Secure Computations on Shared Polynomials and Application to Private Set Operations

Lucas Ottow

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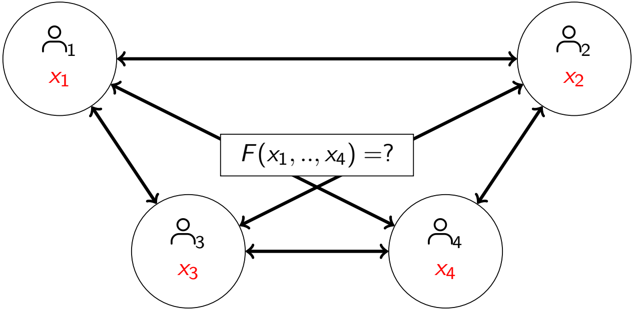
Multiparty Computation



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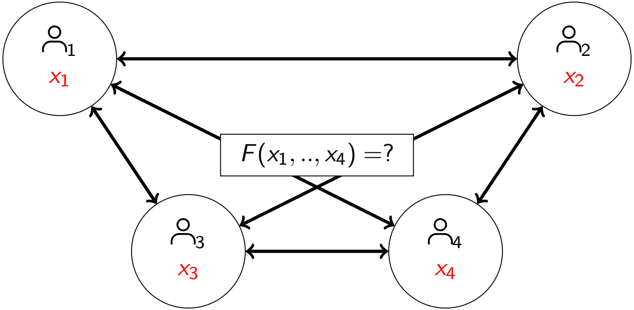
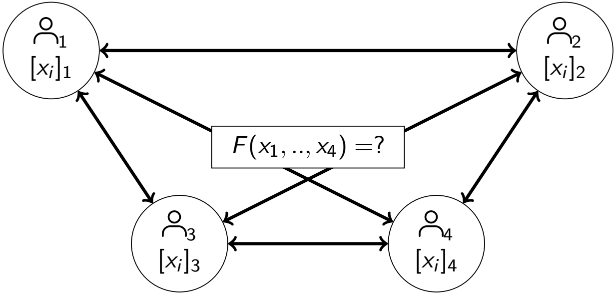


- General result by Yao ('82)

Secret Sharing in MPC



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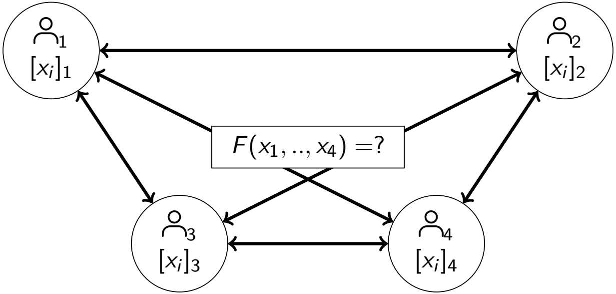

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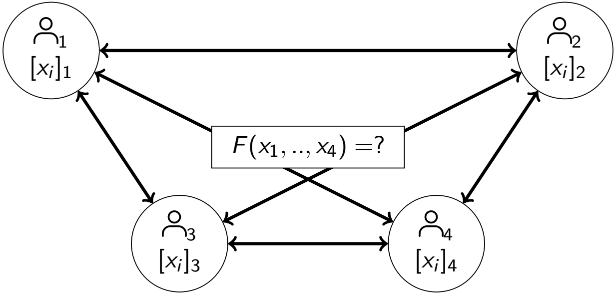

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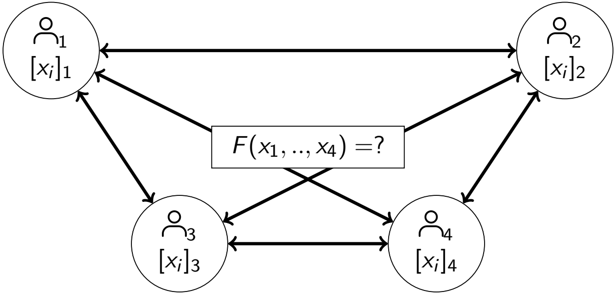

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- Our goal: less secure multiplications (sec. mult.) + constant round

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  - Application to privacy preserving set operations.

Our method: an example

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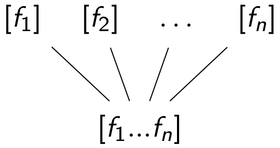

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$1 \times \mathcal{O}(n^2 d)$ sec. mult

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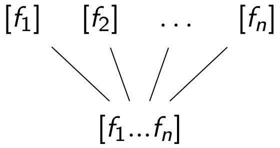

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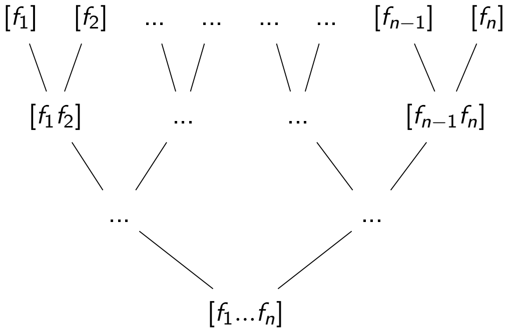

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$\log n \times \mathcal{O}(nd)$ sec. mult

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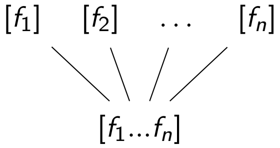

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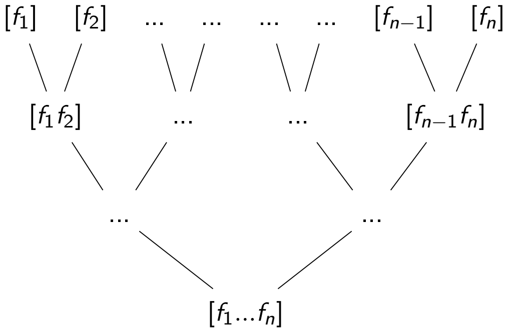

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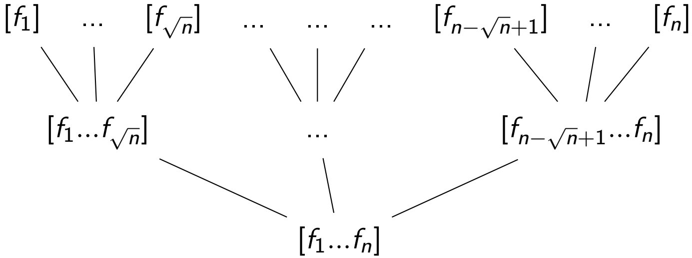

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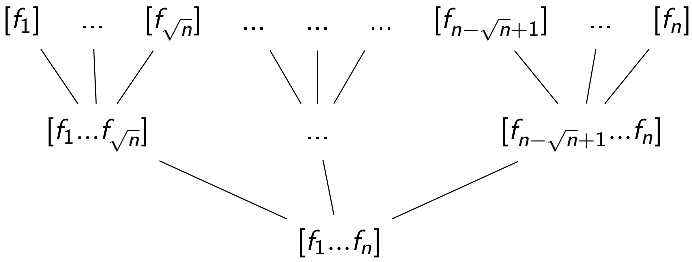

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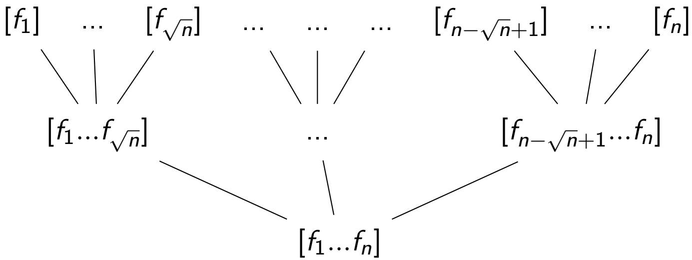

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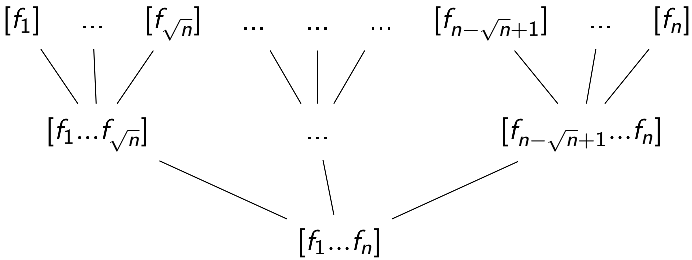

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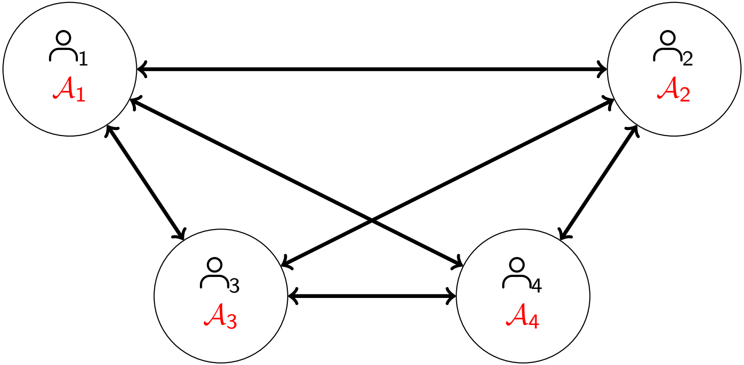
- Generalization: $\mathcal{O}(\tau)$ rounds and $\mathcal{O}(\tau n^{1+1/\tau} d)$ sec. mult.

Privacy Preserving operations



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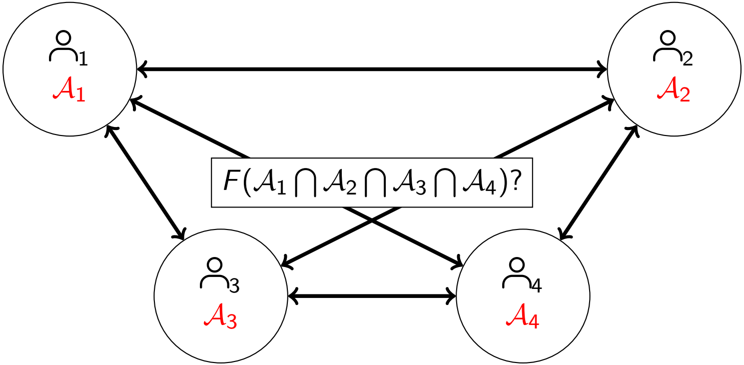

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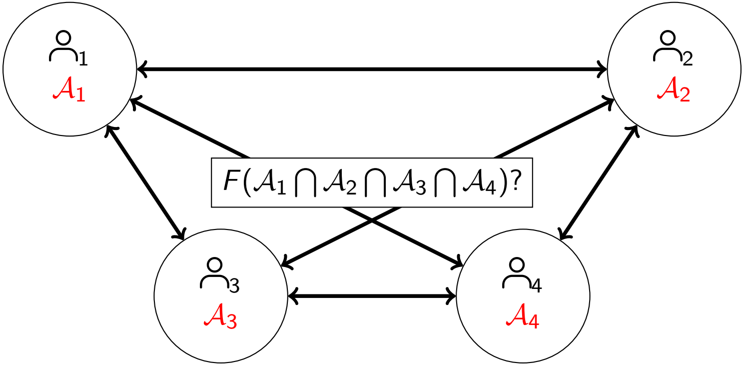

- Example: social network, investigations, ...

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- Main idea: $\mathcal{A}_j$ represented as shared polynomials: $P_j = \prod_{\alpha \in \mathcal{A}_j}(X - \alpha)$.
- Use our evaluation techniques.
- 1st protocol: probabilistic Private Disjointess Test/PSI-emptiness, $\mathcal{O}(mn + \tau n^{1+1/\tau})$
- 2nd protocol: Many other problems without error in $\mathcal{O}(mn \log \log q + \tau mn^{1+1/\tau})$ (using techniques from Damgård et al. (TCC'06))

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Thank you for your attention!