Dynamics of the isothermal compressible Euler system with damping

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We will be interested at the long time asymptotic of the compressible Euler system with damping

$$\begin{cases} \partial_t \rho + \partial_x m = 0 \,, \\ \partial_t m + \partial_x \left(\dfrac{m^2}{\rho} \right) + \partial_x \rho^\gamma + m = 0 \,, \end{cases}$$

in one dimensional space $x \in \mathbb{R}$ and with $\gamma \geq 1$. This system satisfies several conservation laws such as mass conservation and energy dissipation:

$$\|\rho(t)\|_{L^1} = \|\rho_0\|_{L^1} \quad \text{and} \quad \frac{1}{2}\int_{\mathbb{R}} \frac{m^2}{\rho} + \frac{1}{\gamma - 1}\int_{\mathbb{R}} \rho^\gamma + \int_0^t \int_{\mathbb{R}} \frac{m^2}{\rho} \leq E_0 \,.$$

It is now well established that in the isentropic case $\gamma > 1$, solutions of this system strongly converge in $L^1(\mathbb{R})$ to Barenblatt solutions, since the work of [3]. For almost two decades, a lot of effort has been put in order to improve this convergence rate and to extend the range of admissible pressure laws, to finally achieve the full range $\gamma > 1$ in the recent work [2].

In this talk we will especially be interested by the isothermal case $\gamma = 1$ [1]. We will show that this system admits a set of particular Gaussians solutions, whose complete evolution can be explicitly computed through the study of a nonlinear differential equation of order 2. We will then state that under some regularity assumptions, any solution of the isothermal system weakly converges to a universal limit Gaussian profile, as well as its first moments.

We will conclude by discussing several open questions, in particular the achievement of precise convergence rates in the isothermal case.

References

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[3] F. Huang, P. Marcati, and R. Pan. Convergence to the Barenblatt solution for the compressible Euler equations with damping and vacuum. *Archive for Rational Mechanics and Analysis*, 176(1):1–24, 2005.