Boltzmann mean-field game model for knowledge growth

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Subject of this talk will be a Boltzmann mean field game (BMFG) model for knowledge growth, originally introduced by the economists Lucas and Moll [4]. In BMFG the evolution of the agent density with respect to their knowledge level is described by a Boltzmann-type equation. Agents increase their knowledge through binary interactions with others; their increase is modulated by the interaction and learning rate: Agents with similar knowledge learn more in encounters, while agents with very different levels benefit less from learning interactions. The optimal fraction of time spent on learning is calculated by a Bellman equation, resulting in a highly nonlinear forward-backward in time PDE system.

The structure of solutions to the Boltzmann and Bellman equation depends strongly on the aforesaid learning rate in the Boltzmann collision kernel as well as the utility function in the Bellman equation, which relates the individual's productivity to the expected gain in society and encodes risk aversion in the society. We explain the monotonicity behavior of solutions for different learning and utility functions, show existence of solutions and investigate how they impact the existence of so-called balanced growth path solutions, that relate to exponential growth of the overall economy. The talk will be concluded by computational experiments to illustrate our analytical results.

References

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