive scalar field*
12:45	Lunch
14:40	**R. Carles** *Time splitting methods and the semi-classical limit for nonlinear Schrödinger equations*
15:25	**B. Nazaret** *Entropy methods for degenerate diffusions and weighted functional inequalities*

Poster session

A. Ait-Hammou-Oulhaj
Design and analysis of CVFE scheme for Richards equation

J. Berendsen
Gamma Convergence of a Cross Diffusion System with Nonlocal Attraction

A. Bondesan
Numerical scheme for a kinetic model of mixtures with diffusion limit

M. Herda
A finite volume scheme for boundary-driven convection-diffusion equations with relative entropy structure

S. Zermani
Existence and asymptotic behavior of positives solutions of semilinear elliptic system

A. Zhang
Several optimal functional inequalities for fractional operators

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Abstracts

Design and analysis of CVFE scheme for Richards equation

Thu. June 16$^{\text{th}}$
16:00 – 18:00

A. Ait Hammou Oulhaj
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Unsaturated flow in porous media ($\Omega \subset \mathbb{R}^2$) is described by the following Richards equation:

$$\left\{ \begin{array}{lll} \partial_t s(p) - \nabla \cdot (\eta(s(p))\Lambda(\nabla p - \mathbf{g})) = 0 & \text{in} & \Omega \times (0, t_f), \\ s(p)_{|t=0} = s_0 & \text{in} & \Omega, \\ \eta(s(p))\Lambda(\nabla p - \mathbf{g}) \cdot \mathbf{n} = 0 & \text{on} & \partial\Omega \times (0, t_f), \end{array} \right. \tag{1}$$

where p is the pressure, and the saturation s is a nondecreasing function which takes values into $[0, 1]$. The intrinsic permeability $\Lambda : \Omega \to \mathcal{M}_2(\mathbb{R})$ is symmetric and uniformly elliptic, but possibly anisotropic. $\mathbf{g}$ is the gravity acceleration. The mobility function η is a nondecreasing function which satisfies $\eta(0) = 0$ and $\eta(s) > 0$ if $s \neq 0$. Thus (1) is a degenerate parabolic problem.

We propose a nonlinear Control Volume Finite Element scheme (CVFE) in order to approximate the solution of (1). In particular the diffusion terms are discretized by means of a conforming piecewise linear finite element method on a primal triangular mesh, and the others terms are discretized by means of an upwind finite volume method on a barycentric dual mesh. This scheme is based on a suitable upwinding of the mobility, which allows the nonpostive transmissibility coefficients. It is a generalization to the Richards equation of the scheme studied in [1].

The CVFE scheme has some remarkable properties. In particular it permits to handle an anisotropic permeability contrary to the scheme proposed in [2]. First of all, we prove the nonlinear stability of the scheme thanks to an energy estimate, that there exists (at last) one discrete solution and that the saturation belongs to the interval $[0, 1]$. Moreover, the convergence of the method is proved as the discretization steps tend to 0. Finally, some numerical experiments on isotropic and anisotropic cases illustrate the efficiency of the method.

This is a joined work with C. Chainais-Hillairet and C. Cancès.

References

[1] C. Cancès and C. Guichard, *Convergence of a nonlinear entropy diminishing control volume finite element scheme for solving anisotropic degenerate parabolic equations*, Math. Comp. **85**, pp. 549-580 (2016).

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Thu. June 16$^{\text{th}}$
16:00 – 18:00

Gamma Convergence of a Cross Diffusion System with Nonlocal Attraction

J. Berendsen

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We consider a non-linear cross-diffusion system featuring non-local attraction forces. We prove existence of weak solutions $r, b \in [0, 1]$. Furthermore, we consider the Gamma-convergence of the corresponding entropy functional in the limit of vanishing diffusion along the parameter $\varepsilon > 0$. We show that the functional does indeed converge to the one considered by Novaga et al. Finally, numerical experiments show that we also observe phase separation for $\varepsilon > 0$.

Thu. June 16$^{\text{th}}$
11:30 – 12:15

Exponential decay of a finite volume scheme to the thermal equilibrium for drift-diffusion systems

M. Bessemoulin

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We are interested in the large-time behavior of a numerical scheme discretizing drift-diffusion systems for semiconductors. The considered scheme is finite volume in space, and the numerical fluxes are a generalization of the classical Scharfetter-Gummel scheme, which allows to consider both linear or nonlinear pressure laws.

We study the convergence of approximate solutions towards an approximation of the thermal equilibrium state as time tends to infinity, and obtain a decay rate by controlling the discrete relative entropy with the entropy production. This result is proved under assumptions of existence and uniform-in-time L^∞ estimates for numerical solutions, which will be discussed.

This is a joined work with Claire Chainais-Hillairet.

Complete flux schemes for conservation laws of advection-diffusion-reaction type

Fri. June 17^{th}
11:25 – 12:10

J.H.M. ten Thije Boonkkamp

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Complete flux schemes are recently developed numerical flux approximation schemes for conservation laws of advection-diffusion-reaction type; see e.g. [1,2]. The basic complete flux scheme is derived from a local one-dimensional boundary value problem for the entire equation, including the source term. Consequently, the integral representation of the flux contains a homogeneous and an inhomogeneous part, corresponding to the advection-diffusion operator and the source term, respectively. Suitable quadrature rules give the numerical flux.

For time-dependent problems, the time derivative is considered a source term and is included in the inhomogeneous flux, resulting in an implicit semi-discretisation. The implicit system proves to have much smaller dissipation and dispersion errors than the standard semidiscrete system, especially for dominant advection.

Just as for scalar equations, for coupled systems of conservation laws, the complete flux approximation is derived from a local system boundary value problem, this way incorporatin the coupling between the constituent equations in the discretization. Also in the system case, the numerical flux (vector) is the superpostion of a homogeneous and an inhomogeneous component, corresponding to the advection-diffusion operator and the source term vector, respectively. The scheme is applied to multi-species diffusion and satisfies the mass constraint exactly.

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Thu. June 16th
16:00 – 18:00

Numerical scheme for a kinetic model of mixtures with diffusion limit

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We consider a mixture composed with $N \geq 2$ monoatomic ideal gases in a diffusive setting. The evolution of the mixture can be modelled by the multi-species Boltzmann equations

$$\varepsilon\, \partial_t f_i^\varepsilon + v \cdot \nabla_x f_i^\varepsilon = \frac{1}{\varepsilon} \sum_{j=1}^{N} Q_{ij}(f_i^\varepsilon, f_j^\varepsilon) \text{ on } [0, +\infty) \times \Omega \times \mathbb{R}^3 . \tag{2}$$

Under well-prepared assumptions on the evolution of the density functions f_i^ε and using a standard method of moments, it is possible to prove that in the diffusion limit the macroscopic quantities associated to the system (2) satisfy the Maxwell-Stefan system of coupled equations (see [1] and [2] for a more detailed description).

Using a similar idea to the one presented in [3] and [4], we develop a suitable numerical scheme to describe the long-time behaviour of the evolution of a multi-component mixture, both in the kinetic ($\varepsilon = \mathcal{O}(1)$) and in a more diffusive regime ($\varepsilon \ll 1$).

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Time splitting methods and the semi-classical limit for nonlinear Schrödinger equations

Fri. June 17$^{\text{th}}$
14:40 – 15:25

R. Carles

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We consider the time discretization based on Lie-Trotter splitting, for the nonlinear Schrodinger equation, in the semi-classical limit, with initial data under the form of WKB states. Both the exact and the numerical solutions keep a WKB structure, on a time interval independent of the Planck constant. We prove error estimates, which show that the quadratic observables can be computed with a time step independent of the Planck constant. We give a flavor of the functional framework, based on time-dependent analytic spaces.

From particle methods to hybrid semi-Lagrangian schemes

Thu. June 16$^{\text{th}}$
10:15 – 11:00

F. Charles

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Particle methods for transport equations consist in pushing forward particles along the characteristic lines of the flow, and to describe then the transported density as a sum of weighted and smoothed particles. Conceptually simple, standard particle methods have the main drawback to produce noisy solutions or to require frequent remapping.

In this talk we present two classes of particle methods which aim at improving the accuracy of the numerical approximations with a minimal amount of smoothing.

The idea of the Linearly Transformed Particle method is to transform the shape functions of particles in order to follow the local variation of the flow. This method has been adapted and analyzed for the Vlasov- Poisson system and for a compressible aggregation equation. In both cases the error estimate is improved compared to classical particle methods, with the gain of a strong convergence of the numerical solution.

However, for long remapping periods, shapes of particles could become to much stretched out. The second method solve this problem of locality by combining a backward semi-Lagrangian approach and local linearizations of the flow. The convergence properties are improved and validated by numerical experiments. This is a joint work with Martin Campos-Pinto (LJLL, UPMC).

Thu. June 16th
9:30 – 10:15

Implicit-explicit linear multistep methods for stiff kinetic equations

G. Dimarco

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We consider the development of high order asymptotic-preserving linear multistep methods for kinetic equations and related problems. The methods are first developed for BGK-like kinetic models and then extended to the case of the full Boltzmann equation. The behavior of the schemes in the Navier-Stokes regime is also studied and compatibility conditions derived. We show that, compared to IMEX Runge-Kutta methods, the IMEX multistep schemes have several advantages due to the absence of coupling conditions and to the greater computational efficiency. The latter is of paramount importance when dealing with the time discretization of multidimensional kinetic equations.

Thu. June 16th
14:40 – 15:25

Global existence and large-time behaviour for reaction-diffusion models

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Systems of nonlinear reaction-diffusion equations are encountered frequently as models in chemistry, physics, populations dynamics and biology. However, due to the lack of comparison principles for general reaction-diffusion systems, already the existence of global weak/classical solutions poses many open problems, in particular in 3D.

In the absence of comparison principles, so called duality methods have recently proven to be one of the most powerful tools in obtaining global solutions for nonlinear reaction-diffusion systems.

The first part of this talk will present recent advances and results concerning the existence of global solutions via duality methods. The second part of the talk will then consider reaction-diffusion systems, which feature an entropy functional and discuss the convergence to equilibrium states with computable rates for large classes of such reaction-diffusion models.

Uniform asymptotic preserving scheme for hyperbolic systems in 2D

Fri. June 17$^{\text{th}}$
10:15 – 10:50

E. Franck

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In this work, we are interested by the discretization of hyperbolic system with stiff source term. Firstly we consider a simple linear case : the damped wave equation which can be approximative by a diffusion equation at the limit. For this equation we propose a asymptotic preserving scheme which converge uniformly on general and unstructured 2D meshes contrary to the classical extension of the AP which are not consistent in the limit regime on unstructured meshes. After that we propose to extend this method to a nonlinear problem: the Euler equations with friction. At the end the link with the well-balanced scheme (for Euler-Poisson) will be introduced.

A finite volume scheme for boundary-driven convection-diffusion equations with relative entropy structure

Thu. June 16$^{\text{th}}$
16:00 – 18:00

M. Herda

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We propose a finite volume scheme for a class of nonlinear parabolic equations endowed with non-homogeneous Dirichlet boundary conditions and which admit relative entropy functionals. For this kind of models including the porous media equations, Fokker-Planck equations for plasma physics or dumbbell models for polymer flows, it has been proved that the transient solution converges to a steadystate when time goes to infinity. The present scheme is built from the resolution of the stationary equation in order to preserve steady-states and natural Lyapunov functionals which provide a satisfying long-time behavior. After describing the numerical scheme, we present several numerical results which confirm the accuracy and underline the efficiency to preserve the large-time asymptotic.

Wed. June 15th
15:20 – 15:55

Global existence for small data of the viscous Green-Naghdi equations

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We consider the Cauchy problem for the Green-Naghdi equations with viscosity, for small initial data. It is well-known that adding a second order dissipative term to a hyperbolic system leads to the existence of global smooth solutions, once the hyperbolic system is symmetrizable and the so-called Kawashima-Shizuta condition is satisfied. We first show that the Green-Naghdi equations can be written in a symmetric form, using the associated Hamiltonian. This system being dispersive, in the sense that it involves third order derivatives, the symmetric form is based on symmetric differential operators. Then, we use this structure for an appropriate change of variable to prove that adding viscosity effects through a second order term leads to global existence of smooth solutions, for small data. We also deduce that constant solutions are asymptotically stable.

Wed. June 15th
14:35 – 15:20

A multiscale numerical approach for a class of time-space oscillatory problems

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High oscillations may arise in many physical problems: Schrödinger equations, kinetic equations, or more generally high frequency waves. In this talk, we will present a general strategy that allows the construction of uniformly (with respect to the oscillation frequency) accurate numerical schemes in the following situations:

- time oscillations with applications to kinetic and Schrödinger equations.
- time-space oscillations with applications to some high frequency waves and semi-classical quantum models.

Some numerical tests will be presented to illustrate the efficiency of the strategy.

Modelling and numerical approximation for the nonconservative bi-temperature MHD model

Wed. June 15^{th}
12:15 – 12:50

X. Lhebrard

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In order to achieve inertial confinement fusion, one has to improve the knowledge of the laser-plasma interaction. There exists two main ways of describing this phenomenon, the microscopic (kinetic) approach and the macroscopic (hydrodynamic) approach. The kinetic approach is not competitive since it is too expensive in computational time. This is why we investigate an intermediate model in thermal nonequilibrium, which is between the kinetic model and the hydrodynamic model.

In the first stage of the confinement the magnetic field is negligible, the relevant intermediate model is than the nonconservative bitemperature Euler model. Recently in [1], an entropic approximation of this system has been derived thanks to numerical schemes based on an underlying conservative kinetic model.

However in the last stage of the confinement the target is penetrated by relativistic electrons, which induces a strongly variable magnetic field. This is why we want to deal with an intermediate model which takes into account the magnetic field.

In this work we propose to study a bitemperature MHD model. This system consists in four conservation equations for mass, impulsion and magnetic field and two nonconservation equations, that is to say, one for each energy. Physically, the model describes the interaction of a mixture of one species of ions and one species of electrons in thermal nonequilibrium subjected to a transverse variable magnetic field.

A first result is to have been able to established the hydrodynamic model from an underlying kinetic model. More precisely, using an out of equilibrium Chapman-Enskop procedure, the bitemperature MHD model is constructed from a BGK model coupled with Maxwell equations with full Lorentz force, which includes the magnetic field.

Finally, we approximate the weak solutions of the bitemperature MHD model using a kinetic scheme, based on the underlying kinetic model.

This is a joint work with S. Brull and B. Dubroca.

References

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Fri. June 17$^{\text{th}}$
12:10 – 12:45

Non linear stability of Minkowski space-time with massive scalar field

Y. Ma
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In this talk we will present some recent work about the system of Einstein equation coupled with a massive scalar field and the system of $f(R)$ field equation (partially published in [2]). More precisely, on the nonlinear global stability of the Minkowski space-time within these two similar contexts. In a PDE point of view, they are equivalent to the global existence of a special class of quasi-linear wave-Klein-Gordon system with small initial data.

To the author's knowledge there is not so much choice to deal with this kind of system (for a detailed explication of the major difficulty, see for example in [1] page 2), and we apply the "hyperboloidal foliation method" introduced by the author in [1] combined with some newly developed tools such as L^∞ estimates on Klein-Gordon equations in curved space-time and L^∞ estimates on wave equations based on the expression of spherical means. We also adapt some tools developed in classical framework for the analysis of Einstein equation into our hyperboloidal foliation framework, such as the estimates based on wave gauge conditions and the L^∞ estimates on wave equations based on integration along characteristics.

References

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Wed. June 15$^{\text{th}}$
17:05 – 17:40

Numerical convergence rate for the diffusive limit of the p-system with damping

H. Mathis
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We are interested in the study of the diffusive limit of the p-system with damping and its approximation by an Asymptotic Preserving (AP) Finite Volume scheme. Provided the system is endowed with an entropy-entropy flux pair, we give the convergence rate of classical solutions of the p-system with damping towards the smooth solutions of the porous media equation using a relative entropy method. Adopting a semi-discrete scheme, we establish that the convergence rate is preserved by the approximated solutions. Several numerical experiments illustrate the relevance of this result.

Asymptotic analysis for a simplified model of model of dynamical perfect plasticity

Thu. June 16$^{\text{th}}$
15:25 – 16:00

C. Mifsud
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In this talk, we will present an initial boundary value problem for a hyperbolic system under constraints, coming from mechanics. To study the solutions of such a system, we will use a viscous approach that relaxes the constraints. We will explain the asymptotic analysis, when the viscous parameter tends to zero, which leads to an interaction between the boundary condition and the constraints for the constrained system. If time permits, we will show some numerical results.

Entropy methods for degenerate diffusions and weighted functional inequalities

Fri. June 17$^{\text{th}}$
15:25 – 16:00

B. Nazaret
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We will present results on large time asymptotics for some fast diffusion equations with power law weights. We will show that, for such diffusions, new phenomena appear: the asymptotic rates of convergence, obtained by linearization, are not global, the underlying functional inequalities may experience symmetry breaking and the Barenblatt self-similar profiles is not optimal.

Stability results of dissipative systems via the frequency domain approach

Fri. June 17$^{\text{th}}$
9:30 – 10:15

S. Nicaise
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The frequency domain approach goes back to J. Prüss [7] and F. L. Huang [4] that show that a C_0 semigroup $(e^{tA})_{t\geq 0}$ of contractions in a Hilbert space H is exponentially stable if and only if the resolvent of A is uniformly bounded on the imaginary axis. Afterwards Z. Liu and B. Rao [5], C. J. K. Batty and T. Duyckaerts [2], and A. Bátkai, K.-J. Engel, J. Prüss and R. Schnaubelt [1] have given some sufficient conditions on the behavior of the resolvent of A on the imaginary axis that guarantee an almost polynomial decay of the semigroup. Finally an optimal result about the polynomial decay was found by A. A. Borichev and Yu. V. Tomilov [3]. This approach is a powerful tool for the study of the decay of the semigroup associated with concrete dissipative systems since it reduces to the study of the resolvent on the imaginary axis.

In our talk, we will first recall these two results and then illustrate them on two particular dissipative systems, namely a generalized telegraph equation [6] and a dispersive medium model (joint work with C. Scheid).

References

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Wed. June 15$^{\text{th}}$
16:30 – 17:05

Convergence to equilibrium for gradient-like systems with analytic features

M. Pierre

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A celebrated result of S. Lojasiewicz states that every bounded solution of a gradient flow associated to an analytic function converges to a steady state as time goes to infinity. Convergence rates can also be obtained. These convergence results have been generalized to a large variety of finite or infinite dimensional gradient-like flows. The fundamental example in infinite dimension is the semilinear heat equation with an analytic nonlinearity. In this talk, we show how some of these results can be adapted to time discretizations of gradient-like flows, in view of applications to PDEs such as the Allen-Cahn equation, the sine-Gordon equation, the Cahn-Hilliard equation, the Swift-Hohenberg equation, or the phase-field crystal equation.

Dimensional reduction of a multiscale model based on long time asymptotics

Thu. June 16^{th}
12:15 – 12:50

M. Postel

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Depending on their velocity field, some models lead to moment equations that enable one to compute monokinetic solutions economically. We detail the example of a multiscale structured cell population model, consisting of a system of 2D transport equations. The reduced model, a system of 1D transport equations, is obtained by computing the moments of the 2D model with respect to one variable. The 1D solution is defined from the solution of the 2D model starting from an initial condition that is a Dirac mass in the direction removed by reduction. Long time properties of the 1D model solution are obtained in connection with properties of the support of the 2D solution for general case initial conditions. Finite volume numerical approximations of the 1D reduced model can be used to compute the moments of the 2D solution with proper accuracy. The numerical robustness is studied in the scalar case, and a full scale vector case is presented.

A (mainly numerical) study of a hyperbolic model for chemotaxis

Wed. June 15^{th}
11:30 – 12:15

M. Ribot

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The aim of this talk is to give some first results on the behaviour of the solutions of a 1D hyperbolic type chemotaxis system, based on incompressible Euler equation. More precisely, I will completely describe the stationary solutions with vacuum for this system and I will study numerically the stability of these steady states after the presentation of an adapted numerical scheme. A comparison with a limit parabolic system will also be performed.

Wed. June 15th
10:10 – 10:55

Global Existence of Solutions to the 3D Navier-Stokes Equations with Degenerate Viscosities

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We prove the existence of global weak solutions for 3D compressible Navier-Stokes equations with degenerate viscosities. The method is based on the Bresch and Desjardins entropy. The solutions are obtained as limits of the quantic Navier-Stokes system. The main contribution is to derive the Mellet-Vasseur type inequality for the weak solutions, even if it is not verified by the first level of approximation. This provides existence of global solutions in time, for the compressible Navier-Stokes equations, for any γ bigger than one, in three dimensional space, with large initial data, possibly vanishing on the vacuum. This is a joint work with Cheng Yu. The paper will appear in *Inventiones*.

Thu. June 16th
16:00 – 18:00

Existence and asymptotic behavior of positives solutions of semilinear elliptic system

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Thu. June 16th
16:00 – 18:00

Several optimal functional inequalities for fractional operators

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This paper is devoted to optimal functional inequalities for fractional operators on the sphere. Based on spectral estimates, subcritical inequalities are established, with a consequence on the entropy for a heat-type flow. These subcritical inequalities interpolate between fractional Sobolev and fractional logarithmic Sobolev inequalities, and their optimal constants are given by a spectral gap. Our method also provide a remainder terms in the subcritical range. We also consider the interpolation between the fractional logarithmic Sobolev and fractional Poincare inequalities. Finally, weighted inequalities of Caffarelli-Korn-Nirenberg type involving the fractional Laplacian are obtained in the Euclidean space, using the stereographic projection and scaling trick.
