Numerical and Theoritical Study of Patterns in a Chemotaxis Model

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Models of biological interactions are in full swing. They come in different flavors, one of them being systems of partial differential equations, which are then studied by mathematicians. Here, we will be interested in the establishment of stationary solutions to problems of the form

$$\partial_t U = \Delta(\Phi\, U) + f(U)\,,$$

with a computer-assisted method. Starting from a known approximate solution, we get back to study a fixed point problem to solve our initial problem. This fixed point then becomes our existing and unique theoretical solution in a neighbourhood of our approximate solution. The difficulty lies in the choices we make when reducing the problem. In particular, the non-linearity of the equations is a significant obstacle. More precisely we will look at a chemotaxis model where

$$\begin{cases} \partial_t u &= \Delta(\gamma(v)u) &+\sigma u(1-u)\,, \\ \partial_t v &= \varepsilon\Delta v &+u-v\,. \end{cases}$$

In this type of model the search for patterns and the study of their stability is interesting, which justifies our numerical to theoretical approach. We will look at different results according to the chosen function γ : rational fraction, decreasing exponential, power series, ...