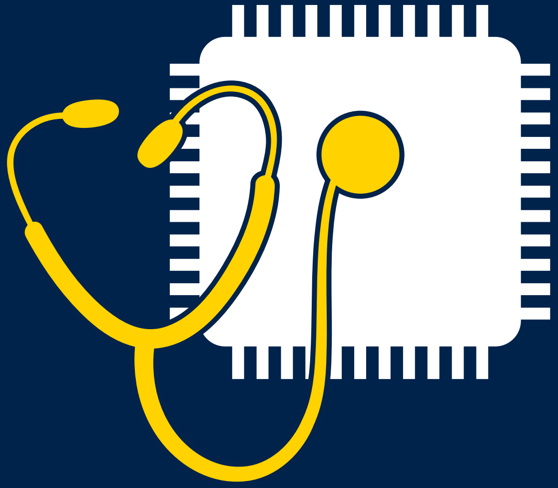
Improving Single-Trace Attacks on the Number-Theoretic Transform for Cortex-M4

Authors: **Guilhèm Assael** [1, 2], Philippe Elbaz-Vincent [2], Guillaume Reymond [1]

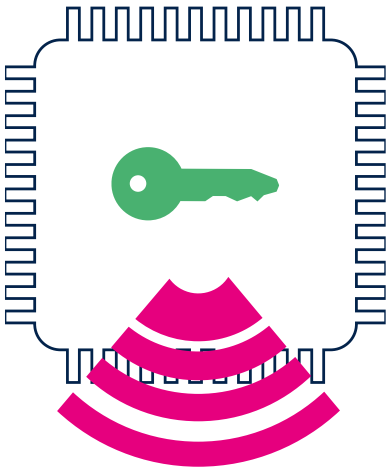
[1] STMicroelectronics Rousset, 13106 Rousset, France

[2] Univ. Grenoble Alpes, CNRS, IF, 38000 Grenoble, France

October 17, 2023



Purpose and summary



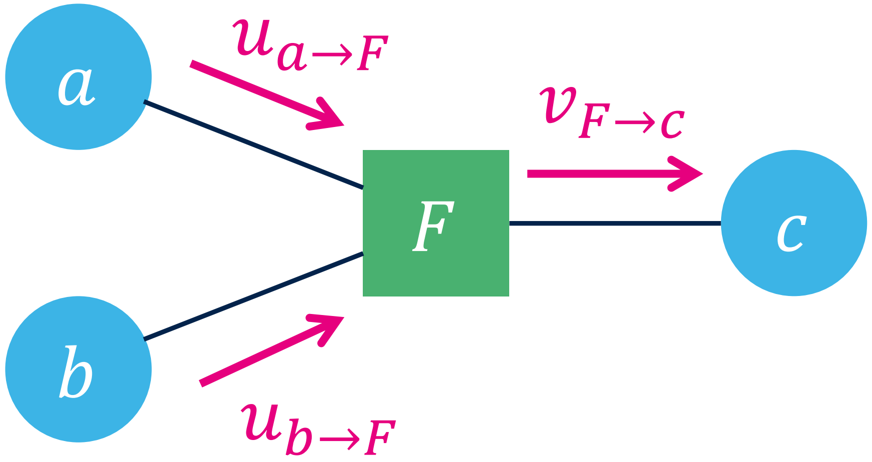
Side-channel attack: determine a **cryptographic secret** from the **environmental leakage** of an electronic device

- Vertical attack: observe several operations using the same secret, then combine the information from these executions
- **Horizontal / single-trace attack**: determine full or partial secret from a single execution of the algorithm

This work was published in [1] Assael, Elbaz-Vincent and Reymond, "Improving Single-Trace Attacks on the Number-Theoretic Transform for Cortex-M4," HOST 2023.

Belief-propagation (BP) attacks

- Optimization technique first applied to side-channel attacks in [2]
- Model the target algorithm as a **factor graph**
  - **Variable nodes**: unknown quantities
  - **Factor nodes**: relations between variables

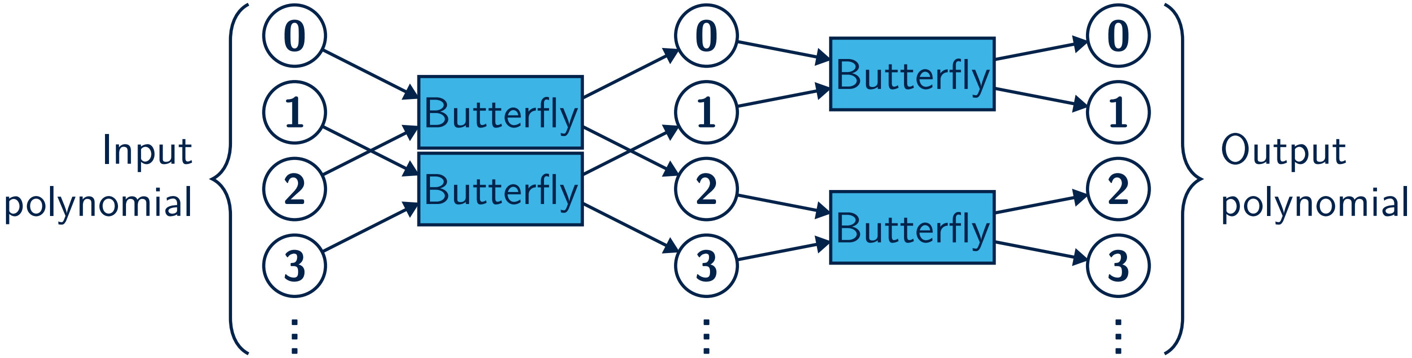


- Pass **messages** (→) between adjacent nodes
  - They represent the belief (estimated probability distribution) on a variable
  - **Variable nodes** compute their **output messages** based on their **input messages**
  - **Factor nodes**' computation is based on the **relation they represent** and their **input messages**
- Iterate message passing until convergence, then extract **marginal probabilities**

[2] Veyrat-Charvillon *et al.*, "Soft analytical side-channel attacks," in Advances in Cryptology – ASIACRYPT 2014.

Kyber NTT for Cortex-M4

- **Kyber** is a lattice-based key-encapsulation mechanism selected by NIST for PQC [3]
- It uses the **Number-Theoretic Transform** (NTT) for polynomial multiplication
  - Kyber NTT is made up of *butterfly* operations on pairs of coefficients, applied in 7 layers



- Microcontrollers based on ARMv7E-M instruction set (*e.g.* Cortex-M4) have **DSP instructions** operating on **packed half-words.** They can implement Kyber NTT **two butterflies at a time** by packing pairs of coefficients into a 32-bit word [4]

[3] Alagic *et al.*, “Status report on the third round of the NIST post-quantum cryptography standardization process,” National Institute of Standards and Technology, 2022.
[4] Huang *et al.*, “Improved Plantard arithmetic for lattice-based cryptography,” TCHES 2022/4.

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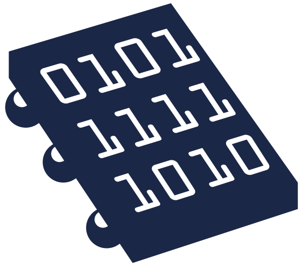
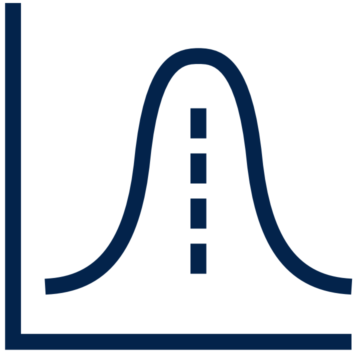
Leakage model



We assume that instructions **leak the result they write** to registers or RAM

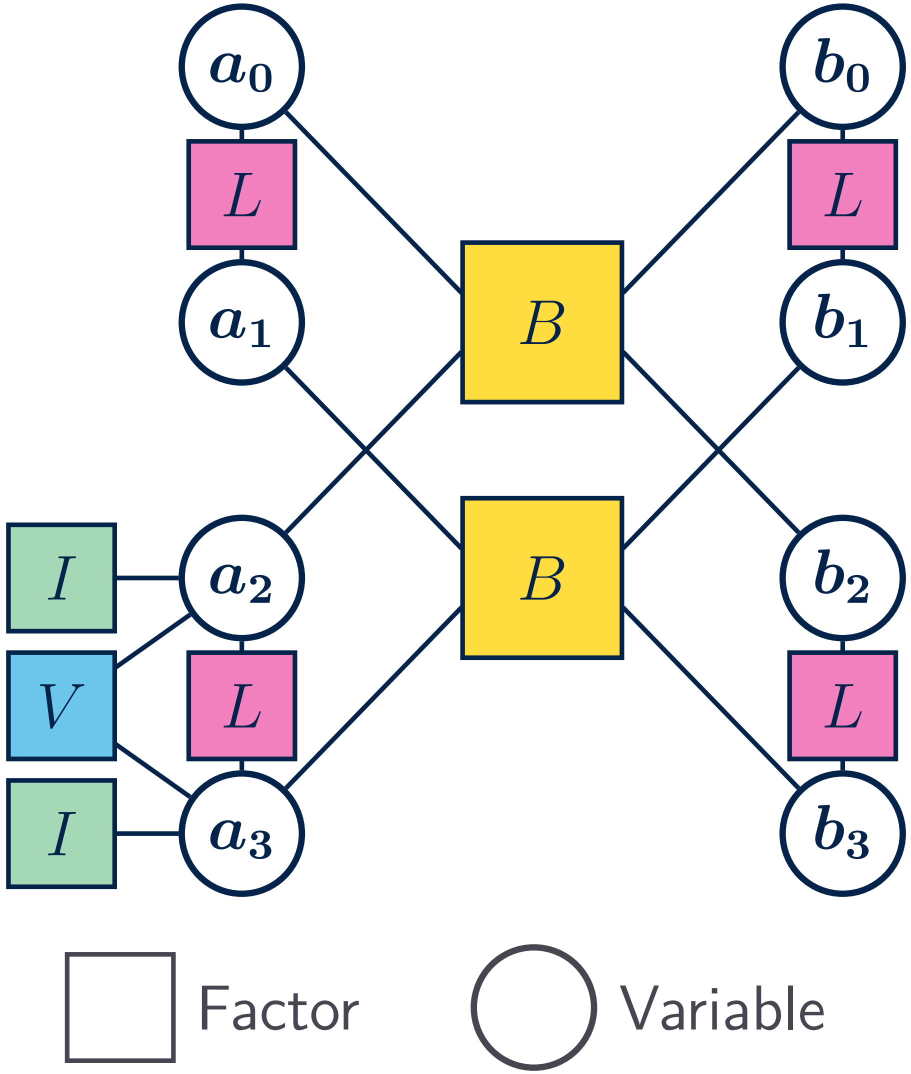
The leakage considered is **Hamming Weight** (between 0 and 32)



Exact Hamming-weight measurements are overlaid with centered **Gaussian noise** having configurable **standard deviation** σ_M

Factor graph for each double butterfly

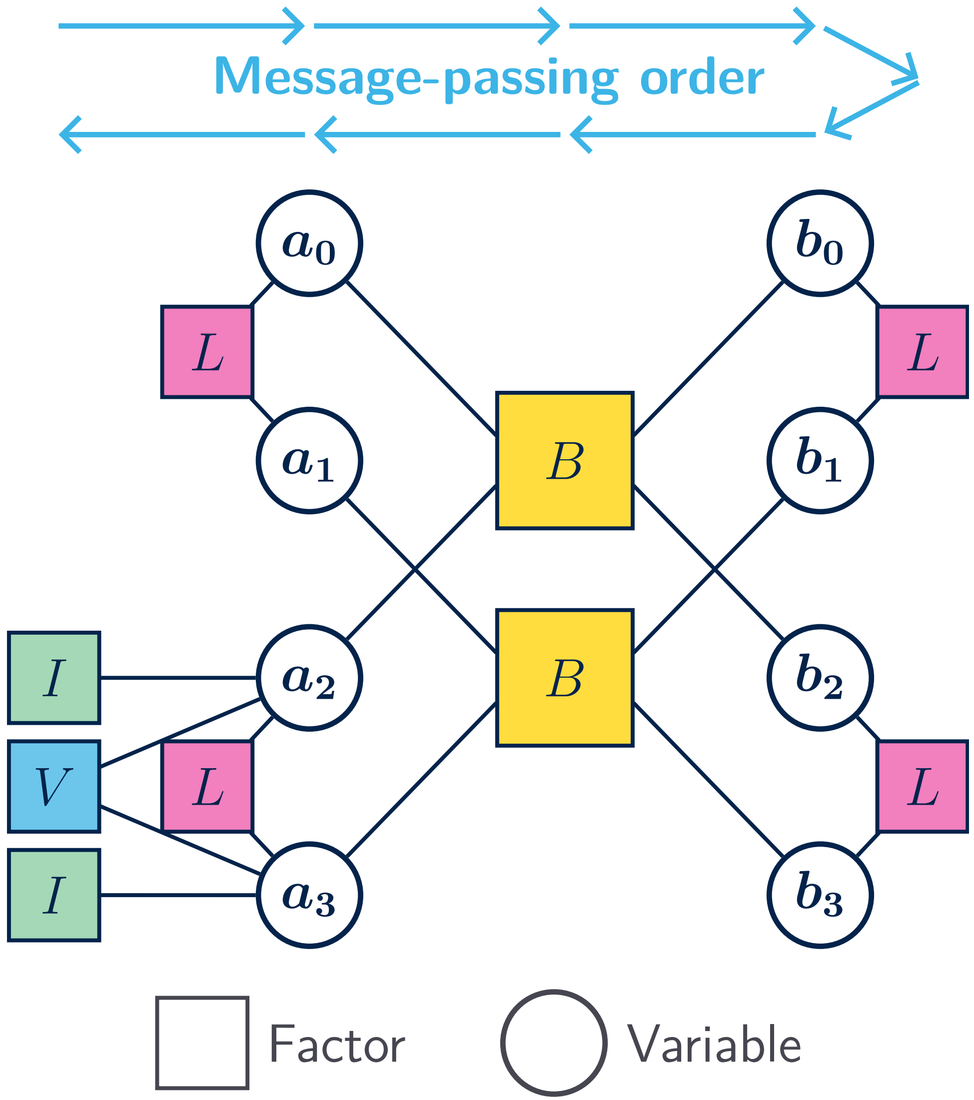
- We model the exact CPU instructions used
- Every 16-bit polynomial coefficient in each NTT layer is modeled by a variable node
- Types of factor nodes used
  - L leakage of load or store operations
  - I leakage from instructions on **single** coefficients
  - V leakage from instructions on **pairs** of coefficients
  - B butterfly equation



Message-passing order

- We use a ping-pong message schedule after [5]: start from the NTT input layer, propagate messages until last layer, and bounce back toward the input
- All same-type factors of each layer can be processed in parallel (up to 128 threads)

L	leakage of load or store operations
I	leakage from instructions on **single** coefficients
V	leakage from instructions on **pairs** of coefficients
B	butterfly equation



[5] Pessl *et al.*, "More practical single-trace attacks on the number theoretic transform," LATINCRYPT 2019.

Message damping and pruning

Pessl *et al*. [5] introduced **message damping** to reduce the risk of messages oscillating across iterations

We apply their technique with the following weighting:

- 95% weight for the message-update rule
- 5% weight for the old message value

Additionally, we perform **message pruning** by zeroing low-probability outcomes to speed up computations

[5] Pessl *et al*, “More practical single-trace attacks on the number theoretic transform,” LATINCRYPT 2019.

General setup

Environment

- Python 3.10, partially JIT-compiled with numba 0.55
- AMD EPYC 7713P @ 2 GHz, running on 32 cores

Target simulation

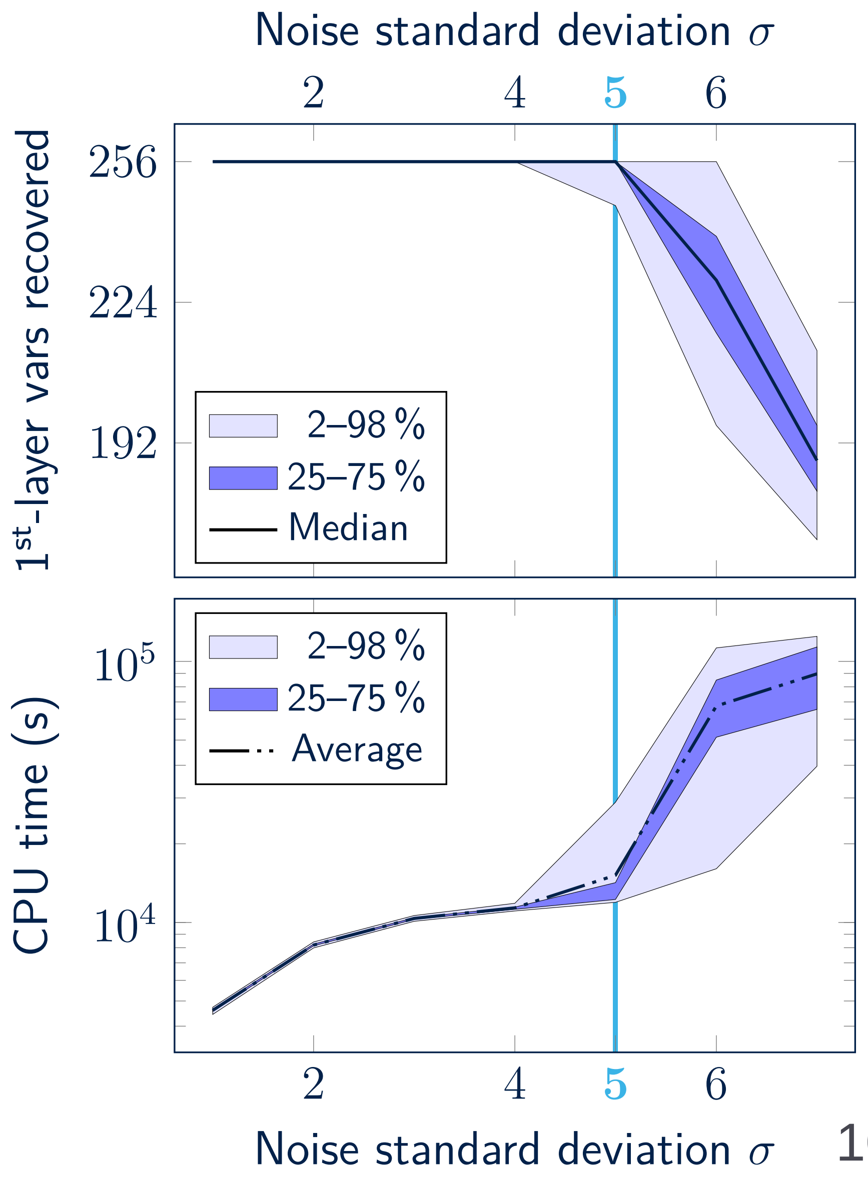
- Sample a **random input polynomial** with 256 coefficients
- Compute its NTT by **simulating the instructions** constituting the butterflies
- Measure the **Hamming weight** of all results written to registers and add **Gaussian noise**

Attack

- Build factor graph and configure factor nodes with the noised measurements
- Run the message passing until **convergence**, **iteration limit** or **failure**
- Compute the marginals of input-layer variables and keep the **highest-probability outcome** of each

Results for binomially-distributed NTT input

- The input polynomial is sampled with coefficients **binomially distributed** in $[-3, 3]$
- The standard deviation σ of measurement noise is **known**
- More than **75%** of the trials reach **perfect success** (all coefficients recovered) up to $\boldsymbol{\sigma = 5}$
- Average CPU time ≤ 3 hours up to $\sigma = 4$

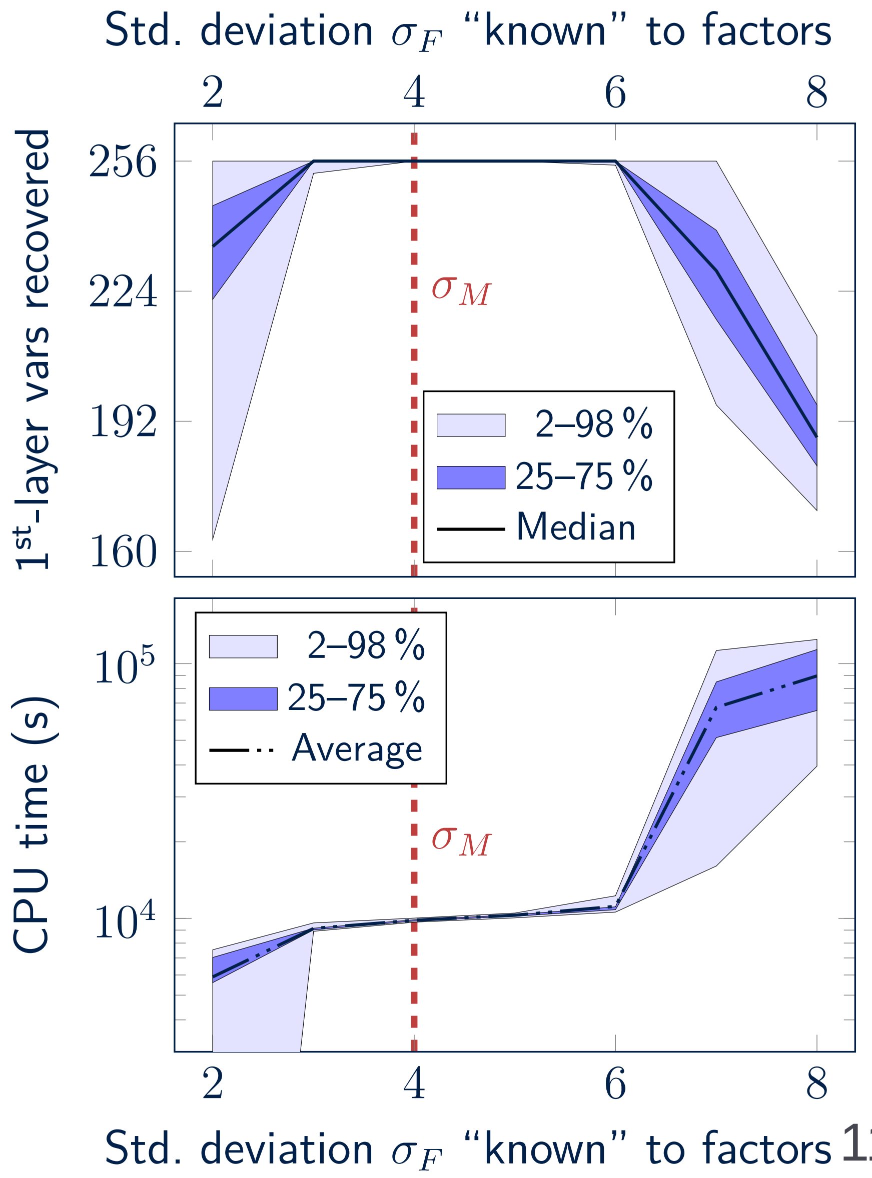


Reality check: what if noise level is unknown?

- The input polynomial is sampled with coefficients **binomially distributed** in $[-3,3]$
- The standard deviation σ_M of measurement noise is **unknown** and **approximated by σ_F**
- **Actual** standard deviation is $\sigma_M = 4$

- More than **90%** of the trials reach **perfect success** (all coefficients recovered) when $\sigma_F \in [3,6]$

- Average CPU time $\leq$ 3 hours for $\sigma_F \in [3,6]$



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Attack exploitation

Threat model: attacker can record side-channel traces and may request the decapsulation of chosen ciphertexts

Message recovery

- Used to recover the encapsulated shared secret
- Recover $\boldsymbol{r}$ during encryption

Key recovery

- Recover $\boldsymbol{s}$ or $\boldsymbol{e}$ during key generation
- Or perform **message recovery** during the decapsulation of chosen ciphertexts, and use as a decryption-failure oracle [6]

Kyber encryption (simplified)

Input: Public key $pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}}) \in R_q^{k\times k} \times R_q^k$
Input: Message $m \in \{0,1\}^n$
Output: Ciphertext $c = (\mathbf{u}, v) \in R_q^k \times R_q$

$\mathbf{r} \leftarrow \mathcal{B}_{\eta_1}^k \qquad \mathbf{e_1} \leftarrow \mathcal{B}_{\eta_1}^k$ // *Binomial sampling*
$e_2 \leftarrow \mathcal{B}_{\eta_2}$ // *Binomial sampling*
$\hat{\mathbf{r}} = \mathrm{NTT}(\mathbf{r})$
$\mathbf{u} = \mathrm{NTT}^{-1}(\hat{\mathbf{A}}^\top \circ \hat{\mathbf{r}}) + \mathbf{e_1}$
$v = \mathrm{NTT}^{-1}(\hat{\mathbf{t}}^\top \circ \hat{\mathbf{r}}) + e_2 + \lceil q/2 \rfloor m$
return $c = (\mathbf{u}, v)$

Kyber key generation (simplified)

Output: Private key $sk = \hat{\mathbf{s}} \in R_q^k$
Output: Public key $pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}}) \in R_q^{k\times k} \times R_q^k$

$\hat{\mathbf{A}} \leftarrow R_q^{k\times k}$ // *Uniform sampling*
$\mathbf{s} \leftarrow \mathcal{B}_{\eta_1}^k \qquad \mathbf{e} \leftarrow \mathcal{B}_{\eta_1}^k$ // *Binomial sampling*
$\hat{\mathbf{s}} = \mathrm{NTT}(\mathbf{s}) \qquad \hat{\mathbf{e}} = \mathrm{NTT}(\mathbf{e})$
$\hat{\mathbf{t}} = \hat{\mathbf{A}} \circ \hat{\mathbf{s}} + \hat{\mathbf{e}}$
return $sk = \hat{\mathbf{s}},\ pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}})$

[6] Hermelink *et al.*, “Fault-Enabled Chosen-Ciphertext Attacks on Kyber,” INDOCRYPT 2021.

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Impact of countermeasures?

✕ Masking

- Not effective as the attacker can recover shares independently and combine them after the attack, or during the attack given modifications to the graph [5]
- **Masking only reduces noise tolerance**

☑ Shuffling

- **Very effective** [5]: **decorrelates timing from the corresponding data**
- Some adaptations of the attack have been proposed [7]

[5] Pessl *et al.*, “More practical single-trace attacks on the number theoretic transform,” LATINCRYPT 2019.
[7] Hermelink *et al.*, “Adapting belief propagation to counter shuffling of NTTs,” TCHES 2023/1.

Summarizing our contribution

We adapt the **belief-propagation attack** to an **optimized Cortex-M4 implementation** of Kyber

We show that **accurate modelling** of the algorithm allows the attack to **tolerate high noise**, up to standard deviation $\sigma = 5$ (for Hamming-weight measurements between 0 and 32)

We highlight that the **attack performs well** when the amplitude of **measurement noise is not precisely known**

References

[1] G. Assael, P. Elbaz-Vincent and G. Reymond, "Improving Single-Trace Attacks on the Number-Theoretic Transform for Cortex-M4," 2023 IEEE International Symposium on Hardware Oriented Security and Trust (HOST), San Jose, CA, USA, 2023, pp. 111-121, doi: 10.1109/HOST55118.2023.10133270.

[2] N. Veyrat-Charvillon, B. Gérard, and F.-X. Standaert, “Soft analytical side-channel attacks,” in Advances in Cryptology – ASIACRYPT 2014, P. Sarkar and T. Iwata, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2014, pp. 282–296.

[3] G. Alagic, D. Apon, D. Cooper, Q. Dang, T. Dang, J. Kelsey, J. Lichtinger, Y.-K. Liu, C. Miller, D. Moody, R. Peralta, R. Perlner, A. Robinson, and D. Smith-Tone, “Status report on the third round of the NIST post-quantum cryptography standardization process,” National Institute of Standards and Technology, 2022.

[4] J. Huang, J. Zhang, H. Zhao, Z. Liu, R. C. C. Cheung, Ç. K. Koç, and D. Chen, “Improved Plantard arithmetic for lattice-based cryptography,” IACR Transactions on Cryptographic Hardware and Embedded Systems, vol. 2022, no. 4, p. 614–636, Aug. 2022.

[5] P. Pessl and R. Primas, “More practical single-trace attacks on the number theoretic transform,” in Progress in Cryptology – LATINCRYPT 2019, P. Schwabe and N. Thériault, Eds. Cham: Springer International Publishing, 2019, pp. 130–149.

[6] J. Hermelink, P. Pessl, T. Pöppelmann, “Fault-Enabled Chosen-Ciphertext Attacks on Kyber,” Progress in Cryptology – INDOCRYPT 2021, INDOCRYPT 2021, P. Schwabe and N. Thériault, Eds. Cham: Springer International Publishing, 2021, pp. 311–334.

[7] J. Hermelink, S. Streit, E. Strieder, and K. Thieme, “Adapting belief propagation to counter shuffling of NTTs,” IACR Transactions on Cryptographic Hardware and Embedded Systems, vol. 2023, no. 1, pp. 60–88, Nov. 2022.

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Thank you

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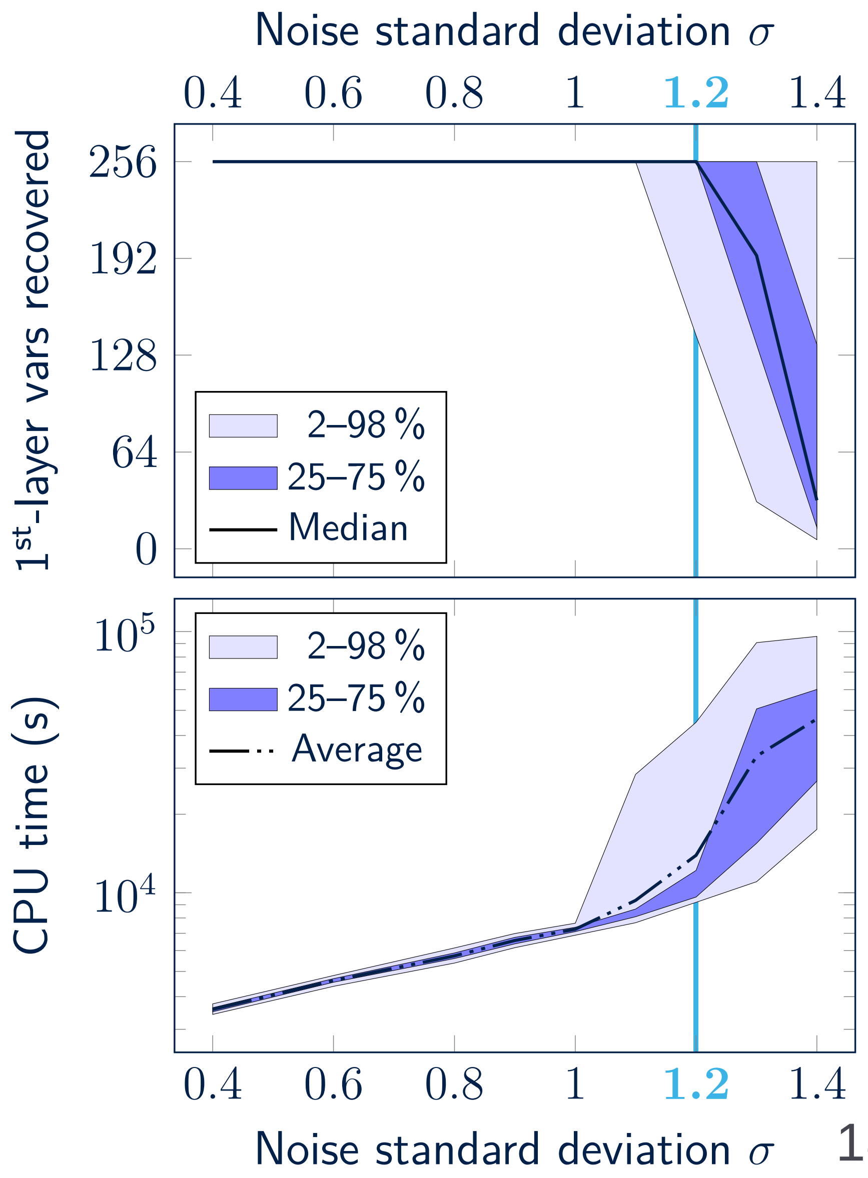
Appendices

Results for uniformly-distributed NTT input

- The input polynomial is sampled with coefficients **uniformly distributed** in $[-\lceil q/2 \rceil, \lfloor q/2 \rfloor]$ ($q = 3329$)
- The standard deviation σ of measurement noise is **known**



- More than **90%** of the trials reach **perfect success** (all coefficients recovered) up to $\boldsymbol{\sigma = 1.2}$
- Average CPU time $\leq$ 3 hours up to $\sigma = 1.1$

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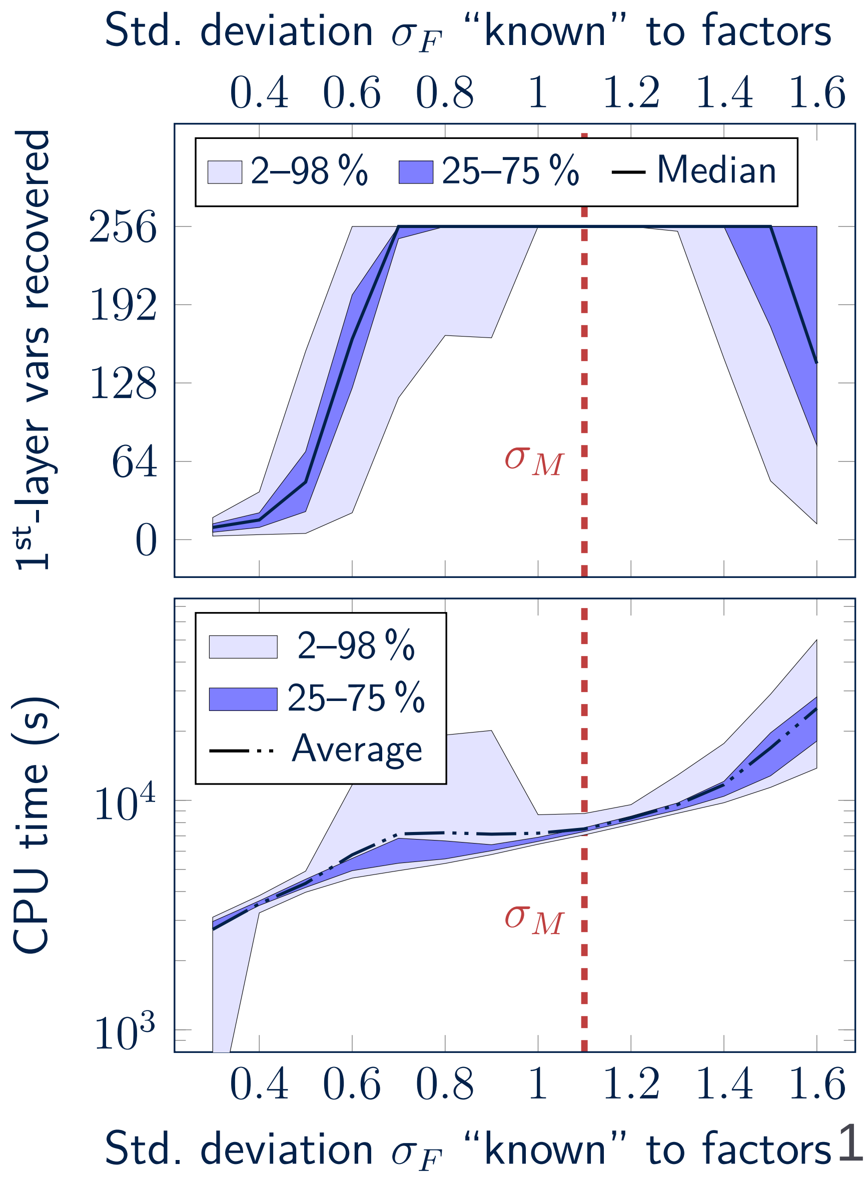
Uniformly-distributed NTT input and unknown noise

- The input polynomial is sampled with coefficients **uniformly distributed** in $[-\lceil q/2 \rceil, \lfloor q/2 \rfloor]$ ($q = 3329$)
- The standard deviation σ_M of measurement noise is **unknown** and **approximated by** σ_F
- **Actual** standard deviation is $\sigma_M = 1.1$

- More than **75%** of the trials reach perfect success (all coefficients recovered) when $\boldsymbol{\sigma_F \in [0.8, 1.4]}$

- Average CPU time $\leq$ 3 hours for $\sigma_F \in [0.8, 1.4]$



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Possible attack paths on IND-CPA Kyber

Kyber key generation (simplified)

Output: Private key $sk = \hat{\mathbf{s}} \in R_q^k$
Output: Public key $pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}}) \in R_q^{k \times k} \times R_q^k$

$\hat{\mathbf{A}} \leftarrow R_q^{k \times k}$ // *Uniform sampling*
$\mathbf{s} \leftarrow \mathcal{B}_{\eta_1}^k \quad \mathbf{e} \leftarrow \mathcal{B}_{\eta_1}^k$ // *Binomial sampling*
$\hat{\mathbf{s}} = \mathrm{NTT}(\mathbf{s}) \quad \hat{\mathbf{e}} = \mathrm{NTT}(\mathbf{e})$
$\hat{\mathbf{t}} = \hat{\mathbf{A}} \circ \hat{\mathbf{s}} + \hat{\mathbf{e}}$
return $sk = \hat{\mathbf{s}}$, $pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}})$

Kyber decryption (simplified)

Input: Private key $sk = \hat{\mathbf{s}} \in R_q^k$
Input: Ciphertext $c = (\mathbf{u}, v) \in R_q^k \times R_q$
Output: Message $m \in \{0,1\}^n$

$w = v - \mathrm{NTT}^{-1}(\hat{\mathbf{s}}^{\mathsf{T}} \circ \mathrm{NTT}(\mathbf{u}))$
$m = \lceil (2/q) w \rfloor \bmod 2$
return m

Kyber encryption (simplified)

Input: Public key $pk = (\hat{\mathbf{A}}, \hat{\mathbf{t}}) \in R_q^{k \times k} \times R_q^k$
Input: Message $m \in \{0,1\}^n$
Output: Ciphertext $c = (\mathbf{u}, v) \in R_q^k \times R_q$

$\mathbf{r} \leftarrow \mathcal{B}_{\eta_1}^k \quad \mathbf{e_1} \leftarrow \mathcal{B}_{\eta_1}^k$ // *Binomial sampling*
$e_2 \leftarrow \mathcal{B}_{\eta_2}$ // *Binomial sampling*
$\hat{\mathbf{r}} = \mathrm{NTT}(\mathbf{r})$
$\mathbf{u} = \mathrm{NTT}^{-1}(\hat{\mathbf{A}}^{\mathsf{T}} \circ \hat{\mathbf{r}}) + \mathbf{e_1}$
$v = \mathrm{NTT}^{-1}(\hat{\mathbf{t}}^{\mathsf{T}} \circ \hat{\mathbf{r}}) + e_2 + \lceil q/2 \rfloor m$
return $c = (\mathbf{u}, v)$

Key-recovery attack

Message-recovery attack

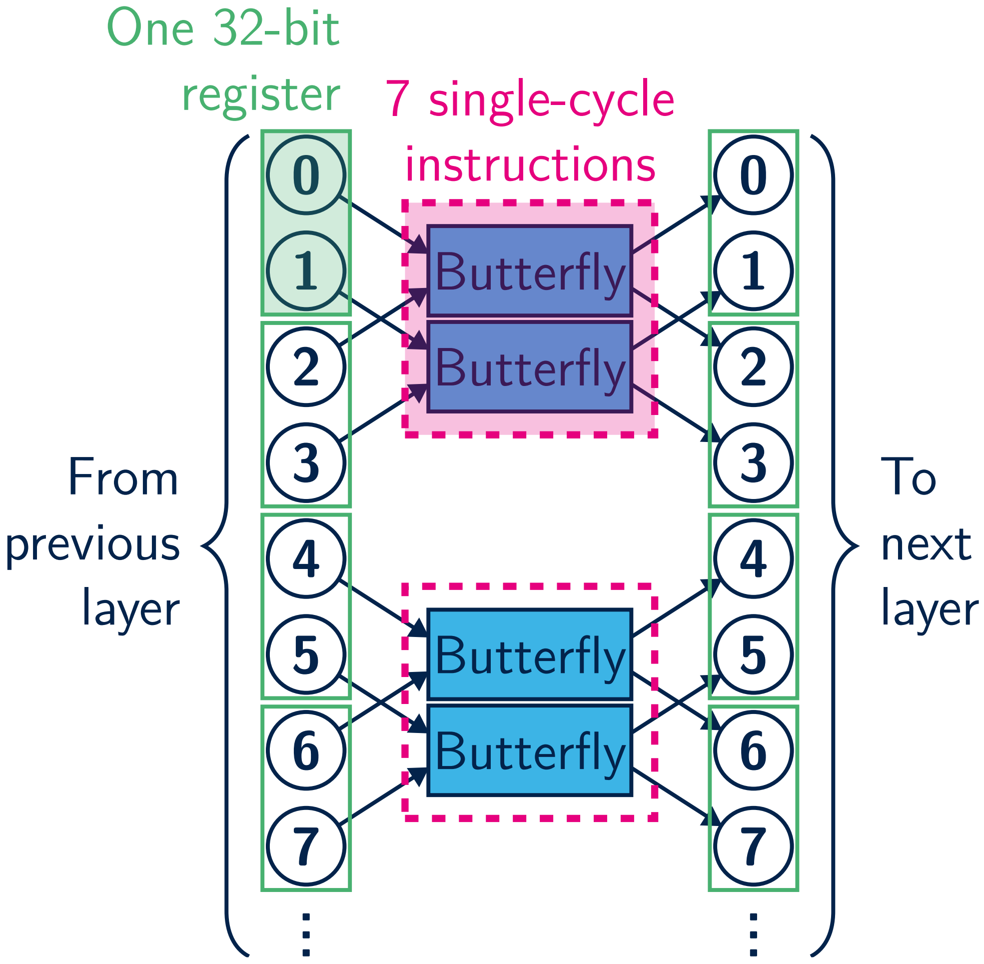
Key-recovery attack through **Inverse** NTT

Optimized NTT implementation for Cortex-M4

- Cortex-M4 microcontrollers have **DSP instructions** operating on **packed half-words**

- They can efficiently implement Kyber NTT **two butterflies at a time** by packing pairs of 16-bit coefficients into a 32-bit word [4]
  Kyber modulus has 12 bits, but coefficients can grow to 16 bits due to lazy reduction

- Previous BP attacks against the NTT did not use the same instructions and were not adapted to packed coefficients



[4] Huang *et al*., "Improved Plantard arithmetic for lattice-based cryptography," TCHES 2022/4.

Assembly implementation of the double butterfly

Double Cooley-Tukey butterfly for Cortex-M4

Input: Packed pairs of signed 16-bit coefficients $a = a_\mathsf{t} \parallel a_\mathsf{b}$, $b = b_\mathsf{t} \parallel b_\mathsf{b}$

Input: 32-bit corrected twiddle factor ζ (from real twiddle factor ζ_0)

Input: q and $q2^\alpha$ in two separate registers

Output: $a \equiv (a_\mathsf{t} + b_\mathsf{t}\zeta_0) \parallel (a_\mathsf{b} + b_\mathsf{b}\zeta_0)$, $b \equiv (a_\mathsf{t} - b_\mathsf{t}\zeta_0) \parallel (a_\mathsf{b} - b_\mathsf{b}\zeta_0)$

smulwb t, ζ, b — $// \ t = \zeta b_\mathsf{b} \gg 16$

smulwt b, ζ, b — $// \ b = \zeta b_\mathsf{t} \gg 16$

smlabb $t, t, q, q2^\alpha$ — $// \ t = t_\mathsf{b} q + q2^\alpha$

smlabb $b, b, q, q2^\alpha$ — $// \ b = b_\mathsf{b} q + q2^\alpha$

pkhtb $t, b, t, \mathrm{asr}\#16$ — $// \ t = t_\mathsf{b} \parallel b_\mathsf{b}$

usub16 b, a, t — $// \ b = (a_\mathsf{t} - t_\mathsf{t}) \parallel (a_\mathsf{b} - t_\mathsf{b})$

uadd16 a, a, t — $// \ a = (a_\mathsf{t} + t_\mathsf{t}) \parallel (a_\mathsf{b} + t_\mathsf{b})$

return a, b

The results written back by instructions are measured and accounted for in factors of types L (2 instances), I and V (2 instances).

The optimized Cortex-M4 implementation of [4] uses Plantard modular reduction.

Lazy reduction is used:

- the **results of modular multiplications** *are* ***reduced*** during the NTT
- the **results of modular additions/subtractions** *are* ***not reduced*** during the NTT: they are allowed to grow up to 16 bits and are only reduced during the subsequent point-wise multiplication.

[4] Huang *et al.*, “Improved Plantard arithmetic for lattice-based cryptography,” TCHES 2022/4.

Details on the attack environment

- **Software environment**
  - All attack phases are run by Python 3.10.6
  - The computation of messages is JIT-compiled using numba 0.55.1

- **Machine and resources**
  - AMD EPYC 7713P CPU @ 2 GHz, using 32 out of 64 cores (the other cores are used by unrelated tasks)
  - RAM usage peaked at 10 GB
  - All reported times are active CPU time, i.e. in core×hours. Real time is less due to parallelism.

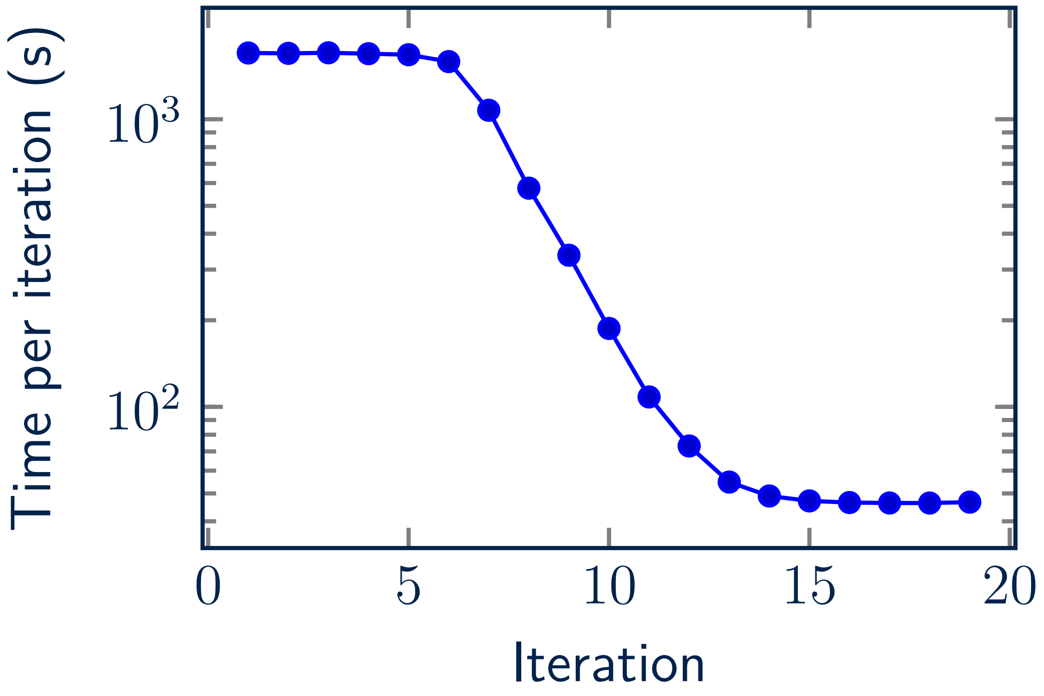
Effects of pruning on runtime

After each message update, we clear all outcomes having probability less than 10^{-8} times the probability of the most-likely outcome

→ Unlikely outcomes are progressively eliminated, making later iterations faster

Due to damping, little to no pruning takes place during the first iterations

Runtime per iteration for a typical execution of the attack against NTT with binomially distributed input, noise $\sigma = 5$



Details on damping and pruning, and stop conditions

- **Damping**: the new value of a message is computed as $m_{i+1} = (1 - \delta)m_i + \delta m'$ where $\delta = 0.95$, m_i is the old value of the message, and m' is given by the message-update equation

- **Pruning**: after each message update, we clear all outcomes having probability less than 10^{-8} times the probability of the most-likely outcome

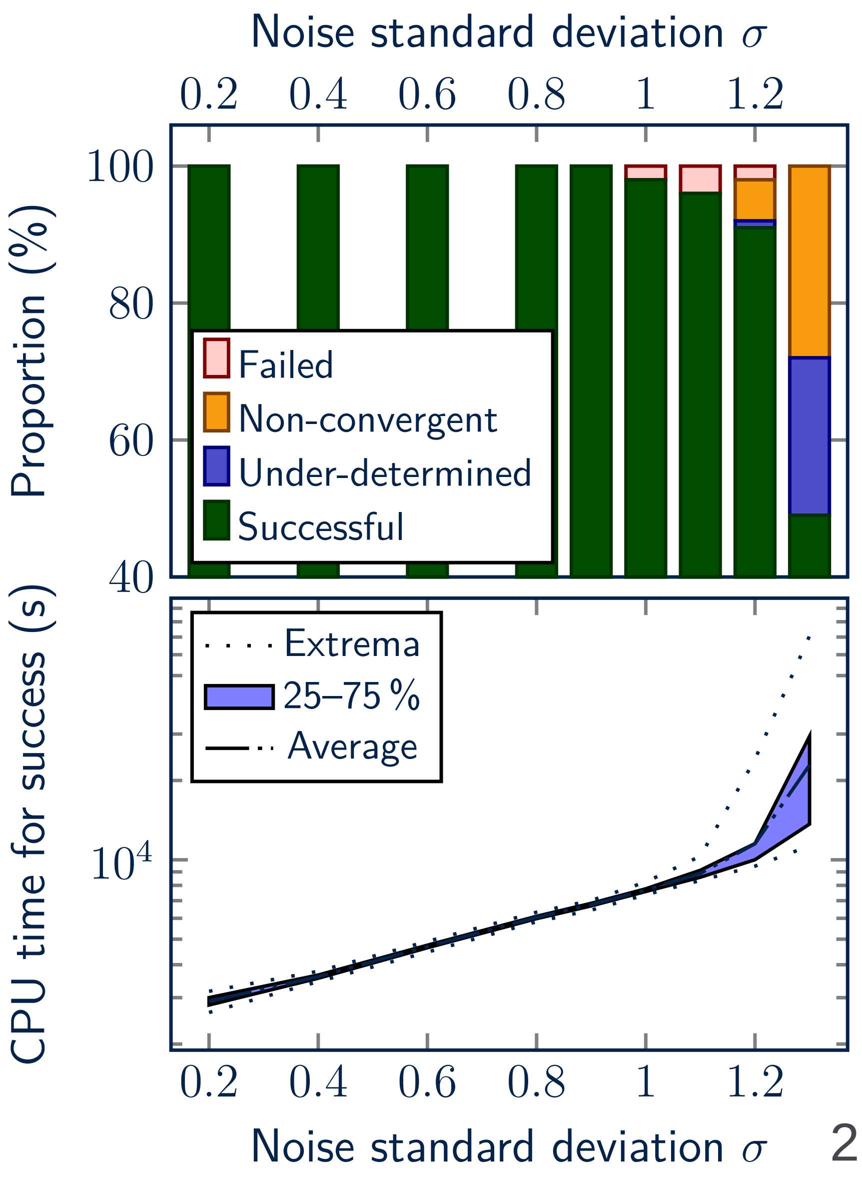
- We stop when reaching any of the **stop conditions**:
  - All messages changed by less than 10^{-5} (absolute value) during the previous iteration, or
  - A message having all-zero outcomes is computed, or
  - 100 iterations have been performed

Possible ending conditions

Four main ending conditions are observed

- **Successful**: belief propagation converges to a single possibility, and the highest-probability outcome is right for all variables. In most such cases, less than 20 iterations have been performed
- **Under-determined**: belief propagation reaches a stable state (messages no longer change) but many outcomes have nonzero probability
- **Non-convergent**: the iteration limit is reached but the messages still change across iterations
- **Failed**: an all-zero message is computed

Situation for a uniform-input NTT→



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