Multimodal analysis and inverse problems for brain imaging

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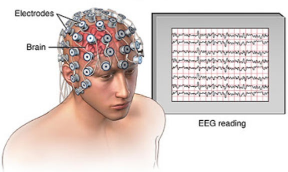
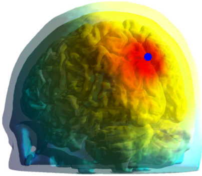
Eletroencephalography



Diffuse optical tomography

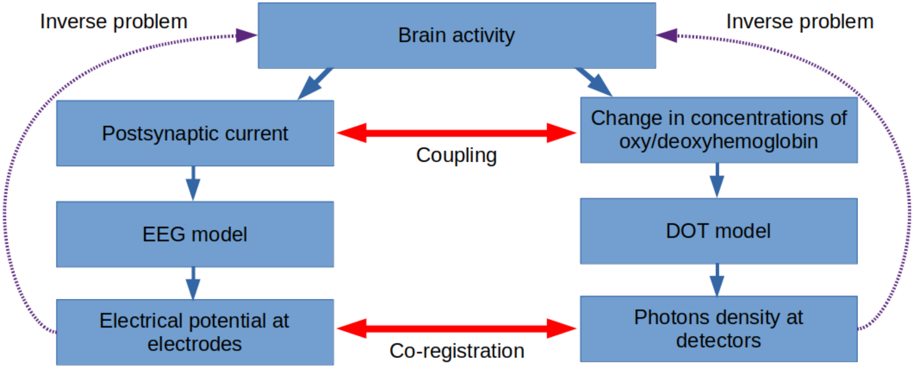


Neurovascular coupling

1. Modelisation and forward problems
   - Neurovascular coupling
   - Computation of EEG measurements
   - Generation of DOT data

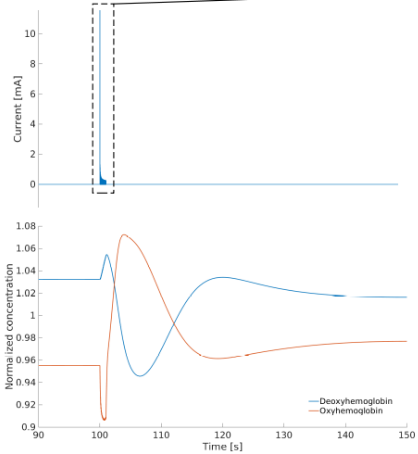
2. Inverse problems
   - EEG source localization
   - DOT parameters identification
   - Numerical coupling

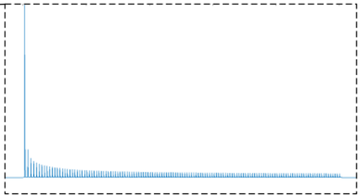
Neurovascular coupling

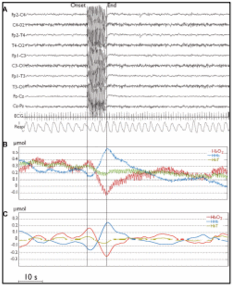


- Generation of coupled measurements
- Resolution of associated inverse problems

Coupled measurements







[1]

- EEG depends on current
- DOT depends on concentrations

1. Roche-Labarbe, NIRS-measured oxy- and deoxyhemoglobin changes associated with EEG, 2008

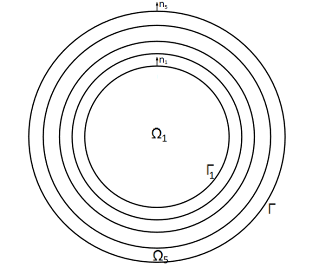
Stationary problem

Common forward problem

The electric potential u is solution of

$$\begin{cases} \nabla \cdot (\sigma \nabla u) & = & F & \text{in } \Omega \\ \sigma \partial_n u & = & 0 & \text{on } \partial\Omega \end{cases}$$

with σ the electrical conductivity of the head tissues.



Dipolar current source

$$F = \sum_{m=1}^{M} \mathbf{q}_m \cdot \nabla \delta_{S_m}$$

where $\mathbf{q}_m \in \mathbb{R}^d$ is the moment and $S_m \in \Omega_2$ the position of the source of neuronal activity constrained to grey matter.

Subtraction approach

Decomposition of the potential

The substraction approach constists in decomposing the potential into a potential $\tilde{u}$ which contains the singularity and a regular lifting w :

$$u = \tilde{u} + w.$$

Singular potential

The singular potential is solution of a Poisson equation and is given by

$$\tilde{u}(\mathbf{x}) = \frac{1}{2^{d-1}\pi\sigma_2}\mathbf{q} \cdot \frac{\mathbf{x} - S}{|\mathbf{x} - S|^d},\ \forall x \in \mathbb{R}^d \setminus \{S\}.$$

Regular part

The regular part is obtained by solving the variational problem

$$\int_\Omega \sigma \nabla w \cdot \nabla v \mathrm{d}x = \int_\Omega (\sigma_2 - \sigma) \nabla \tilde{u} \cdot \nabla v \mathrm{d}x - \int_{\partial\Omega} \sigma_2 \partial_n \tilde{u} v \mathrm{d}s.$$

Time-dependent EEG model

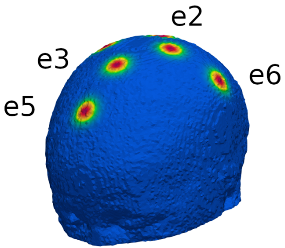
Elliptic equation

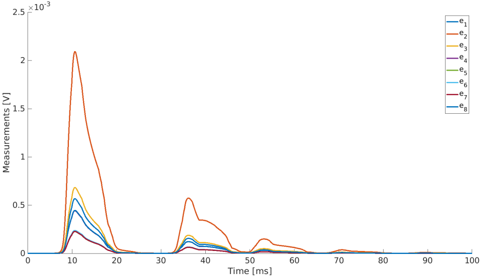
By means of a dimensional analysis and classical arguments we have

$$\nabla \cdot (\sigma \nabla u) = \nabla \cdot \mathbf{j}^p = \sum_{m=1}^{M} \mathbf{q}_m(t) \cdot \nabla \delta_{S_m(t)} \quad \text{in } (0, T) \times \Omega$$

Modelisation of moments

$\mathbf{q}_m$ is generated by a system of ODEs (e.g. Hodgkin-Huxley).





DOT forward problem

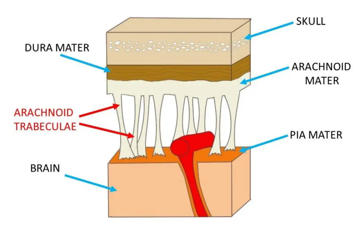
Forward problem

The photons density ϕ is solution of

$$\begin{cases} -\nabla \cdot (\kappa \nabla \phi) + \left(\mu_a + \frac{i\omega}{c}\right)\phi & = & q & \text{in } \Omega \\ \phi + 2\chi\kappa\partial_n\phi & = & 0 & \text{on } \partial\Omega \end{cases}$$

Parameters

- μ_a [1/mm] absorption
- μ_s' [1/mm] reduced scattering
- $\kappa = \frac{1}{3(\mu_a + \mu_s')}$ [mm] diffusion
- χ internal reflexion
- ω source frequency



- $\mu^{AT+CSF} = \delta\mu^{AT} + (1-\delta)\mu^{CSF}$

Time-dependent forward solution

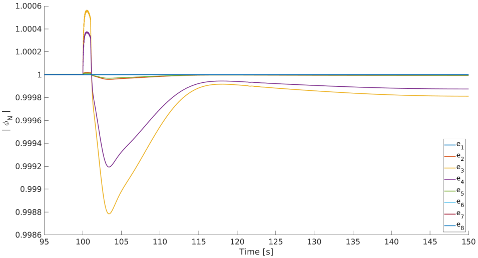
Time-depend DOT forward problem

The photons density ϕ is solution of

$$\begin{cases} -\nabla \cdot (\kappa(t,\mathbf{x})\nabla\phi(t,\mathbf{x})) + \left(\mu_a(t,\mathbf{x}) + \frac{i\omega}{c}\right)\phi(t,\mathbf{x}) & = & q \text{ in } (0,T)\times\Omega \\ \phi(t,\mathbf{x}) + 2\chi\kappa(t,\mathbf{x})\partial_n\phi(t,\mathbf{x}) & = & 0 \text{ on } (0,T)\times\partial\Omega \end{cases}$$

Parameters generation

μ_a and κ come from a system of ODEs.



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Identifiability for L^2 source term

Theorem

(H_1) $\mathbf{q}_m \in L^2(0,T)^d$ and $S_m \in C^0([0,T])^d$, $\forall m \in \{1,\dots,M\}$.

(H_2) At time t, the points $(S_m(t))_m$ are mutually distinct

(H_3) All sources are located in the same convex subdomain Ω_{p_0}

Then, the locations and the moments are uniquely determined from a single measurement $g =: u_{|(0,T),\partial\Omega}$.

Lead field matrix

- Consider a source space $\mathcal{S} = \{\mathcal{S}_1, \dots, \mathcal{S}_R\} \subset \Omega$
- $u_m^{(i)}$ is the solution of the problem with the source term

$$F_m^{(i)} = \mathbf{e}^{(i)} \cdot \nabla\delta(\cdot - \mathcal{S}_m)$$

- $(\mathbf{p}_\ell)_{\ell=1,\dots,L}$ are the measurement positions
- $u_m(\mathbf{p}_\ell) = \left(u_m^{(1)}(\mathbf{p}_\ell), u_m^{(2)}(\mathbf{p}_\ell)\right)$ and $\mathbf{q}_m = \left(\mathbf{q}_m^{(1)}, \mathbf{q}_m^{(2)}\right)$

Distributed source model

The measurements $\mathbf{u}_{EEG}$ of u at points $(\mathbf{p}_\ell)$ are then given by

$$\mathbf{u}_{EEG} = \begin{pmatrix} u_1(\mathbf{p}_1) & u_2(\mathbf{p}_1) & \cdots & u_R(\mathbf{p}_1) \\ u_1(\mathbf{p}_2) & u_2(\mathbf{p}_2) & \cdots & u_R(\mathbf{p}_2) \\ \vdots & \vdots & \ddots & \vdots \\ u_1(\mathbf{p}_L) & u_2(\mathbf{p}_L) & \cdots & u_R(\mathbf{p}_L) \end{pmatrix} \begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_R \end{pmatrix} := \mathbb{L}\mathbf{q}$$

Lead field basis [2]

Let w_h be the nodal vector solution of the discrete problem

$$\mathbb{K}w_h = \mathbf{F},$$

with $\mathbf{F}$ depending on of the singular potential $\tilde{u}$. We can obtain the measurements by means of a restriction matrix $\mathbb{R} \in \mathcal{M}_{L,R}$,

$$\mathbf{w}_{EEG} = \mathbb{R}w_h = \mathbb{R}\mathbb{K}^{-1}\mathbf{F}.$$

To compute $\mathbb{B} = \mathbb{R}\mathbb{K}^{-1}$ advantageously, notice that

$$\mathbb{B} = \mathbb{R}\mathbb{K}^{-1} \Leftrightarrow \mathbb{B}^t = \mathbb{K}^{-t}\mathbb{R}^t \Leftrightarrow \mathbb{K}^t\mathbb{B}^t = \mathbb{R}^t \Leftrightarrow \mathbb{K}\mathbb{B}^t = \mathbb{R}^t$$

We can now apply the subtraction approach

$$\mathbf{u}_{EEG} = \mathbb{R}(\tilde{u}_h + w_h) = \mathbb{B}(\mathbb{K}\tilde{u}_h + \mathbb{K}w_h) = \mathbb{B}(\mathbb{K}\tilde{u}_h + \mathbf{F})$$

Finally we can define $\mathbb{L}$ such as

$$\mathbb{L} = \mathbb{B}(\mathbb{K} + \tilde{\mathbb{K}})\mathbb{A}$$

where $\tilde{\mathbb{K}}$ is such that $\mathbf{F} = \tilde{\mathbb{K}}\tilde{u}_h$ and $\mathbb{A}$ satisfies $\tilde{u}_h = \mathbb{A}\mathbf{q}$.

2. Wolters,Efficient Computation of lead field bases and influence matrix for the FEM-based EEG and MEG inverse problem, 2004

EEG inverse problem

Minimization problem

$$\begin{cases} \text{Find } \mathbf{q}_m \in \mathbb{R}^{(dR)} \text{ such that} \\ \mathbf{q}^* = \arg \min_{\mathbf{q} \in \mathbb{R}^{(dR)}} \|\mathbb{L}\mathbf{q} - \mathbf{u}_{EEG}\|^2 + \alpha \|\mathbf{q}\|^2. \end{cases}$$

sLORETA approach

- Minimum norm estimates

$$\mathbf{q}^* = \mathbb{L}^t(\mathbb{L}\mathbb{L}^t + \alpha\mathbb{I})^{-1}\mathbf{u}_{EEG}$$

- sLORETA

$$\tilde{\mathbf{q}}_i^* = (\mathbf{q}_i^*)^t([\mathbf{S}^*]_{ii})^{-1}\mathbf{q}_i^*$$

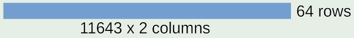
with $\mathbf{S}^*$ defined by

$$\mathbf{S}^* = \mathbb{L}^t(\mathbb{L}\mathbb{L}^t + \alpha\mathbb{I})^{-1}\mathbb{L}$$

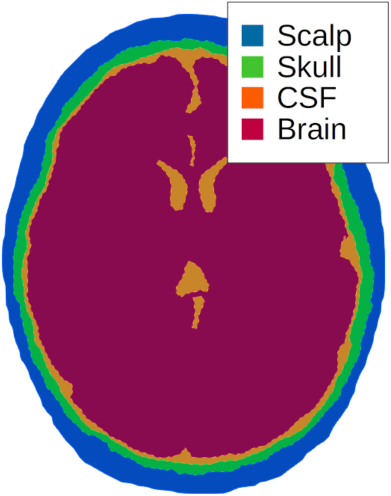
Parameters and informations

Characteristics of the problem

- Mesh comes from a MRI image
- 64 electrodes
- 11643 sources
- Lead field matrix shape

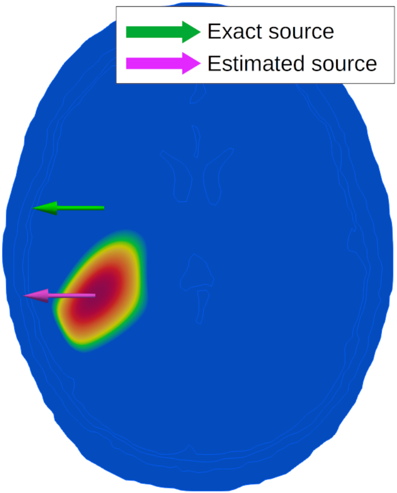


- Computation LFM $\sim$ 5h
- Resolution $\sim$ 4.2s
- 2% noise on measurements



Layers of the mesh

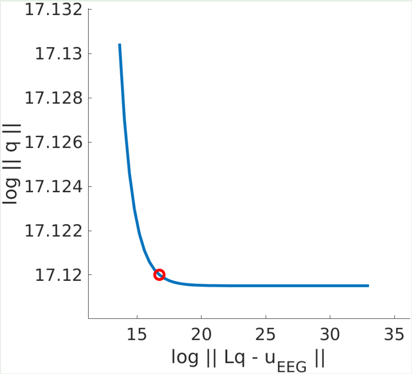
Source localization



Exact and estimated source

Results

- Computation of L-curve



- 2.5 cm error on location
- Wrong amplitude

DOT inverse problem

Cost function

The inverse problem consists in minimizing the cost function

$$J(\mu_a, \kappa) = \left\| \frac{\phi[\mu_a, \kappa] - \Phi}{\Phi} \right\|_{\partial\Omega}^2 + \frac{\alpha_\mu}{2} \left\| \frac{\mu_a - \mu_{a,0}}{\mu_{a,0}} \right\|_{\Omega}^2 + \frac{\alpha_\kappa}{2} \left\| \frac{\kappa - \kappa_0}{\kappa_0} \right\|_{\Omega}^2$$

Cost function gradient

The cost function gradient is given by

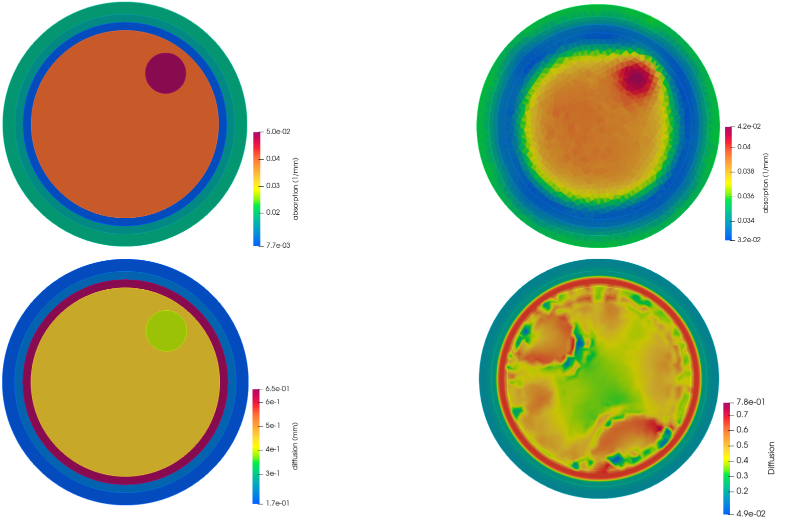
$$\frac{\partial J}{\partial \nu}(\nu) = \mathfrak{Re} \left\langle \frac{\partial \phi}{\partial \nu}(\nu), \frac{\phi(\nu) - \Phi}{|\Phi|^2} \right\rangle_{\partial\Omega}$$

Adjoint method

To reduce the computation of the gradient, we solve the adjoint problem

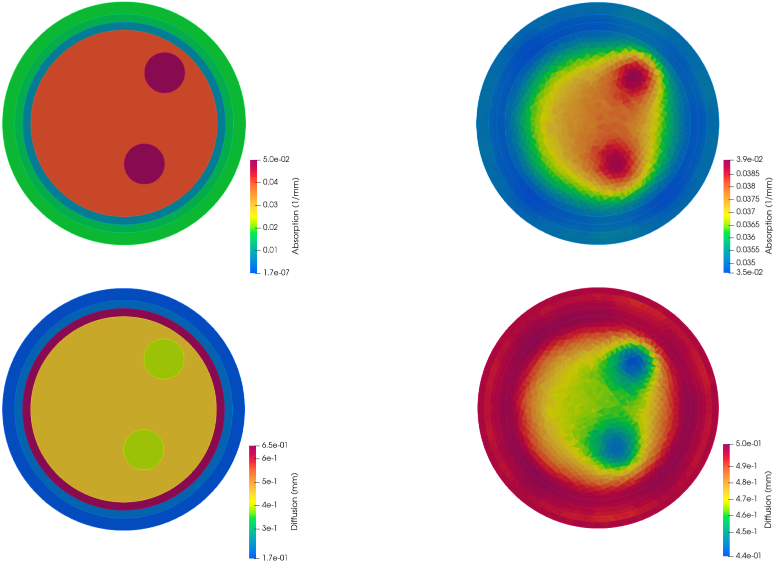
$$\int_\Omega \kappa \nabla v \cdot \nabla \bar{p} \mathrm{d}x + \int_\Omega \left(\mu_a + \frac{i\omega}{c} \right) v \bar{p} \mathrm{d}x + \frac{1}{2\chi} \int_{\partial\Omega} v \bar{p} \mathrm{d}s = -\mathfrak{Re} \left\langle v, \frac{\phi - \Phi}{\Phi} \right\rangle_{\partial\Omega}$$

Difficulties to find regularization parameters



Identification of absorption and diffusion coefficients

Parameter identification



Identification of absorption and diffusion coefficients

Coupling EEG to DOT

Informations given by EEG

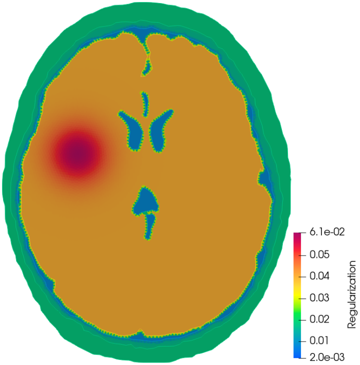
- Estimated position $\tilde{S}_m$
- Estimated moment $\tilde{\mathbf{q}}_m$

In regularization term

We can use these informations to constrain the reconstruction around an estimated function

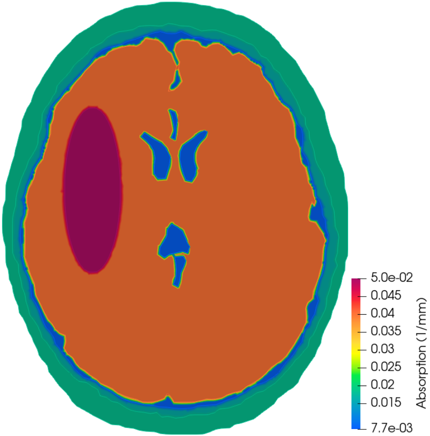
$$J(\mu_a) = \frac{1}{2}\left\|\frac{\phi - \Phi}{\Phi}\right\|^2 + \frac{\alpha}{2}\left\|\frac{\mu_a - \tilde{\mu}_{a,0}}{\tilde{\mu}_{a,0}}\right\|^2$$

with $\tilde{\mu}_{a,0} = \mu_{a,0} + A\exp\left(-\frac{1}{B}\|\mathbf{x} - \tilde{S}_m\|^2\right)$

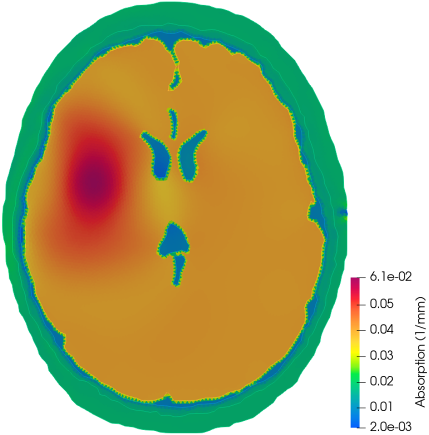


Regularization term

Absorption coefficient reconstruction



Absorption coefficient



Absorption coefficient reconstruction

From DOT to EEG

Informations given by DOT

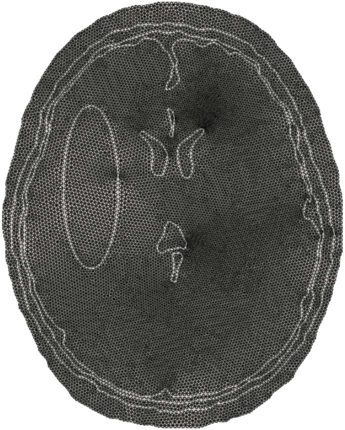
- Perturbation location
- Value in the perturbation

Source space reduction

We can use the location to include it in the mesh and reduce the source space :

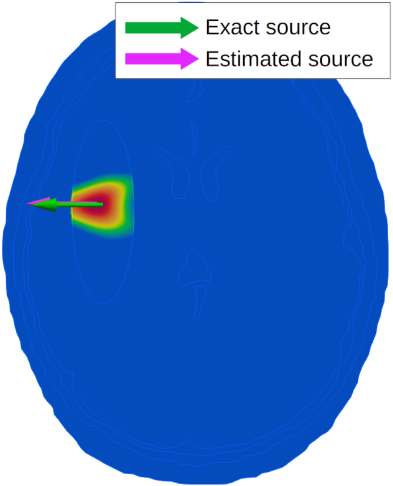
- Thresholding the solution
- Mesh the thresholding
- Use the new mesh for EEG problem

Source space before reduction : 11600
Source space after reduction : 750



Include the perturbation in the mesh

Source space reduction



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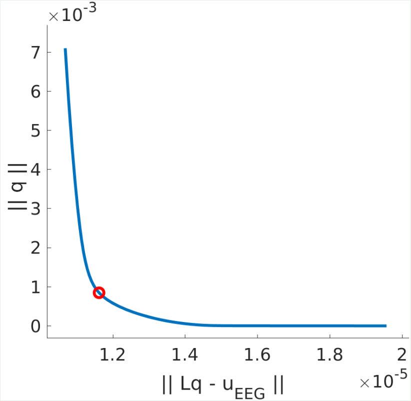
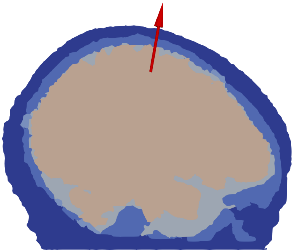


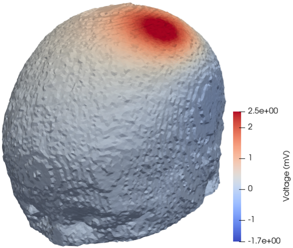
- 0.5 mm error on location
- Wrong amplitude

Conclusion and perspectives

Conclusion

- Time-depending models
- Coupled measurements
- Numerical inverse problem coupling





Perspectives

- 3D inverse problems
- Method to find a DOT regularization parameter
- Time depend resolution of (coupled) inverse problems [3]

3. Gramfort, Time-Frequency Mixed-Norm Estimates : Sparse M/EEG imaging with non-stationary source activations, 2013