Malleable Commitments from Group Actions and Zero-Knowledge Proofs for Circuits based on Isogenies

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Overview

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- Threshold signatures
- Fully Homomorphic Encryption
- Zero-knowledge proofs
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How ?

1. Construct a proof of knowledge for a NP-complete statement.
2. Reduce any other NP problem to this.

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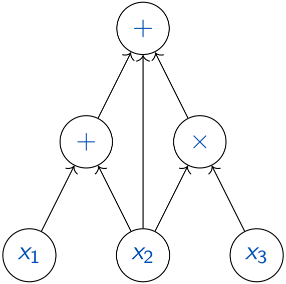

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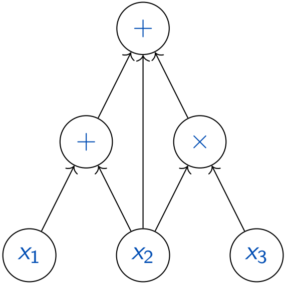

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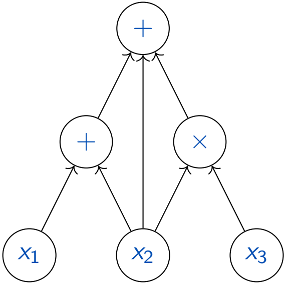

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Theorem

The satisfiability problem for arithmetic circuits is NP-complete.

Commitment Schemes

Definition (Commitment Scheme)

A *commitment scheme* is a tuple $(\mathcal{M}, \mathcal{R}, \mathcal{C}, \text{Commit}, \text{Verify})$ where $\text{Commit} : \mathcal{M} \times \mathcal{R} \to \mathcal{C}$ and $\text{Verify} : \mathcal{M} \times \mathcal{R} \times \mathcal{C} \to \{0, 1\}$ are PPTA algorithms

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Related security notions :

Hiding

An attacker cannot retrieve m or r from c.

Binding

It's hard to find $(m, r) \neq (m', r')$ that give the same commitment.

Homomorphism and malleability

- Efficient solutions use homomophic property :

$$\forall m, m', r, r', \;\; \mathsf{Commit}(m + m', r + r') = \mathsf{Commit}(m, r) \cdot \mathsf{Commit}(m', r')$$

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A GAMC is a commitment scheme satisfying :

- $\mathcal{M}$ and $\mathcal{R}$ are groups. $\mathcal{C}$ is a set.
- We have a group action $\star : (\mathcal{M} \times \mathcal{R}) \times \mathcal{C} \to \mathcal{C}$
- $C_0 := \mathsf{Commit}(0_{\mathcal{M}}, 0_{\mathcal{R}})$
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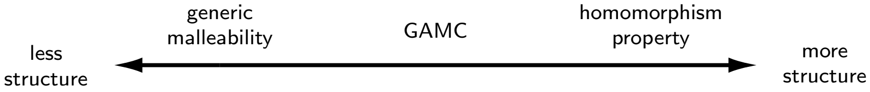
In our case :

- $\mathcal{M}$ and $\mathcal{R}$ are groups of isogenies (with composition).
- $\mathcal{C}$ is a set of elliptic curves (up to isomorphism).



How to use GAMC

Several GAMC ⇝ Proof systems for additions and multiplications ⇝ Proof system for arithmetic circuits

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- $\mathfrak{a}$ and $\mathfrak{b}$ are ideals in $\mathcal{C}\ell(\mathcal{O})$: the *ideal class group* (of $\mathbb{Z}[\pi]$).
- E_0 is a curve in SS_p : the set of supersingular curves "over $\mathbb{F}_p$".

An instance of an GAMC

In the CSIDH setting :

- $\mathcal{M} = \mathcal{R} := \mathcal{C}\ell(\mathcal{O})$ are groups (of ideals).
- $\mathcal{C} := \mathsf{SS}_p \times \mathsf{SS}_p$.
- $C_0 := (E_0, E_1)$

Malleability is given by

$$(\mathfrak{m}, \mathfrak{r}) \star (E, E') := (\mathfrak{r} \cdot E, \mathfrak{m}\mathfrak{r} \cdot E')$$

Conclusion

Contributions :

- New framework for generic NP statements ZK proofs.
- Proof-of-concept construction.

Performances :

- Strong security assumptions and no trusted setup.
- Proof system for an arithmetic circuit $= O(|\mathcal{M}|)$ malleability computations.
- Size of the proof $= O(\lambda|\mathcal{M}|)$ bits.

Future work :

- Cannot use higher security parameters than CSIDH-512.
- The size of the message space is limited.