Spacetime Symmetries and Fractons

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Based on 2212.06848, 2107.13884, 2105.01084, 2112.00531

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Outline

- Very Brief introduction to Fractons
- Fractons hydrodynamics
- The local fractonic symmetry group

What is a fracton? Is an excitation with reduced mobility.

Spin liquids

Elasticity

Carrol Theories

Quantum Hall

Fractons

Galileons

...

Haah code

X-cube model

Pretko, et al

Son, et al

Gromov

Vijay, Haah, Fu

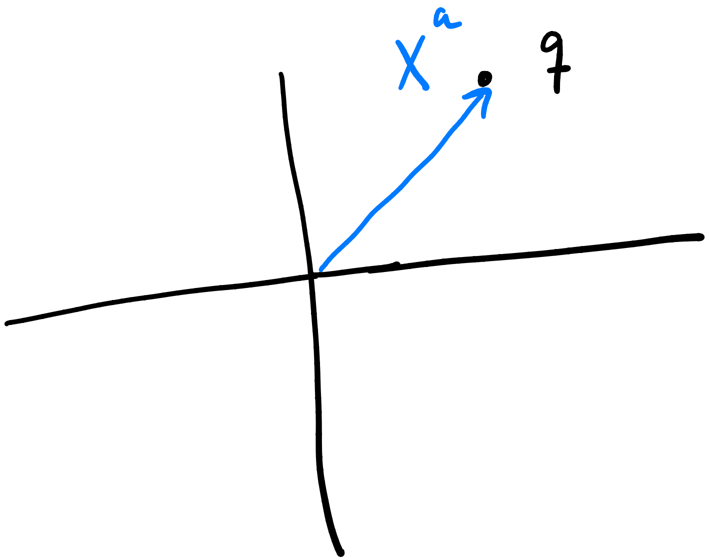
...

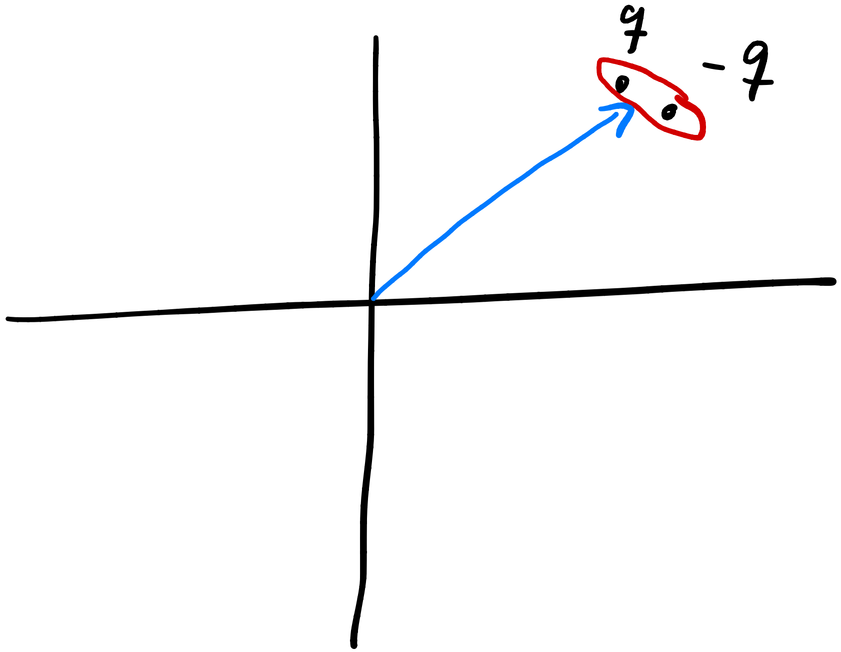
Gapless phases

Are constructed with a scalar (o vector) charge and certain amount of its moments.

Simple case:

U(1) charge + Dipole moment





Q

$$Q = \int d^D x \, \bar{\rho}(x)$$

$$\dot{Q} = 0 \Rightarrow \boxed{\partial_t \bar{\rho} + \partial_a J^a = 0}$$

$$D^a = \int d^D x \left(x^a \bar{\rho}(x) + \bar{\rho}^a(x) \right)$$

$$\dot{D}_a = 0 \Rightarrow \boxed{\partial_t \bar{\rho}^a(x) + \partial_b \bar{K}^{ba} = J^a}$$

this conservation equations can be improved to

$$\boxed{\begin{gathered} \partial_t \rho + \partial_a \partial_b J^{ab} = 0 \\ \rho^a = x^a \rho \;; \quad K^{ab} = x^a \partial_c J^{cb} - J^{ab} \end{gathered}}$$

MDMA

Translations on $Q \Rightarrow \delta_\xi Q = 0$ $\Rightarrow$
on $D_a \Rightarrow \delta_\xi Q_a = \xi Q$

$$[P_a, D_a] = \delta_{ab} Q$$

Introduction of rotations extends the algebra as

$$[L, P] \sim P \quad ; \quad [L, D_a] \sim D_a \quad ; \quad [L, L] \sim L$$

<u>Non-Abelian space symmetry group</u> !

Galileons 1905.05190
Quantum Hall 2103.09826

Carroll algebra Levy-Leblond 1965 ; . . .
Warped Lifschitz 1909.01157

Field theory realizations

$$\Phi$$

$$\delta\Phi = \alpha + \beta_a x^a$$

$$S = \int d^d x \, \mathcal{L}(\dot{\Phi}, \partial^2\Phi, \partial\dot{\Phi})$$

$$\Psi$$

$$\delta\Psi = i(\alpha + \beta_a x^a)\Psi$$

$$S = \int d^d x \Big[|\partial_t \psi|^2 - V(|\psi|^2) - c_1 (\nabla|\psi|^2)^2$$
$$- c_2 \left[\psi^{*2} (\psi \nabla^2 \psi - \nabla\psi \cdot \nabla\psi) + h.c. \right] - c_3 |\psi \partial_i \partial_j \psi - \partial_i \psi \partial_j \psi|^2 \Big]$$

Hydrodynamics

* Effective finite temperature dissipative low energy theory
* d.o.f are conserved charges
* e.o.m. Local conservation equations
* Local thermal equilibrium
* Currents expanded in derivatives of hydro variables with phenomenological coefficients (constrained by symmetries!)

Conserved charges : ε, n, P_i

E.O.M

$$\dot{\varepsilon} + \partial_i j^i_\varepsilon = 0$$

$$\dot{n} + \partial_i \partial_j J_{ij} = 0$$

$$\dot{P}_i + \partial_j T_{ji} = 0$$

Symmetry constraints

Rot. invariance $\Rightarrow$ $T_{ij} = T_{ji}$

$$[D_i, P_j] = \delta_{ij} Q \quad \Rightarrow$$

$$\delta_\beta P_i = -n\beta_i \quad ; \quad \delta_\beta T_{ij} = \partial_k J_{ij}\beta_k - \partial_k J_{kj}\beta_i - \partial_k J_{ki}\beta_j$$

Symmetry constraints & thermodynamics

Usually $\mathcal{E}(s, n, P_i)$ however $\delta_\beta \mathcal{E} = 0 \Rightarrow$ energy density cannot depend on P_i.

Notice $\delta_\beta \left(\frac{P_i}{n} \right) = \beta_i \Rightarrow \delta_\beta\, \partial_i \left(\frac{P_j}{n} \right) = 0$

transforms as a Goldstone

therefore we assume different values for $V_{ij} \equiv \partial_i \left(\frac{P_j}{n} \right)$ label different thermodynamic states (analogy superfluid)

$$d\mathcal{E} = T\,dS + \mu\,dn + F_{ij}\,dV_{ij}$$

Constitutive relations and derivative expansion.

Thermodynamic variables ϵ, n, s, v_{ij} are $\mathcal{O}(\nabla) \sim 0$. Therefore $\mathcal{O}(P_i) \sim -1$ the equations are truncated as follow

$$\dot{\varepsilon} = -\partial_i j_\varepsilon + \mathcal{O}(\nabla)^{2n+3}$$

$$\dot{n} = -\partial_i \partial_j J^{ij} + \mathcal{O}(\nabla)^{2n+3}$$

$$\dot{P}_i = -\partial_j T^{ji} + \mathcal{O}(\nabla)^{2n+2}$$

Here we understand the currents as polynomials of the densities and their derivatives

Constitutive relations at $\underline{\underline{\text{zeroth order}}}$

$$j^i_\varepsilon = (\epsilon + P)V^i - F_{ij}\partial_t\left(\frac{P_j}{n}\right) + \alpha\,\partial_i \frac{1}{T}$$

$$J^{ij} = -F_{ij}$$

$$T^{ij} = P\delta_{ij} + V_i P_j + V_j P_i + \partial_k F_{ij}\frac{P_k}{n} + F_{ij}V_{kk}$$

$$s^i = sV_i + \frac{\alpha}{T}\partial_i \frac{1}{T}$$

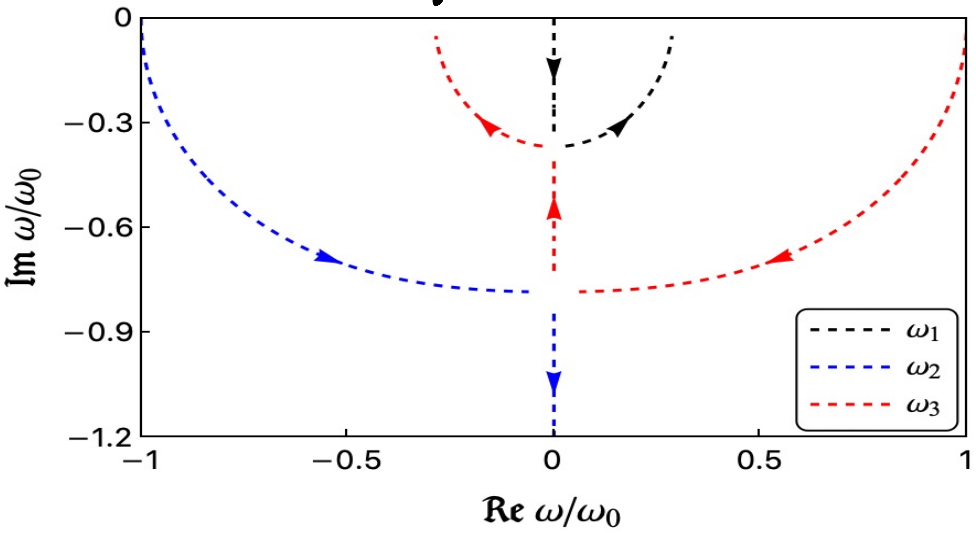
The condition

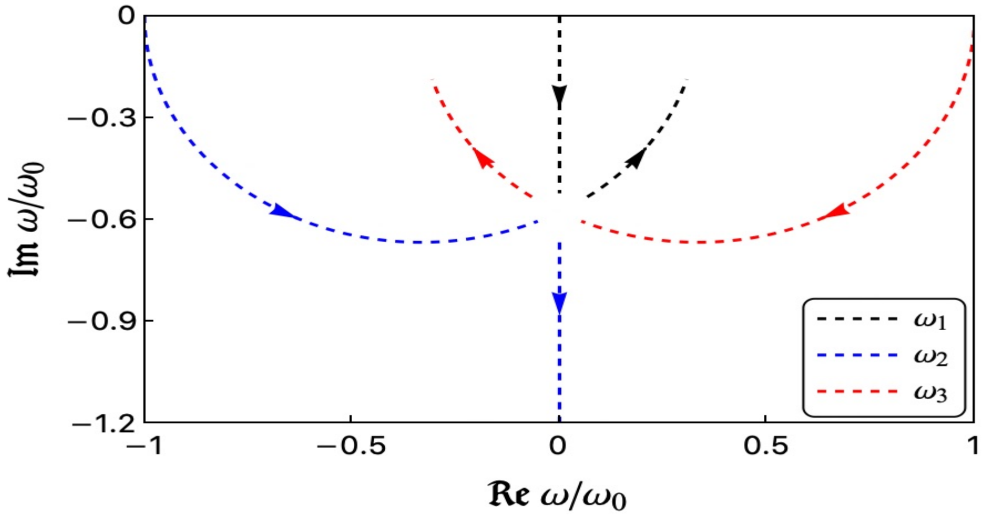
$$\dot{s} + \partial_i s^i \geqslant 0 \implies \alpha \geqslant 0$$

Effective $\underline{\underline{\text{hydro}}}$ velocity

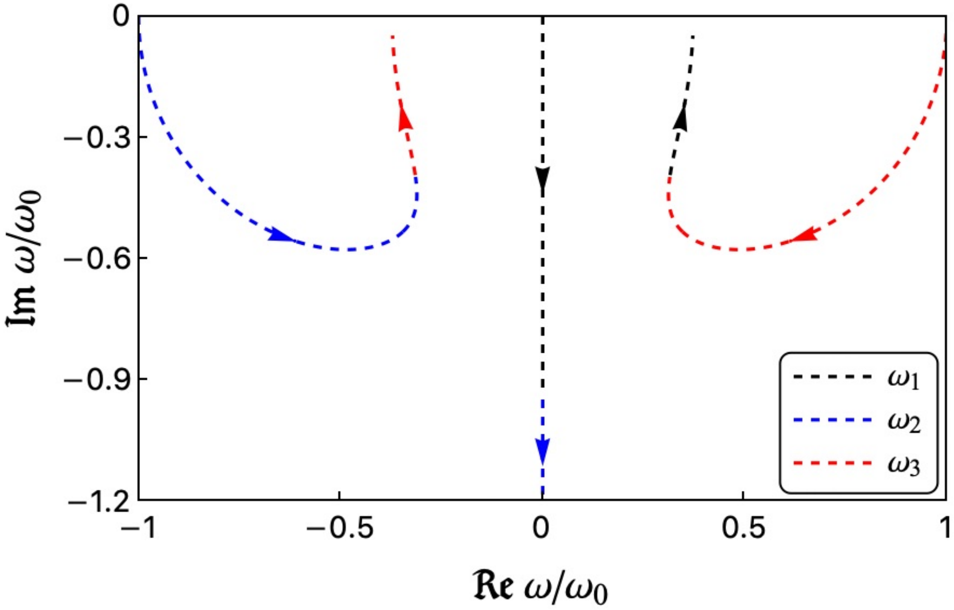
$$V_i = -\frac{\partial_j F_{ji}}{n}$$

Longitudinal modes around $q_i = 0$





a_1/a_2 fixed



$$\left(\frac{\omega}{\omega_0}\right)^3 + i a_2 \left(\frac{\omega}{\omega_0}\right)^2 - \frac{\omega}{\omega_0} - i a_1 = 0$$

$\omega_0 \sim K^2$; $a_1 \sim \alpha$, $a_2 \sim \alpha$

$\frac{a_1}{a_2}$ depends on thermodynamic parameters only

On the other hand the shear (transverse) mode does not disperse

$$\omega = \mathcal{O}(k^4)$$

If we introduce the first order corrections we find 12 transport coefficients. the shear mode becomes

$$\omega \sim i\eta k^4$$ ← subdiffusive.

analogue to shear viscosity

the longitudinal modes receive also k^4 corrections

What happens if the symmetry is local?

Scalar charge Fracton theory vs electromagnetism

Symetric gauge fields

$$S = \int d^{D+1}x \left(E^2 - B^2 + \rho\phi + J^{ij}A_{ij}\right)$$

$$\phi \rightarrow \phi - \partial_t \alpha$$
$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$$

Eq of motion +

$$\boxed{\partial_t \rho + \partial_i \partial_j J^{ij} = 0}$$

Not obvious what to do to make it diff covariant

Electrodynamics

$$S = \int ({}^*F \wedge F + {}^*J \wedge A)$$

$$A \rightarrow A - d\Lambda$$

$$d{}^*F = {}^*J$$

+

$$\boxed{d{}^*J = 0}$$

Diff covariant

1606.08857 ; 1711.11044 ; See also 1807.00827 and 2103.09826

$ISO(D+1)$

$\tilde{c} \longrightarrow 0$

MDMA

$= \text{Heisenberg} \times SO(D)$

$[P_a, J_{b\,D+1}] = \delta_{ab} P_{D+1}$

Translations

transverse rotations

transverse translations

Poincarè $ISO(D,1)$

$c \longrightarrow 0$

Carroll algebra

$= \text{Heisenberg} \times SO(D)$

$[P_a, C_b] = \delta_{ab} H$

Translations

Carroll boosts

Time translations

The gauge theory

$$\mathcal{A} = A^i T^i \longrightarrow F = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$$

$$\delta \mathcal{A} = d\Lambda + [\mathcal{A}, \Lambda]$$

$$\mathcal{A} = e^I P_I + \frac{1}{2}\omega^{ab} L_{ab} + \omega_a Q^a + v Q$$

$$\mathcal{R} = T^I P_I + F_a Q^a + f Q + \frac{1}{2} R^{ab} L_{ab}$$

$$f = dv + e^a \omega_a \quad ; \quad F_a = D\omega_a$$

$$T^0 = de^0 \ ; \ T^a = De^a \quad ; \quad R^{ab} = D\omega^{ab}$$

$$Ds^a = ds^a - \omega^{ab} s^b$$

Under gauge transformations

$\delta e^I = \xi^I$; $\delta \omega_a = D\beta_a$; $\delta v = d\lambda + \beta_a e^a$;

$\delta \omega^{ab} = D\gamma^{ab}$

the curvatures transform as

$\delta F_a \sim \beta_b R^b{}_a$; $\delta f \sim \beta_a T^a$

setting $T^a = 0 \Rightarrow \delta f = 0$

But F_a is not invariant

Notice that $\delta v = d\alpha + \beta_a e^a$ allows us to set $v = A_0 e^0$

Left-over tranformations are

$$\beta_a = -E^\mu{}_a \partial_\mu \Lambda$$

For simplicity we impose

$$T^I = 0$$

f is invariant but does not contain "time derivatives"

F_a is not invariant!

The last constraint we introduce is

$f=0 \Rightarrow$

$$v = A_0 \tau \qquad \omega_a = -E^\mu{}_a \partial_\mu A_0 Z + \underset{\text{Simmetric}}{A_{ab}} e^b$$

$$\delta A_{ab} = E^\mu{}_a E^\nu{}_b \nabla_\mu \nabla_\nu \lambda$$

Symmetric if and only if torsion vanishes

$$\nabla_\mu \xi_\nu = \partial_\mu \xi_\nu - \Gamma^\alpha_{\mu\nu} \xi_\alpha \quad \text{with}$$

$$\Gamma^\alpha{}_{\mu\nu} = t^\alpha \partial_\mu \tau_\nu + E^\alpha{}_a D_\mu e_\nu{}^a$$

If I want a local theory I must introduce a Stuckelberg field ϕ

$$\tilde{F}_a = F_a + E^{\mu}{}_b \partial_\mu \phi R^b{}_a \quad ; \quad \delta\phi = \lambda$$

therefore we can construct a diff + gauge invariant action

$$S = \int d^{n+1}x \sqrt{G} \left[t^\mu t^\alpha h^{\nu\rho} \tilde{F}_{\mu\nu a} \tilde{F}_{\alpha\rho a} - \frac{1}{2} h^{\mu\alpha} h^{\rho\beta} \tilde{F}_{\mu\rho a} \tilde{F}_{\alpha\beta a} \right]$$

An invariant action for the matter fields will have the form

$$S_M = \int \Omega \, \mathcal{L}_M(A_0, A_{ab})$$

and gauge invariance implies

$$\boxed{\partial_0(e\rho) + \partial_a \partial_b (e J^{ab}) = 0}$$

$$\rho = \frac{\partial \mathcal{L}_M}{\partial A_0} \quad ; \quad J^{ab} = \frac{\partial \mathcal{L}_M}{\delta A_{ab}}$$

For the derivation of energy momentum conservation see 2105.01084, 2111.03973

The Fracton electric and magnetic fields in flat space are

$$F_{0b}{}^{a} \equiv \mathcal{E}_b{}^{a} \qquad B_{ab}{}^{c}$$

$$t^\mu E^\nu{}_a F_{\mu\nu\, a} = \partial_a \partial_b A_0 + \partial_0 A_{ab}$$

$$E^\mu{}_a E^\nu{}_b F_{\mu\nu\, a} = \partial_a A_{bc} - \partial_b A_{ac}$$

$$A_0 \longrightarrow A_0 - \partial_t \lambda$$

$$A_{ab} \longrightarrow A_{ab} + \partial_a \partial_b \lambda$$

As expected !!

Conclusions

- Non invariance of momentum $\Rightarrow$ modification of "equilibrium"
  Consistent hydro theory with exotic modes!!
- No gauge invariance without a "Higgs" field for local dynamical theories
- Symmetric gauge field strictly speaking are not gauge fields

thanks