Sharp-interface limits in the dynamics of phase transitions: from the Allen-Cahn equation to liquid crystals

Tim LAUX

University of Bonn

tim.laux@hcm.uni-bonn.de

The large-scale behavior of phase transitions has a long history. In this talk, I want to present two recent projects which establish convergence results based on a new relative entropy for phase-field models. With Julian Fischer and Theresa Simon, we prove optimal convergence rates for the Allen-Cahn equation to mean curvature flow before the onset of singularities. The proof does not rely on the maximum principle and does not require understanding the spectral properties of the linearized Allen-Cahn operator.

With Yuning Liu, we consider the dynamics in the Landau-de Gennes theory of liquid crystals. We show that at the critical temperature, a scaling limit can be derived: The interface between the isotropic and nematic phases moves by mean curvature flow. Furthermore, in the nematic phase, the director field is a harmonic map heat flow with homogeneous Neumann boundary conditions. To derive the equations, we combine the relative entropy method with weak convergence methods.