Hydrodynamic limits for kinetic equations: a spectral and unified approach

Pierre GERVAIS

Université de Turin

pierre.gervais@ens.fr

The Boltzmann equation, introduced by J.C. Maxwell and L. Boltzmann at the end of the 19th century, describes the evolution of a gas at the molecular and statistical level, when the particles tend to move in a straight line but their velocities are influenced by interactions between pairs of particles. More precisely, instead of considering the exact position and speed of each of the particles constituting the gas, we are interested in their statistical distributions for a typical particle (we speak of a mesoscopic or kinetic point of view). In the case of plasmas, a variant was introduced by L. Landau in 1936, and other variants were progressively presented to take into account quantum effects, relativistic effects, etc.

In 1900, D. Hilbert presented a list of 23 problems, the sixth one being the "Problem of axiomatization of physics". In the case of fluid mechanics, it consists of the derivation of hydrodynamic equations (macroscopic point of view) from kinetic equations (mesoscopic point of view), which must themselves be derived from Newton's equations applied to all the particles making up the gas (microscopic point of view).

From the 90s up to today, starting from the founding papers of C. Bardos, F. Golse and D. Levermore, several works focused on the derivation of the Navier-Stokes equations from the Boltzmann and Landau equations, or their variants, for weak solutions or strong solutions. However, the different models were studied independently, and most of them assumed very strong integrability conditions for the initial data (of the form $e^{|v|^2} f \in L^2$).

In collaboration with B. Lods, we develop a unified theory allowing to derive the Navier-Stokes equations from most of the kinetic models mentioned above, with explicit rates and to consider initial data with low integrability (of the form $(1 + |v|^k) f \in L^2$).