Covariance modulated optimal transport: geometry and gradient flows

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This talk is about a novel variant of optimal mass transport: we consider the dynamical Benamou-Brenier formulation with a mobility that is modulated by the covariance of the density. This gives rise to a modulated Wasserstein metric which provides a rigorous gradient flow formulation of the mean-field limit in certain ensemble Kalman methods for solving inverse problems. In combination with the abstract machinery of contractive metric gradient flows, the new metric is an effective tool to study the rate of convergence of these methods.

I shall present several analytic results about the modulated Wasserstein metric. The first is the splitting representation, which allows to write the modulated metric as the sum of two simpler metrics, one measuring the distance in terms of first and second moments, the other one measuring in terms of shapes. The second result is about geodesic convexity and the related rates of convergence in gradient flows. Specifically, we prove exponential equilibration in linear Fokker-Planck equations with Gaussian steady states at a rate that does not depend on the covariance of the Gaussian. The third and only partial result is about geodesics, that we prove to exist for sufficiently close densities, or densities with multiple reflection symmetries. We also characterize geodesics in terms of particle trajectories, that are no longer straight line as in the genuine Wasserstein metric, but follow more complicated curves that satisfy second order ordinary differential equations.

This is joint work with Andre Schlichting, Matthias Erbar, Franca Hoffmann and Martin Burger.