Probabilistic Analysis of LLL-based decoder of Interleaved Chinese Remainder Codes

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LIRMM

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Chinese Remainder Theorem $N = \prod_{i=1}^{n} p_i$

$$\begin{aligned} \mathbb{Z}_N &\longrightarrow \mathbb{Z}_{p_1} \times \ldots \times \mathbb{Z}_{p_n} \\ a &\longmapsto ([a]_{p_1}, \ldots, [a]_{p_n}) \end{aligned}$$

Chinese Remainder Theorem & Redundancy

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Redundancy

If we know that $0 \leq a < K < N \Rightarrow$
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Example

$$N = 3 \cdot 5 \cdot 7 = 105$$
$$K = 3 \cdot 5 = 15$$
$$a = 6 \leftrightsquigarrow (0, 1, 6) \in \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_7$$

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6 is the only integer $a \in [0, 14]$ with

$$\begin{cases} [a]_3 = 0 \\ [a]_5 = 1 \end{cases}$$

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Chinese Remainder code

Let $p_1 < \ldots < p_n$, let $k < n$ thus $K = \prod_{i=1}^{k} p_i < N = \prod_{i=1}^{n} p_i$.

$$\mathcal{C} = CR(N, K) := \{([C]_{p_1}, \ldots, [C]_{p_n}) : 0 \leq C < K\}$$

Evaluation - Interpolation codes

	$A = \mathbb{F}_q[x]$: Reed-Solomon codes	$A = \mathbb{Z}$: Chinese Remainder codes
	$f \in \mathbb{F}_q[x]_{<k}$	$C \in [0, K)$
Encoding	$\pi_{(x-\alpha_1)} \quad \dots \quad \pi_{(x-\alpha_n)}$	$\pi_{p_1} \quad \dots \quad \pi_{p_n}$
	$\vec{c} = (f(\alpha_1), \dots, f(\alpha_n))$	$\vec{c} = ([C]_{p_1}, \dots, [C]_{p_n})$
Channel	$\vec{e} = (e_1, \dots, e_n)$	$\vec{e} = (e_1, \dots, e_n)$
	$\vec{r} = (r_1, \dots, r_n) \leftrightarrow R(x) \in \mathbb{F}_q[x]_{<n}$	$\vec{r} = (r_1, \dots, r_n) \leftrightarrow R \in \mathbb{Z}_N$
Decoding		
	$f(x) \in \mathbb{F}_q[x]_{<k}$	$C \in [0, K)$

Polyalphabetic code? Different metric!

$$CR(N, K) := \pi_{p_1} \times \ldots \times \pi_{p_n}([0, K)) \subseteq \mathbb{Z}_{p_1} \times \ldots \times \mathbb{Z}_{p_n}$$

Weighted Distance

$\Lambda_{r,c} = \prod_{i:r_i \neq c_i} p_i$ Error locator, $d_{r,c} = \log_2(\Lambda_{r,c}) = \sum_{i:r_i \neq c_i} \log_2(p_i)$

Reed Solomon codes	Chinese Remainder codes
Linear	Not Linear
monoalphabetic	polyalphabetic
Hamming metric	Weighted distance
MDS: $d = n - k + 1$	$d > \log_2\left(\frac{N}{K}\right)$

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Burst Errors Channels

$$\ldots \left| c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5} \right| c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}, c_{2,5} \left| c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5} \right| \ldots$$

Interleaving

Burst Errors Channels (length bursts $\ell \approx 3$)

$$\ldots \left| c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5} \right| c_{2,1}, \cancel{c_{2,2}}, \cancel{c_{2,3}}, \cancel{c_{2,4}}, c_{2,5} \left| c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5} \right| \ldots$$

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$$\dots \left| c_{1,1} \right.$$

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$$\ldots \left| c_{1,1}, c_{2,1} \right.$$

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$$I_\ell(\mathcal{C}) = \left\{ \begin{pmatrix} \cdots & \vec{c}_1 & \cdots \\ \cdots & \vec{c}_2 & \cdots \\ & \vdots & \\ \cdots & \vec{c}_\ell & \cdots \end{pmatrix} : \vec{c}_i \in \mathcal{C} \right\}$$

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- Localized errors on common coordinates
- Increases the decoding radius beyond d_u
- Involves a failure probability analysis

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Decoding of Interleaved CR codes [LSN13][1]

$0 \le C_i < K$

$$(C_1, \dots, C_\ell) \mapsto \begin{pmatrix} [C_1]_{p_1} & \cdots & [C_1]_{p_n} \\ [C_2]_{p_1} & \cdots & [C_2]_{p_n} \\ & \vdots & \\ [C_\ell]_{p_1} & \cdots & [C_\ell]_{p_n} \end{pmatrix} \xrightarrow[\text{Channel}]{\overbrace{\begin{pmatrix} e_{1,1} & \cdots & e_{1,n} \\ & \vdots & \\ e_{\ell,1} & \cdots & e_{\ell,n} \end{pmatrix}}^{t \text{ columns} \sim \mathcal{U}\left(\mathbb{Z}_{p_i}^{\ell}\right)}} \begin{pmatrix} [R_1]_{p_1} & \cdots & [R_1]_{p_n} \\ [R_2]_{p_1} & \cdots & [R_2]_{p_n} \\ & \vdots & \\ [R_\ell]_{p_1} & \cdots & [R_\ell]_{p_n} \end{pmatrix} \in D_{\boldsymbol{C},\xi_r}$$

$$\Lambda = \prod_{i \in \xi_r} p_i$$

[1] Li, Wenhui, Vladimir Sidorenko, and Johan SR Nielsen. "On decoding interleaved chinese remainder codes." 2013 IEEE International Symposium on Information Theory. IEEE, 2013.

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$$\Lambda C_i = \Lambda R_i \mod N \quad \begin{cases} \psi_i = \Lambda C_i \\ \varphi = \Lambda \end{cases} \quad \psi_i = \varphi R_i \mod N \qquad (\varphi, \psi_1, \dots, \psi_\ell) \in \mathcal{L} = \begin{pmatrix} 1 & R_1 & \cdots & R_\ell \\ 0 & N & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & N \end{pmatrix} \subseteq \mathbb{Z}^{\ell+1}$$

[1] Li, Wenhui, Vladimir Sidorenko, and Johan SR Nielsen. "On decoding interleaved chinese remainder codes." 2013 IEEE International Symposium on Information Theory. IEEE, 2013.

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$0 \le C_i < K$

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Channel

$$\Lambda = \prod_{i \in \xi_r} p_i$$

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$\Lambda \le 2^\tau$

Short elements of $\mathcal{L}$

↓

$$(\Lambda, \Lambda C_1, \ldots, \Lambda C_\ell) \in S_R = \{(\varphi, \psi_1, \ldots, \psi_\ell) \in \mathcal{L} : 0 < \varphi < 2^\tau, |\psi_i| < 2^\tau K\} \subseteq \mathcal{L}$$

LLL

[1] Li, Wenhui, Vladimir Sidorenko, and Johan SR Nielsen. "On decoding interleaved chinese remainder codes." 2013 IEEE International Symposium on Information Theory. IEEE, 2013.

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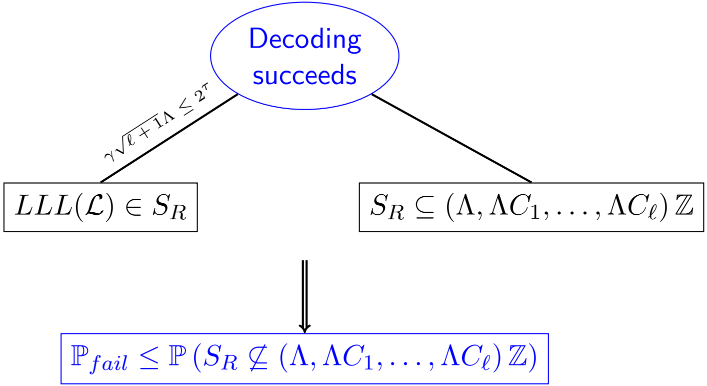
Idea of the analysis

$$(\Lambda, \Lambda C_1, \ldots, \Lambda C_\ell) \in S_R = \{(\varphi, \psi_1, \ldots, \psi_\ell) \in \mathcal{L} : 0 < \varphi < 2^\tau, |\psi_i| < 2^\tau K\} \subseteq \mathcal{L}$$



Our Result [AAGL][2]

Theorem (Decoding ICR codes)

Set

$$d_{\max} := \frac{\ell}{\ell+1}\left[\log(N) - \log(K) - \log\left(6\gamma\sqrt{\ell+1}\right)\right].$$

Choose $d_t < d_{\max}$, *set* $\tau_t := d_t + \log\left(\gamma\sqrt{\ell+1}\right)$.
Consider $R \sim D_{C,\mathcal{E}_r}$ *such that* $\log \Lambda_r \leq d_t$.
Then the decoding algorithm on random input R *outputs the center codeword* C *of the distribution* $D_{C,\mathcal{E}_r}$, *with a probability of failure* $\mathbb{P}_f$ *upper-bounded by*

$$\mathbb{P}_f \leq 2^{-(\ell+1)(d_{\max}-d_t)} + \exp\left(\frac{n}{p_1^{\ell-1}}\right) - 1.$$

[2] Abbondati, Matteo, et al. "Probabilistic Analysis of LLL-based Decoder of Interleaved Chinese Remainder Codes." ITW 2023-IEEE Information Theory Workshop. 2023.

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Rational Extensions of Evaluation-Interpolation Codes [P][3]

Field of fractions of $\mathbb{F}_q[x]$: $\mathbb{F}_q(x)$

RF Codes

$$\mathbb{F}_q(x)$$
$$\cup|$$
$$\left\{ \frac{f}{g} \in \mathbb{F}_q(x) : d^o(f) < d_f, d^o(g) < d_g, \gcd(f,g) = 1 \right\}$$

$$ev_{\alpha_1} \quad \ldots \quad ev_{\alpha_n}$$

$$\vec{c} = \left(\frac{f}{g}(\alpha_1), \ldots, \frac{f}{g}(\alpha_n) \right)$$

Field of fractions of $\mathbb{Z}$: $\mathbb{Q}$

RN Codes

$$\mathbb{Q}$$
$$\cup|$$
$$\left\{ \frac{f}{g} \in \mathbb{Q} : |f| < F, 0 < g < G, \gcd(f,g) = 1 \right\}$$

$$ev_{p_1} \quad \ldots \quad ev_{p_n}$$

$$\vec{c} = ([f]_{p_1}[g]^{-1}_{p_1}, \ldots, [f]_{p_1}[g]^{-1}_{p_n})$$

[3] Pernet, Clément. High performance and reliable algebraic computing. Diss. Université Joseph Fourier, Grenoble 1, 2014.

State of the Art

	Previously	Us
IRN	$\emptyset$	$d < \frac{\ell}{\ell+1}\left[\log(N) - \log(FG) - \log\left(6\gamma\sqrt{\ell+1}\right)\right]$ $\mathbb{P}_f \leq 2^{-(\ell+1)(d_{\max}-d_t)} + \exp\left(\frac{n}{p_1^{\ell-1}}\right) - 1$
IRF	$t < \frac{\ell}{\ell+1}(n - d_f - d_g + 1)$ $\mathbb{P}_f \leq \frac{d_g+t}{q}$ [GLZ][4]	$t < \frac{\ell}{\ell+1}(n - d_f - d_g + 1)$ $\mathbb{P}_f \leq \frac{1}{q}\frac{1}{q^{(\ell+1)(t_{max}-t)}}\exp\left(\frac{t}{q}\right) + \frac{2t}{q^\ell}$

[4] Guerrini, E., Lebreton, R., & Zappatore, I. (2020). Enhancing simultaneous rational function recovery: adaptive error correction capability and new bounds for applications. arXiv preprint arXiv:2003.01793.

Thank you for your attention!