Lightweight FHE-based Protocols Achieving Results Consistency for Data Encrypted under Different Keys

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$$[m] \xrightarrow[\mathrm{Eval}(f,\,[m])]{} [f(m)]$$

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$$\begin{tikzcd}
m \arrow[r, "f"] \arrow[d, bend left, "\mathrm{Enc}(m)"] & f(m) \\
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What for?

Data confidentiality *during* calculations

Typically, for Cloud Computing

A Multi-user / Multi-key Framework

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- Real context → several data from different contributors
- A user cannot trust the other users
- One key-pair per user ⇒ Multiple keys
- Need of multi-user settings based on FHE
- Medical secrecy, analysis of sensitive data, ...

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Analysis of sensitive data

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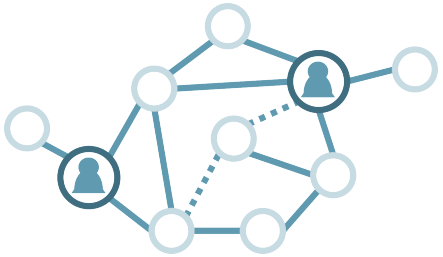
Analysis of sensitive data



Secure discovery of friends in social networks

Prerequisite: Ensure Results Consistency

How can users detect those encrypted messages intended for them?

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👉 Detect key inconsistency with a padding of zeroes

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How can users detect those encrypted messages intended for them?

Detect key inconsistency with a padding of zeroes

$\hookrightarrow$ Decrypting zero with a wrong key yields a random number

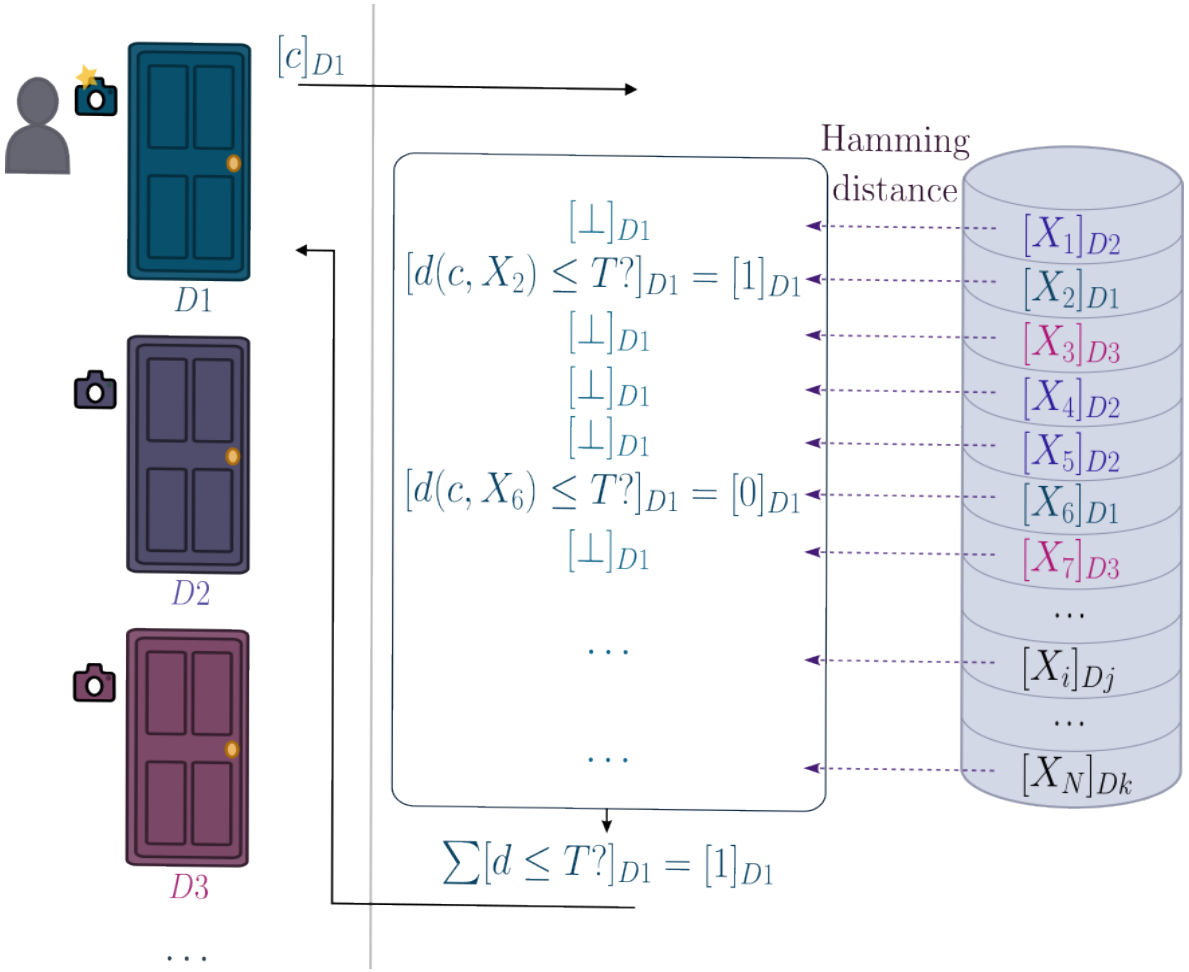
$\hookrightarrow$ Zero padding makes it possible to sort valid results from random ones

$\hookrightarrow$ Assuming a uniform distribution over $\mathbb{F}_q$
$\Rightarrow$ When decrypting $[\underbrace{00\cdots0}_{\ell}]$, the probability of a false positive is $1/q^\ell$

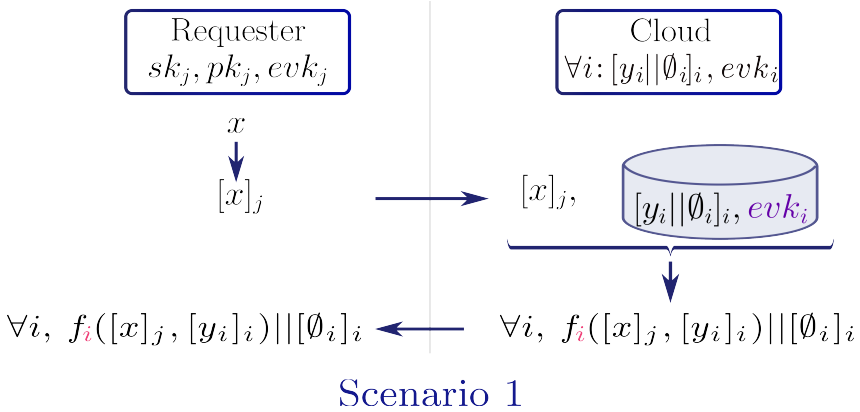
A Use Case: Regulate Accesses to Secured Rooms

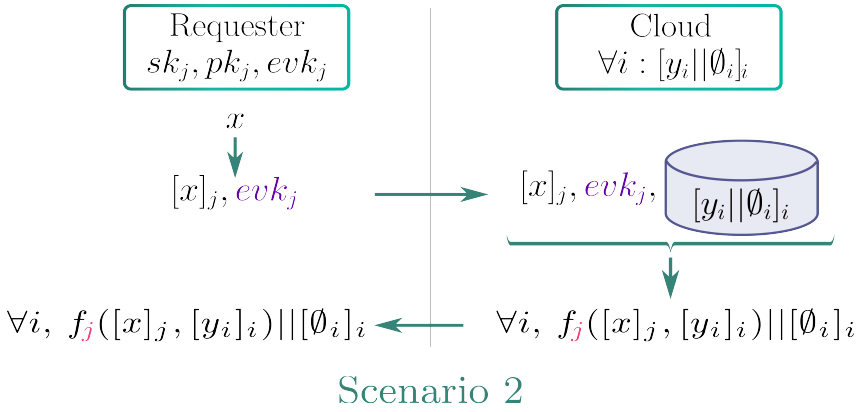
The server:

- ↪ grants or denies room access, according to user's accreditation.
- ↪ must learn nothing about the admission policy or room attendance.



An Incremental Approach: Scenarii 1 and 2

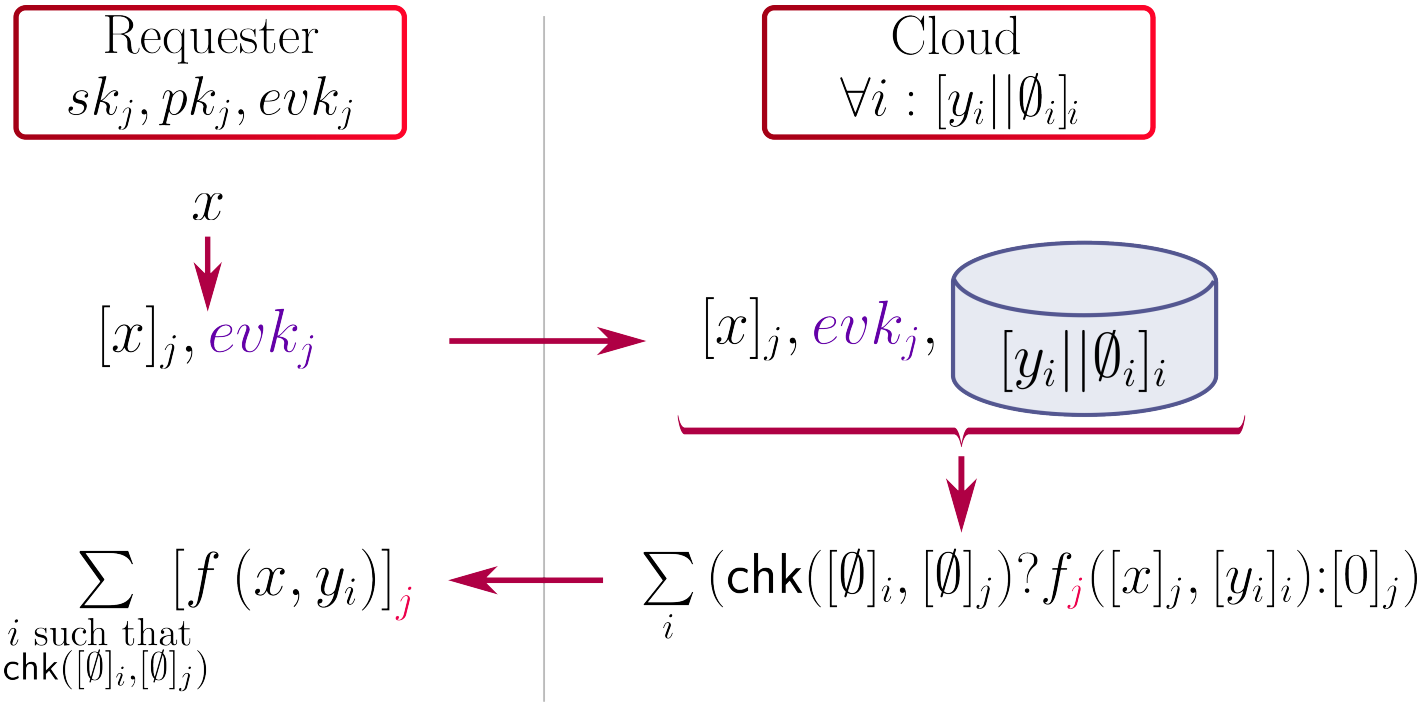




☑ Produce consistent results derived only from data encrypted under the requester's key.

☑ Avoid transmissions of evks by the contributors

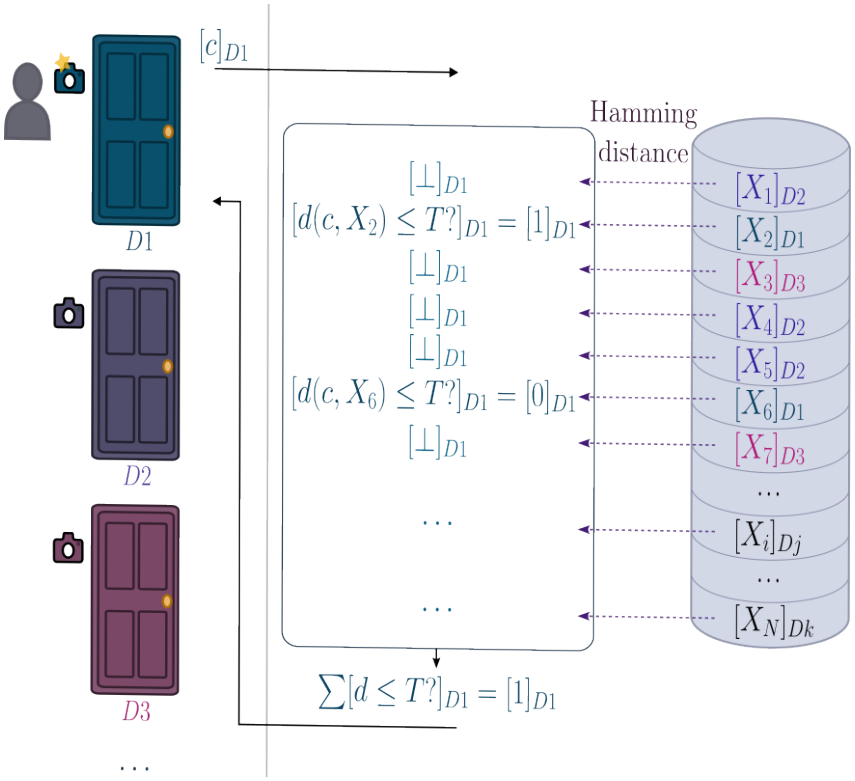
An Incremental Approach: Scenario 3



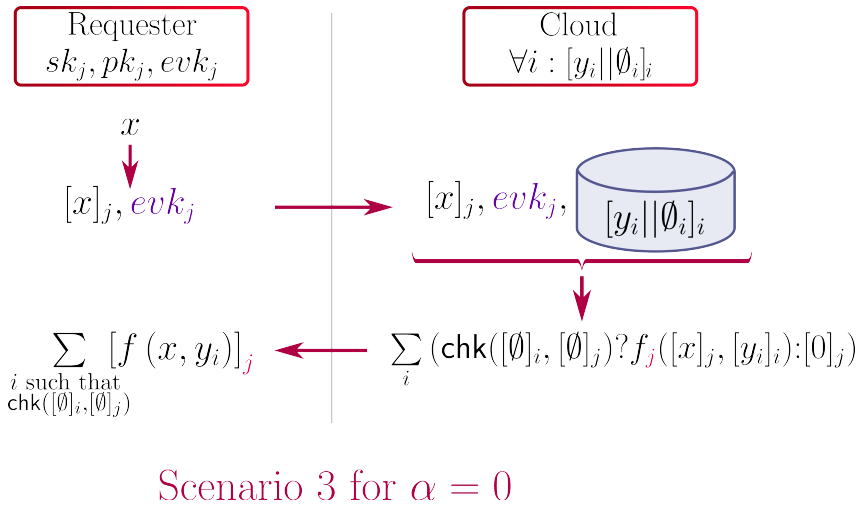
Scenario 3 for $\alpha = 0$

- ☑ Result compactness
- ☑ No need for post-decryption padding verification for the requester

Our Use Case is Addressed by Scenario 3



(a) Use case



(b) Scenario 3 - last scenario

$$[b] ? [m] : [\alpha] \iff ([m] - [\alpha]) \cdot [b] + [\alpha] = \begin{cases} [m] & \text{if } b = 1 \\ [\alpha] & \text{if } b = 0 \end{cases}$$

Key Forcing Subtility with TFHE

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TFHE is not natively designed for multi-user operations

Key Forcing Subtility with TFHE

FHE and noise

FHE encryption	$\rightarrow$	add noise for security
Homomorphic operations	$\rightarrow$	increase the noise
Bootstrapping	$\rightarrow$	to reduce the noise
		= homomorphic decryption

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TFHE Bootstrapping key

Bootstrapping key = evaluation key

Bootstrapping key = encryption of the private key

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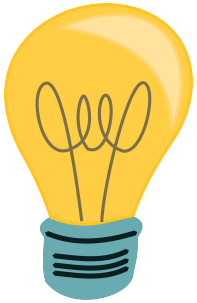
Multi-user framework

$\hookrightarrow$ The encryption key used does not always match the bootstrapping key.

Key Forcing Subtility with TFHE



$$
\begin{aligned}
& \mathrm{JFK}(\varkappa_{\mathrm{bk}_{RK}}, [a]_{\mathrm{pk}_{RK}}, [b]_{\mathrm{pk}_{RK}}) \\
= \; & [a]_{\mathrm{pk}_{RK}} \; \varkappa_{\mathrm{bk}_{RK}} \; \underbrace{([1]_{\mathrm{pk}_{RK}} \; \&_{\mathrm{bk}_{RK}} \; [b]_{\mathrm{pk}_{RK}})}_{[b]_{\mathrm{pk}_{RK}}} \\
= \; & [a]_{\mathrm{pk}_{RK}} \; \varkappa_{\mathrm{bk}_{RK}} \; [b]_{\mathrm{pk}_{RK}} \\
= \; & [a \varkappa b]_{\mathrm{pk}_{RK}}
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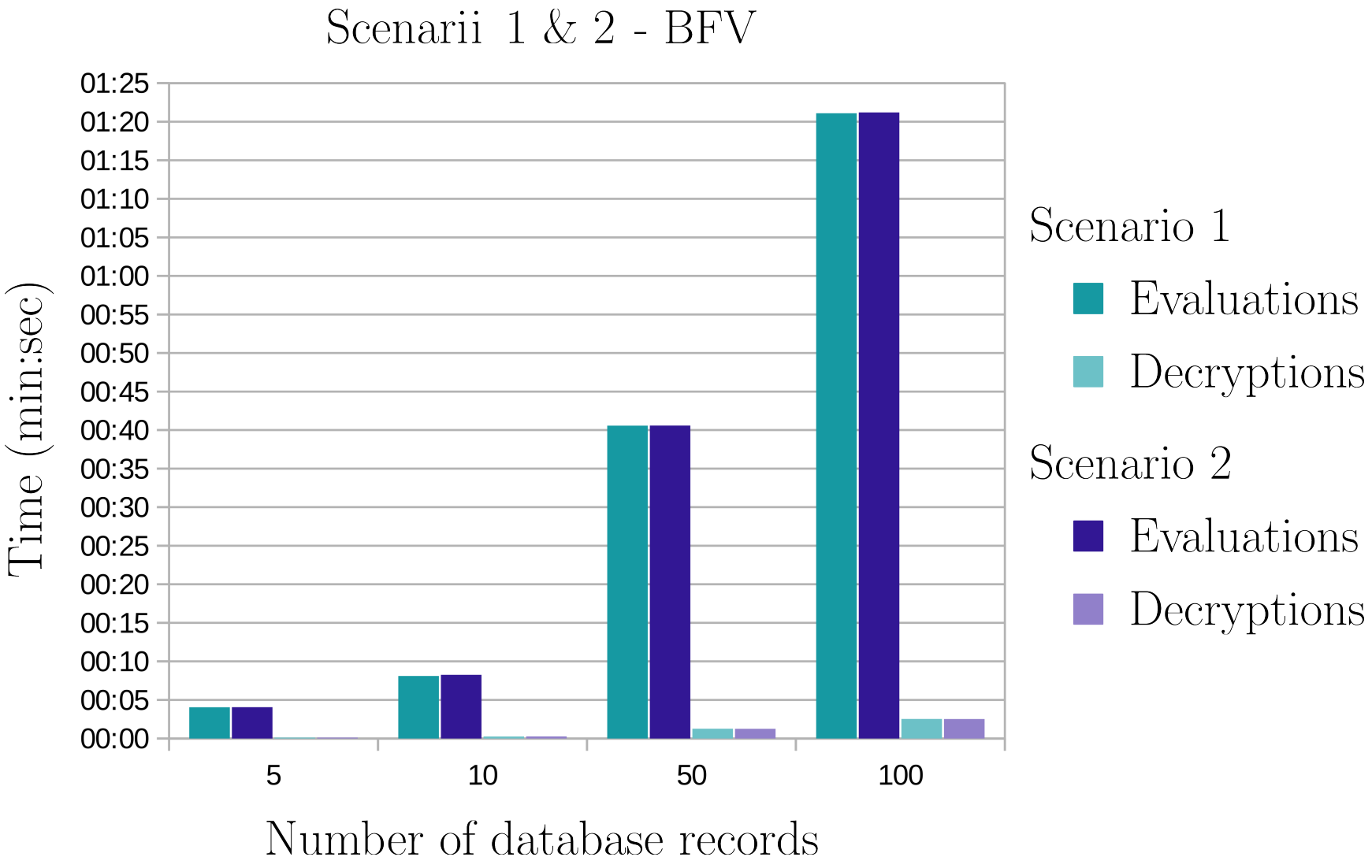
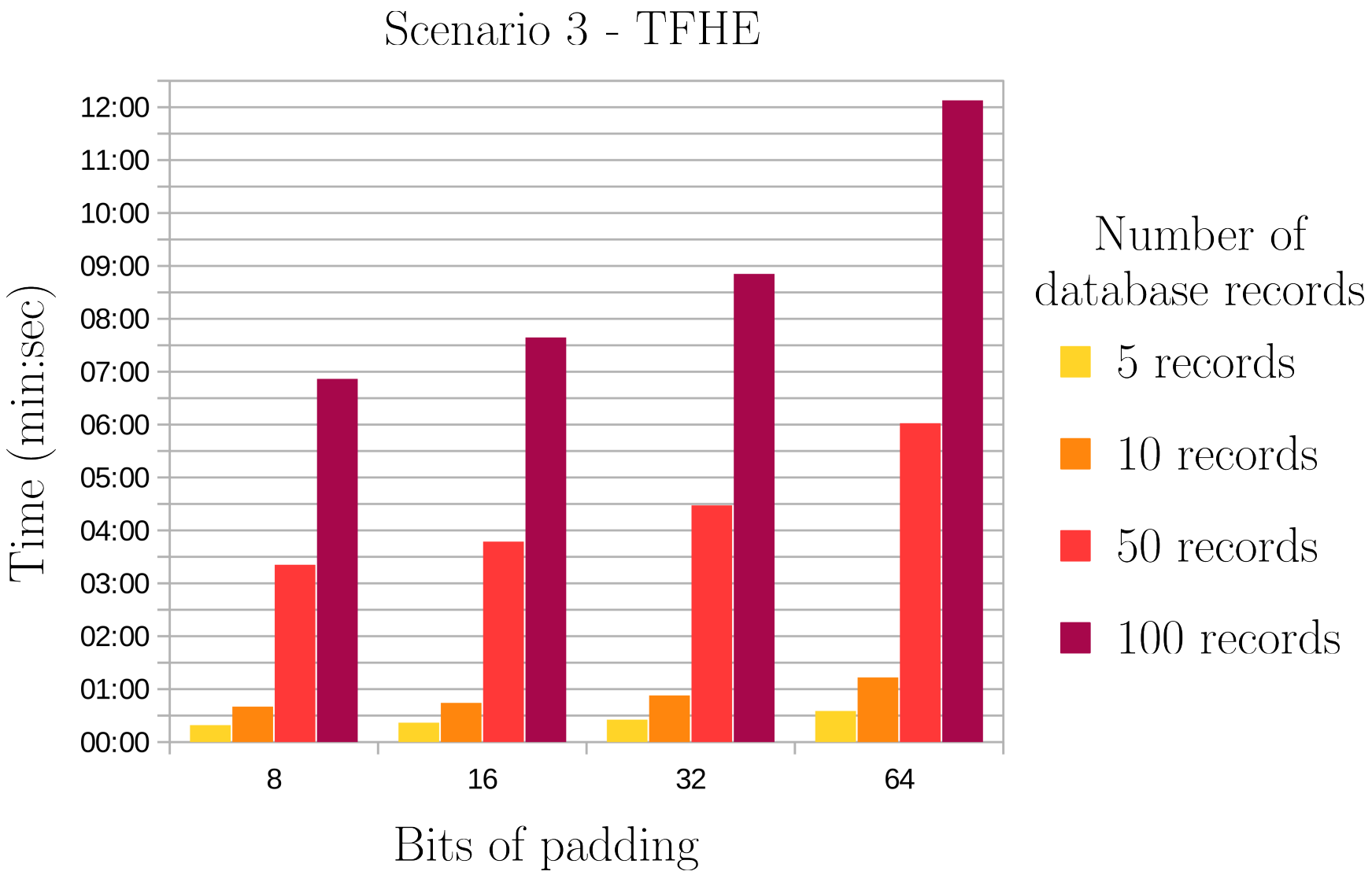
Time Analysis of Scenarii 1 and 2



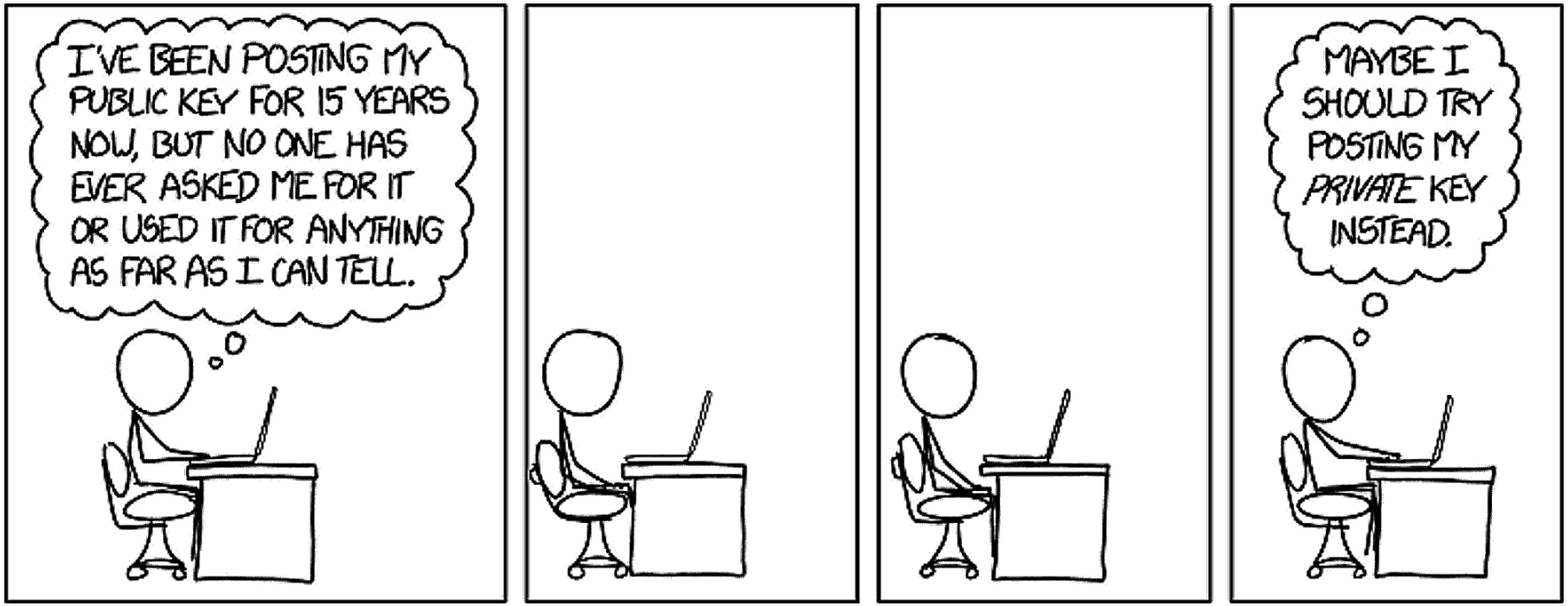
Time Analysis of Scenario 3



Key Takeaways

- Scenarii of increasing practical relevance and usefulness
- Lightweight multi-user setting (one requester per request)
- Calculations on data from different contributors under different keys
- Data and results confidentiality
- Results consistency
- Untraceability of contributors by requesters

Thank you for your kind attention!



credit: xkcd.com