# OBLIVIOUS LWE SAMPLING IN QUANTUM POLYNOMIAL TIME

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# Sampling hard instances of lattices

**Short Integer Solution (SIS)** [Ajtai 96]:

$$\mathbf{A} \sim \mathcal{U}(\mathbb{Z}_q^{m\times n})$$

Given $\mathbf{A}$, find a short vector $\mathbf{x}$ such that $\mathbf{A}^T\mathbf{x} = \mathbf{0} \mod q$

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Don't know the secret

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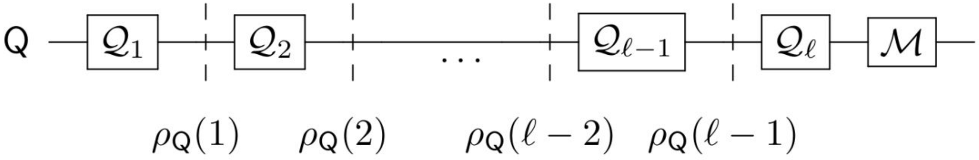
**Lattice Knowledge Assumption**: if $\mathcal{A}$ is an LWE sampler, it is witness-aware.

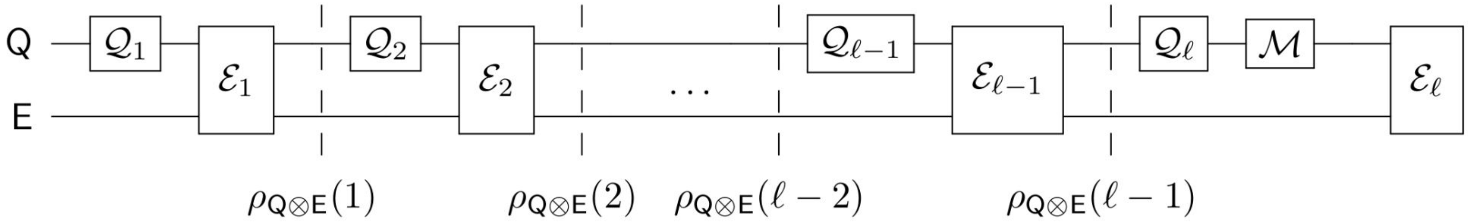
**Our contribution**: A quantum witness-oblivious LWE sampler

Several cryptographic protocols are built based on the lattice knowledge assumption [LMS11,GMNO18,ISW21].

# OBLIVIOUSNESS

# What is an aware/oblivious sampler?



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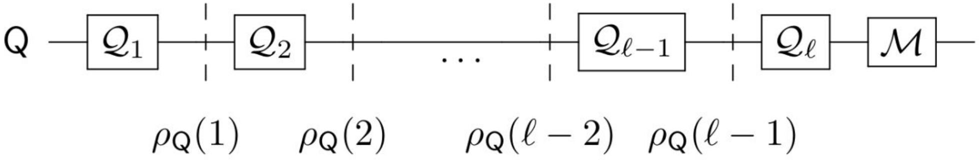


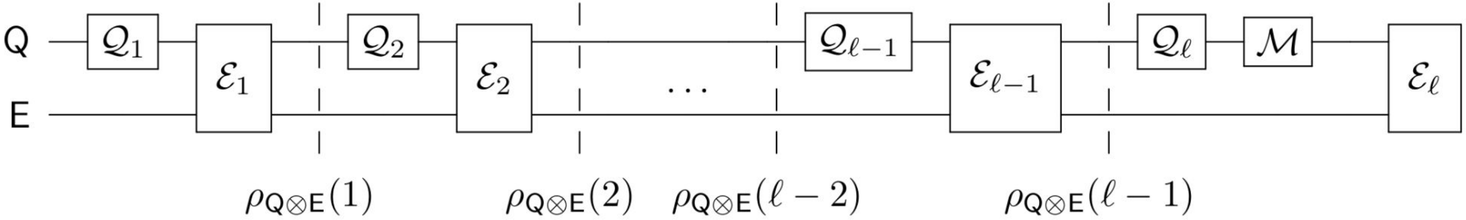


For all steps it must hold that: $\mathrm{Tr}_\mathsf{E}(\rho_{\mathsf{Q}\otimes\mathsf{E}}(i)) = \rho_\mathsf{Q}(i)$

# THE OBLIVIOUS SAMPLER

# LWE state [Regev 05, SSTX 09]

**Idea**: build uniform superposition of LWE samples, then measure

$$|\mathrm{LWE}\rangle_{n,q,f}^{m} \propto \sum_{\mathbf{s}\in\mathbb{Z}_q^n} \sum_{\mathbf{e}\in\mathbb{Z}_q^m} f(\mathbf{e})|\mathbf{A}\mathbf{s}+\mathbf{e}\rangle \quad \text{with} \quad |f|^2 := (\chi_\sigma)^{\otimes m}$$

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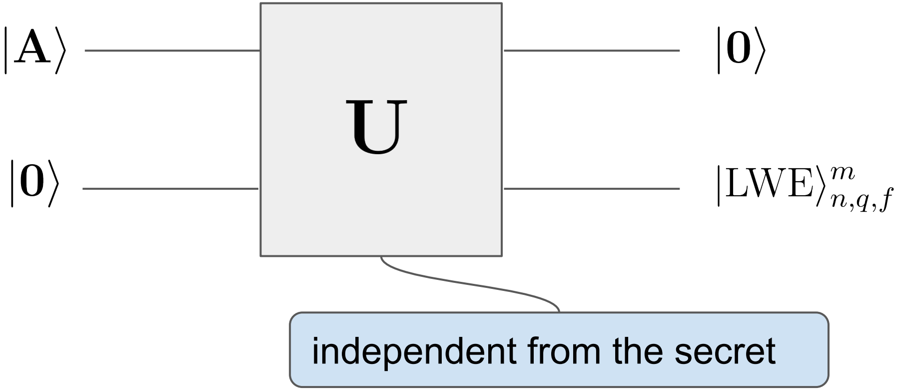

# Building LWE state with [Regev 05, SSTX 09]

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**Not doable in polynomial time**

# TWO INGREDIENTS

# First ingredient: decomposing

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$$\boxed{|\psi_j\rangle := \sum_{e\in\mathbb{Z}_q} f(e)|j+e\rangle}$$

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Given $|\psi_j\rangle$ for unknown $j$

Find $j$ without error or return $\perp$ in the case of failure

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$$|\psi_j\rangle := \sum_{e \in \mathbb{Z}_q} f(e)|j+e\rangle \xrightarrow{\text{Optimal POVM [CB98]}} p_{\text{succ}} = q \cdot \min_y |\hat{f}(y)|^2$$

# First ingredient: Gaussian elimination

$$\sum_{\mathbf{s}\in\mathbb{Z}_q^n} |\mathbf{s}\rangle \otimes |\psi_{\mathbf{a}_1^T\mathbf{s}}\rangle \otimes |\psi_{\mathbf{a}_2^T\mathbf{s}}\rangle \otimes \cdots \otimes |\psi_{\mathbf{a}_{m-1}^T\mathbf{s}}\rangle \otimes |\psi_{\mathbf{a}_m^T\mathbf{s}}\rangle$$

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$$\downarrow$$

$$\perp$$

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$\downarrow$ $\perp$ $\qquad$ $\downarrow$ $\mathbf{a}_2^T\mathbf{s}$

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Optimal POVM $\downarrow$

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$$|\psi_{\mathbf{a}_1^T\mathbf{s}}\rangle \longrightarrow \perp \qquad |\psi_{\mathbf{a}_2^T\mathbf{s}}\rangle \longrightarrow \mathbf{a}_2^T\mathbf{s} \qquad |\psi_{\mathbf{a}_{m-1}^T\mathbf{s}}\rangle \longrightarrow \mathbf{a}_{m-1}^T\mathbf{s}$$

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$$\xrightarrow{\text{Optimal POVM}}$$

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Apply Gaussian Elimination to recover $\mathbf{s}$

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Apply Gaussian Elimination to recover $\mathbf{s}$

Require $m \gtrsim \dfrac{n}{p_{\mathrm{succ}}} = \dfrac{n}{q} \cdot \dfrac{1}{\min_y |\hat{f}(y)|^2}$

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Optimal POVM

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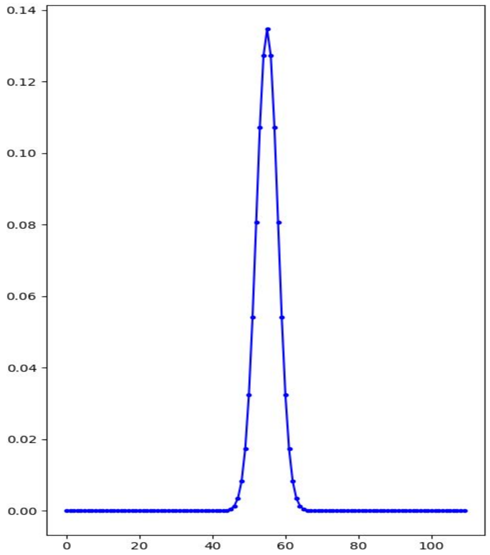
**What we actually do: Naimark's dilation.**

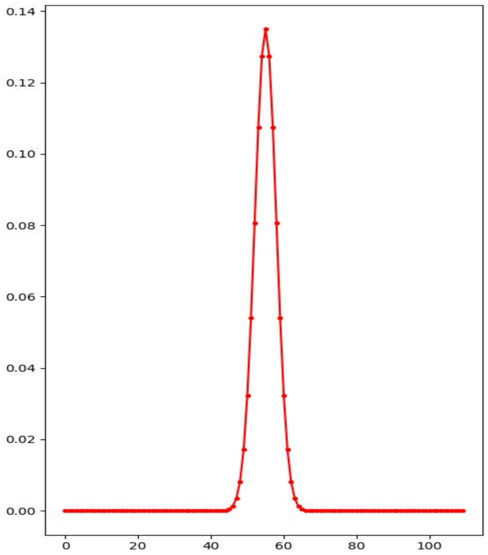
# Summary of the first ingredient

LWE state conditioned on: $m \gtrsim \frac{n}{q} \cdot \frac{1}{\min_y |\hat{f}(y)|^2}$ where $|f|^2 := \chi_\sigma$ (Gaussian)

# Second ingredient: the issue of Gaussian

LWE state conditioned on: $m \gtrapprox n \cdot \mathcal{O}(e^{(q/\sigma)^2})$




Modulus $q$: 110

Standard deviation $\sigma$ : 10.5

Blue: $|f|^2$

Red: $|\hat{f}|^2$

# Second ingredient: adding phases

**idea**: add minus one phases

$$\sum_{\mathbf{s}\in\mathbb{Z}_q^n}\sum_{\mathbf{e}\in\mathbb{Z}_q^m}(-1)^{\theta(\mathbf{e})}f(\mathbf{e})|\mathbf{A}\mathbf{s}+\mathbf{e}\rangle$$

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New condition: $m \gtrsim n \cdot \sigma$

# Wrapping up

**Theorem:** We construct a quantum witness-oblivious LWE sampler when the standard deviation is polynomially large.

# Conclusion

- Obliviously sampling instances of LWE
  - New techniques and definitions: robust definition of obliviousness, (-1) phases
  - Maybe a new example of quantum vs classical gap.

- Breaking the security of several proof systems

Thank you!