A HIGH-ORDER SCHEME FOR ADVECTION-DIFFUSION PRESERVING POSITIVITY AND LONG-TIME BEHAVIOUR

Julien MOATTI

Inria Lille

julien.moatti@inria.fr

We present a high-order scheme to discretise anisotropic advection-diffusion models. The scheme under consideration is based on a HHO / HDG+ spatial discretisation [1], which is devised to handle anisotropic diffusion tensors on general polyhedral meshes with arbitrary approximation orders. Adapting the ideas of the low-order schemes of [2, 3] to the high-order framework, we introduce a nonlinear arbitrary order scheme which:

(i) preserves the positivity of the discrete solution;

(ii) mimics the long-time behaviour of the continuous solution, i.e. that the discrete solution converges exponentially fast towards a discrete equilibrium when time tends to infinity.

The analysis of this scheme relies on a discrete entropy/entropy dissipation relation, which mimics the entropy structure of the continuous model.

We will illustrate the main properties of the scheme with numerical examples, and give numerical evidences of the interest of using high-order approximations compared to low-order ones.

References

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