Quantum thermodynamics and transport

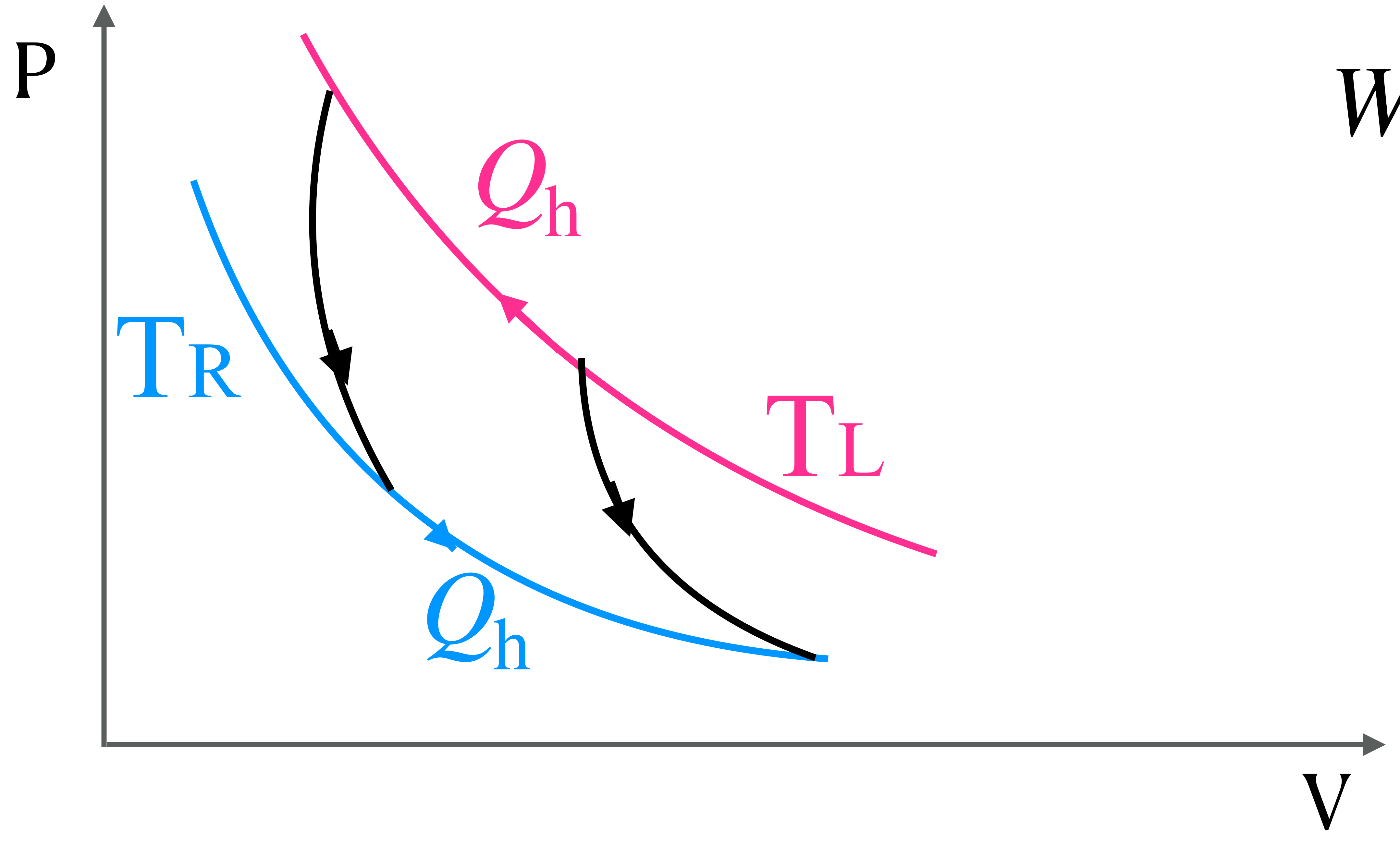
Focus on heat-work conversion

Lecture 3

Liliana Arrachea (2023)

Thermodynamic cycles

Carnot heat engine

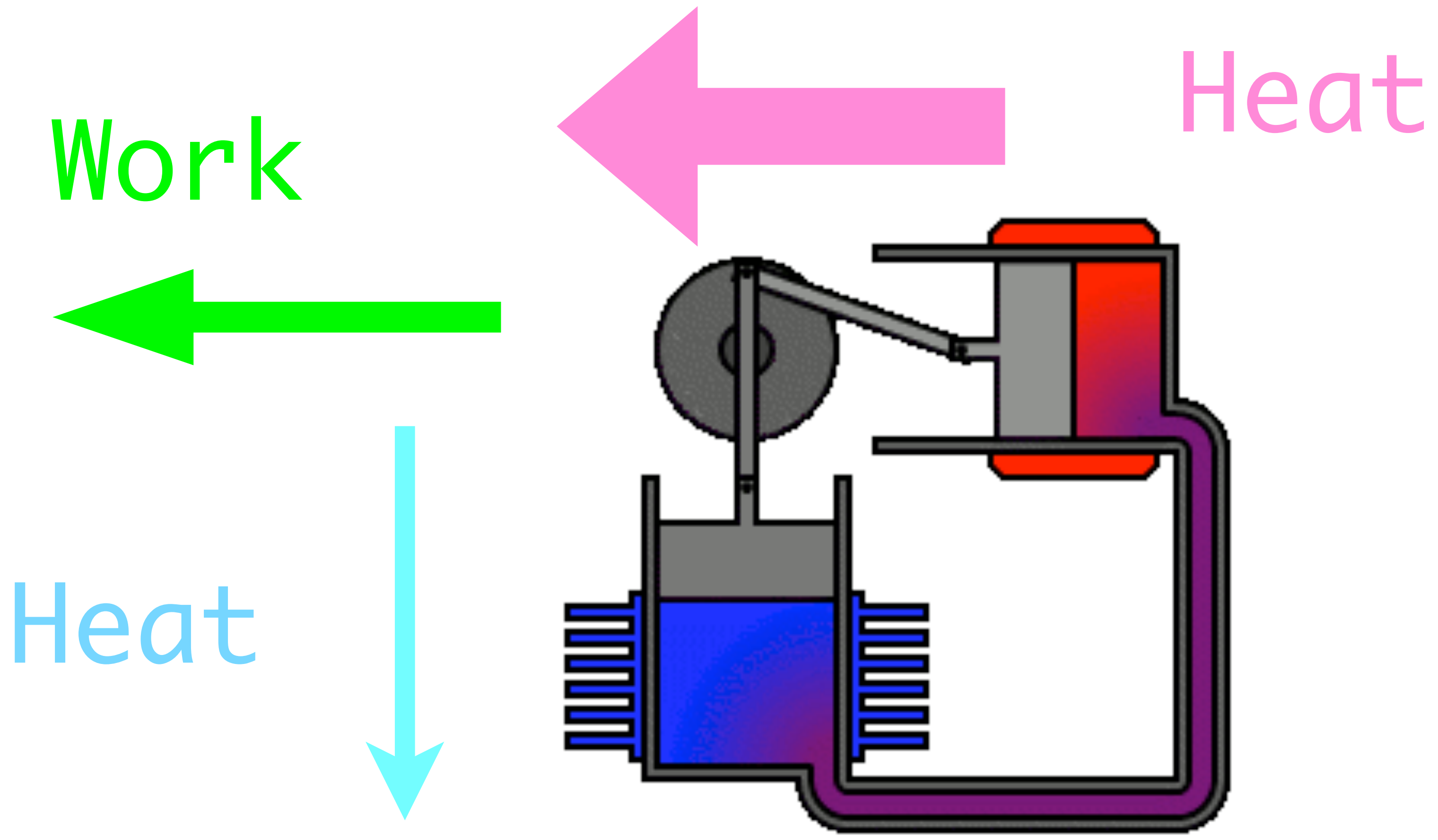


$$W = Q_h + Q_c.$$

$$\sum_\alpha \Delta S_\alpha = \sum_\alpha \frac{Q_\alpha^{(qs)}}{T_\alpha} = 0.$$

Heat-work conversion

$$W^{(cons)} = Q_h^{(qs)} \frac{(T_h - T_c)}{T_h}$$



Finite-time efficiency

$$\eta = \frac{W}{Q} \leq \eta_C$$

Power

$$P = \frac{W}{\tau_C}$$

Single ion heat engine with maximum efficiency at maximum power

O. Abah,[1] J. Roßnagel,[2] G. Jacob,[2] S. Deffner,[1,3] F. Schmidt-Kaler,[2] K. Singer,[2] and E. Lutz[1,4]
[1] *Department of Physics, University of Augsburg, D-86159 Augsburg, Germany*
[2] *Institut für Quantenphysik, Universität Mainz, 55128 Mainz, Germany*
[3] *Department of Chemistry and Biochemistry and Institute for Physical Sciences and Technology, University of Maryland, College Park, MD 20742, USA*
[4] *Dahlem Center for Complex Quantum Systems, FU Berlin, D-14195 Berlin, Germany*

PRL 109, 203006 (2012)

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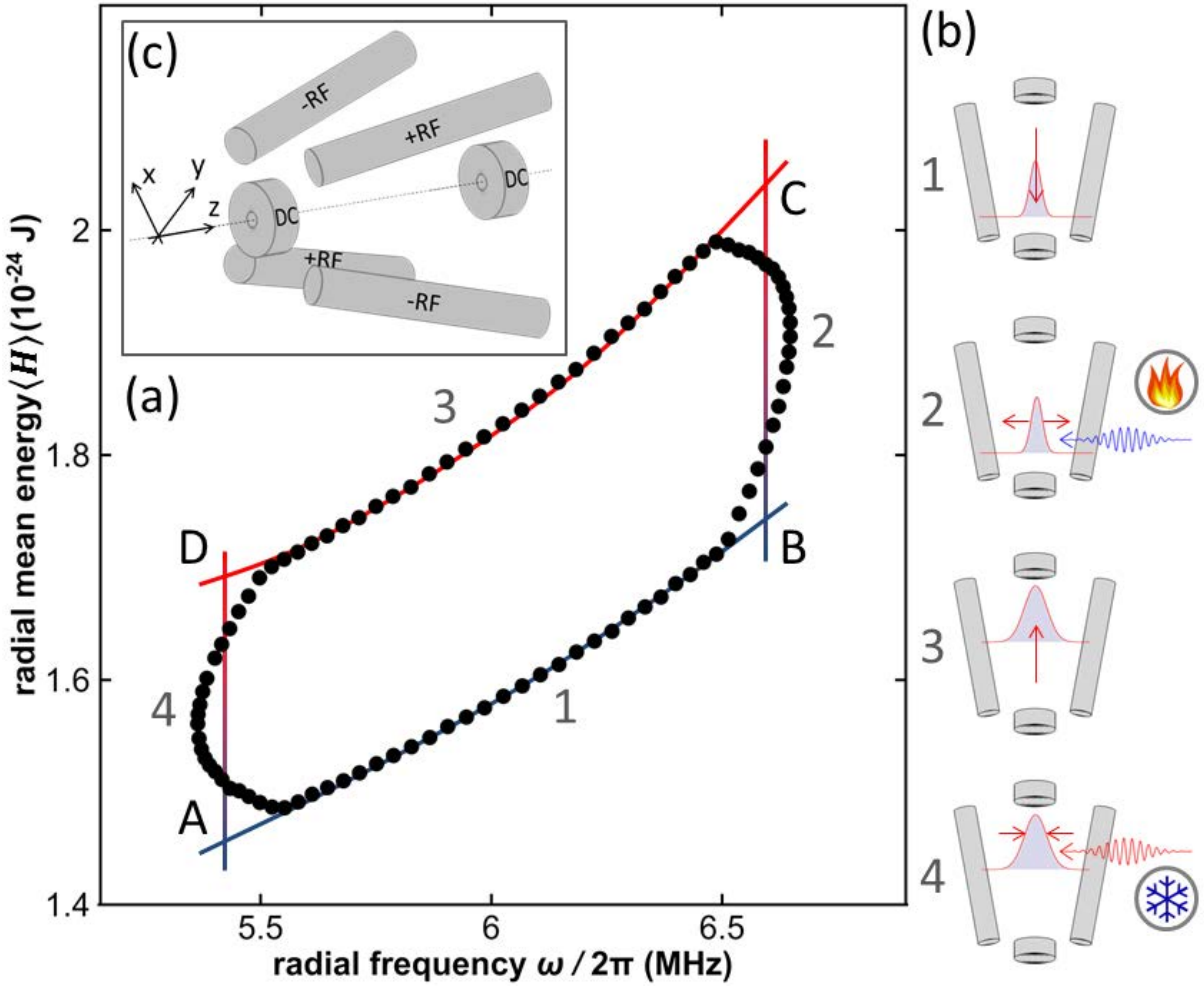
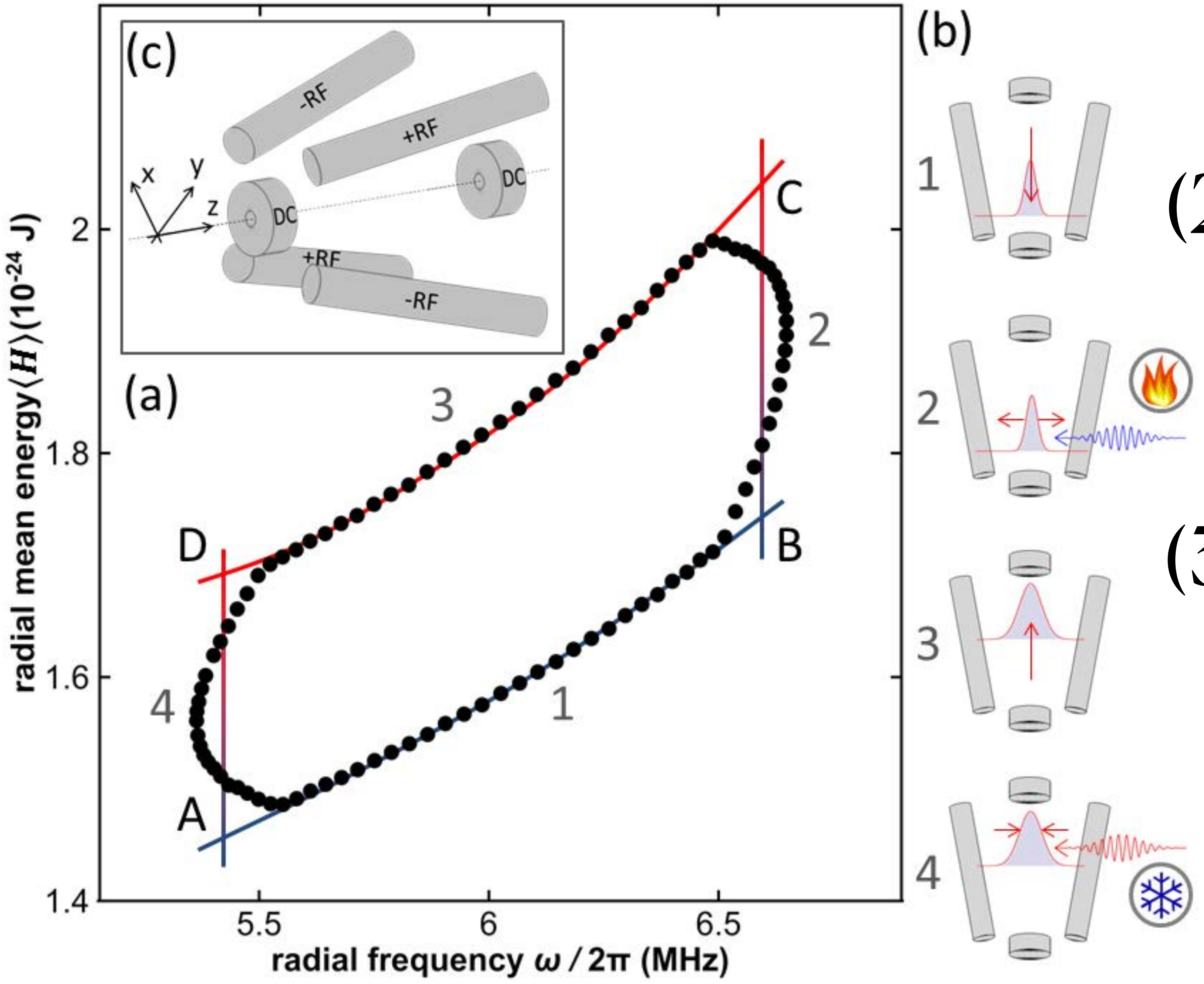


$$H(t) = \frac{p^2}{2m} + \frac{1}{2}\, m\omega^2(t)\, x^2,$$

(1) Isentropic compression: $A(\omega_1, \beta_1) \rightarrow B(\omega_2, \beta_1)$



$$\langle W_1 \rangle = \langle H \rangle_B - \langle H \rangle_A = \left(\frac{\hbar\omega_2}{2} Q_1^* - \frac{\hbar\omega_1}{2} \right) \coth\left(\frac{\beta_1 \hbar\omega_1}{2} \right)$$

(2) Hot isochore: $B(\omega_2, \beta_1) \rightarrow C(\omega_2, \beta_2)$

$$\langle Q_2 \rangle = \langle H \rangle_C - \langle H \rangle_B = \frac{\hbar\omega_2}{2} \left[\coth\left(\frac{\beta_2 \hbar\omega_2}{2} \right) - Q_1^* \coth\left(\frac{\beta_1 \hbar\omega_1}{2} \right) \right]$$

(3) Isentropic expansion: $C(\omega_2, \beta_2) \rightarrow D(\omega_1, \beta_2)$

$$\langle W_3 \rangle = \langle H \rangle_D - \langle H \rangle_C = \left(\frac{\hbar\omega_1}{2} Q_2^* - \frac{\hbar\omega_2}{2} \right) \coth\left(\frac{\beta_2 \hbar\omega_2}{2} \right)$$

(4) Cold isochore: $D(\omega_1, \beta_2) \rightarrow A(\omega_1, \beta_1)$

$$\langle Q_4 \rangle = \langle H \rangle_A - \langle H \rangle_D = \frac{\hbar\omega_1}{2} \left[\coth\left(\frac{\beta_1 \hbar\omega_1}{2} \right) - Q_2^* \coth\left(\frac{\beta_2 \hbar\omega_2}{2} \right) \right]$$

$$\eta = -\frac{W_1 + W_3}{Q_2} \qquad P = -(W_1 + W_3)/\tau_{\mathrm{cyc}}$$

Q_j^* =1 in quasistatic processes

A single-atom heat engine

Science 352, 325 (2016)

Johannes Roßnagel,[1,*] Samuel Thomas Dawkins,[1] Karl Nicolas Tolazzi,[1]
Obinna Abah,[2] Eric Lutz,[2] Ferdinand Schmidt-Kaler,[1] and Kilian Singer[1,3]

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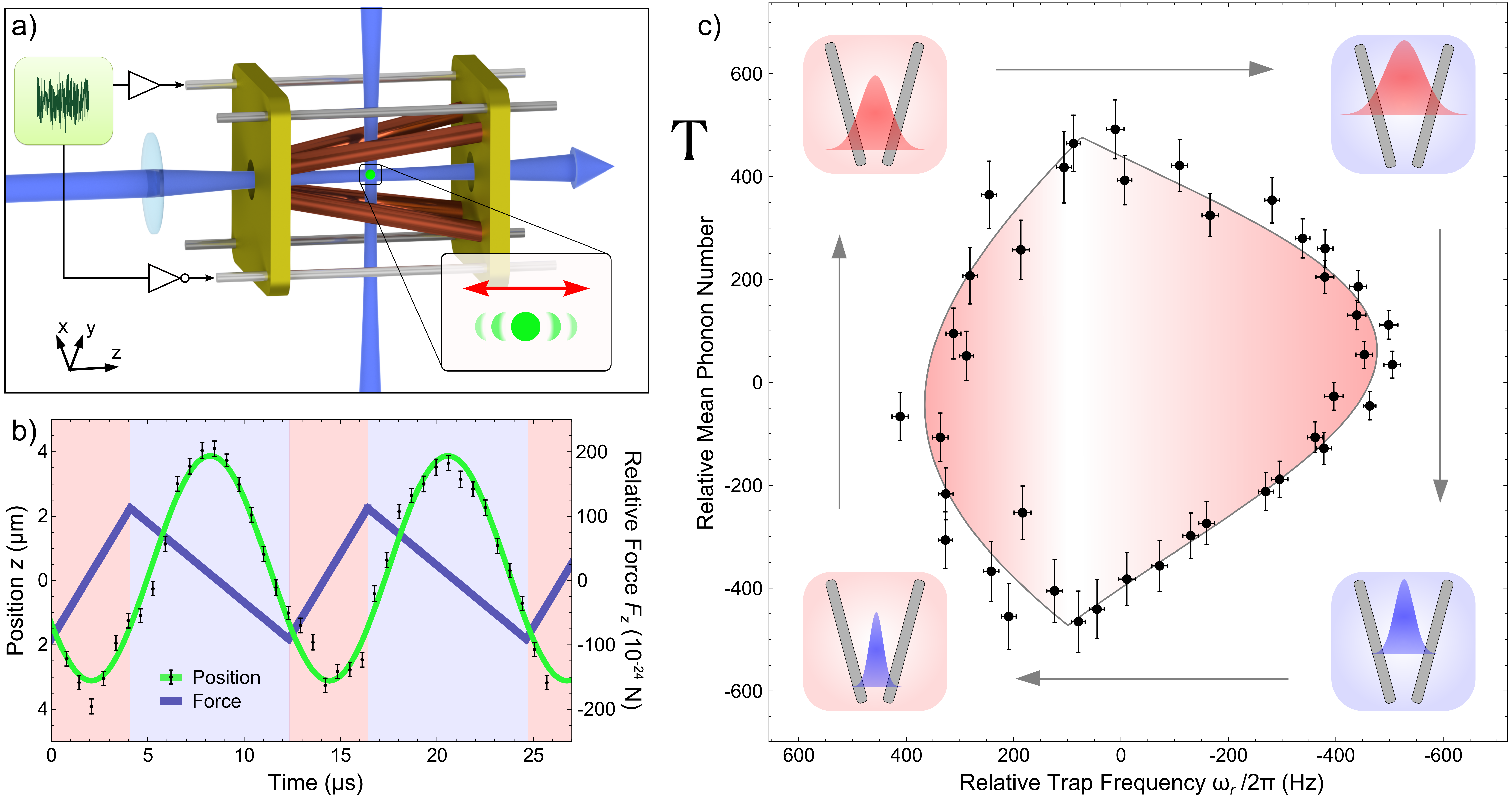
[2] *Department of Physics, Friedrich-Alexander Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany*
[3] *Experimentalphysik I, Universität Kassel, Heinrich-Plett-Str. 40, D-34132 Kassel, Germany*

(Dated: October 14, 2015)

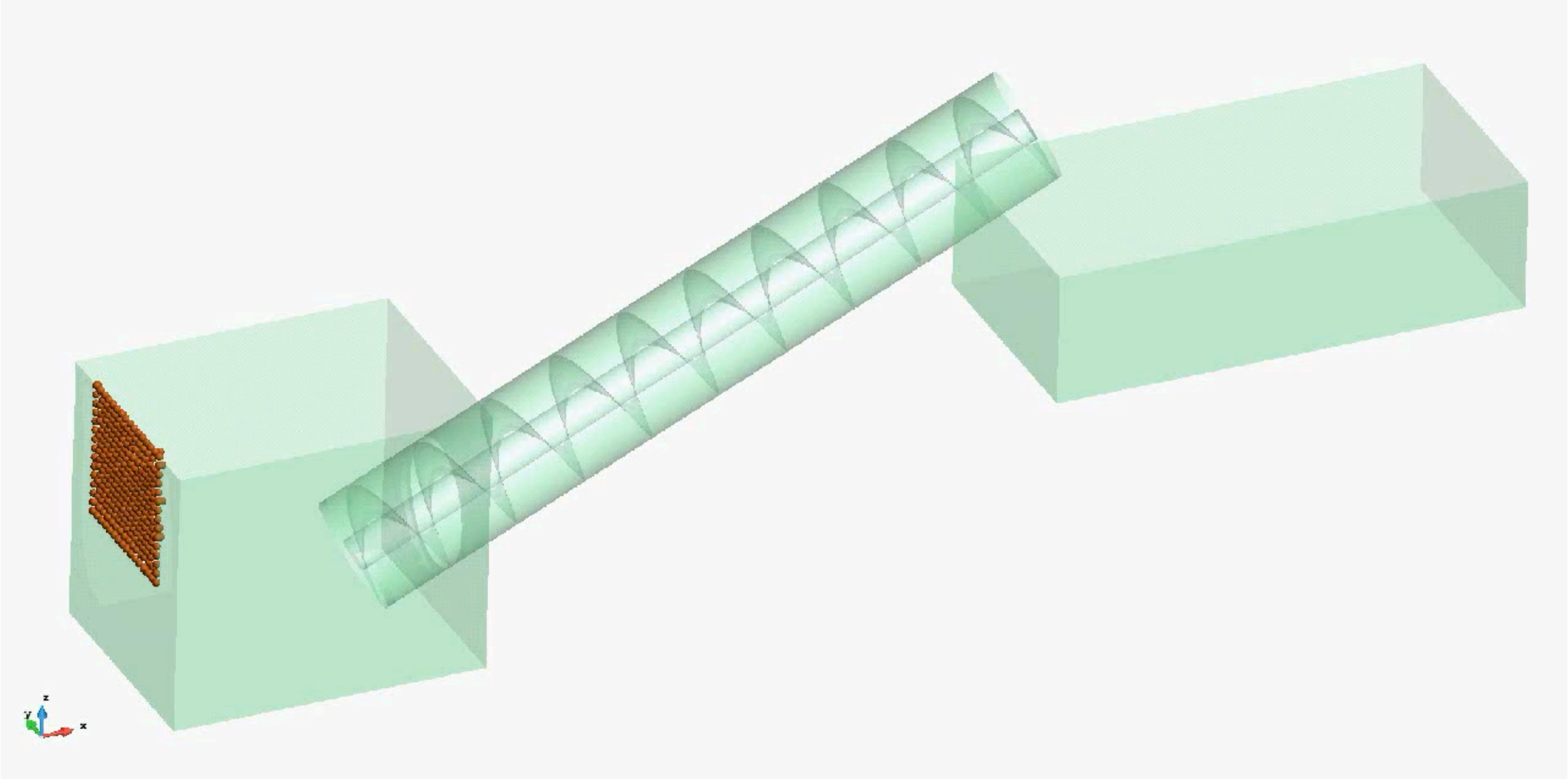


Simplifications of the “Otto description”

- Perfect separation of strokes where only work or only heat is exchanged.
- The time-dependent processes of switching on and off the contacts to the baths.

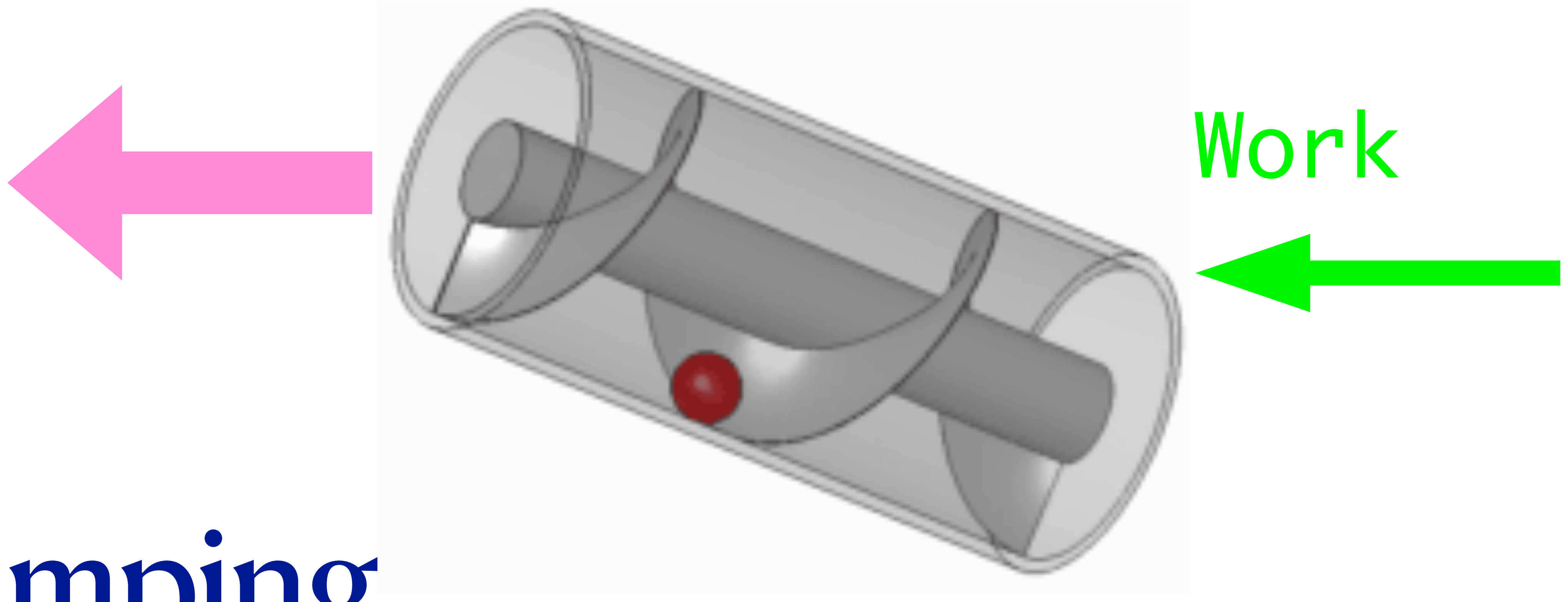
Pumping

Archimides screw



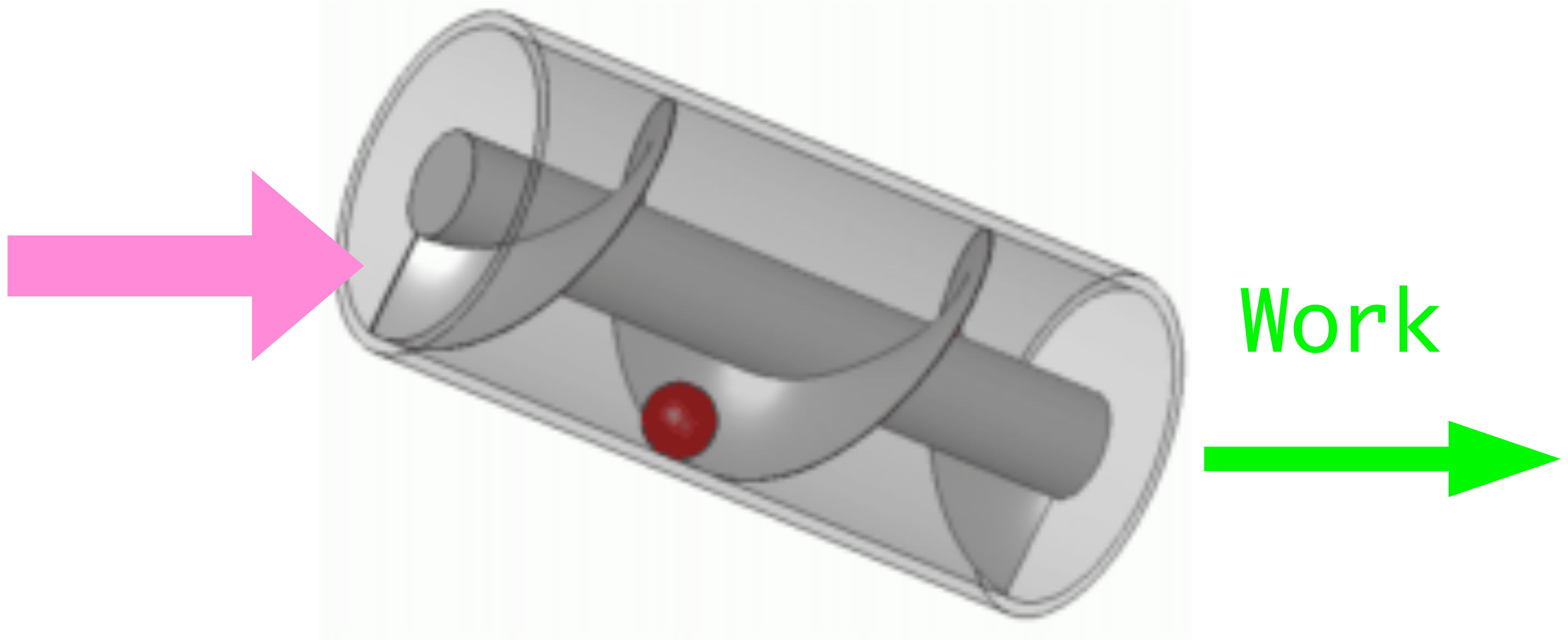

Archimedes
Pump
CREEK
RIVER

Two operations



Pumping

Generator



Charge pumping

EUROPHYSICS LETTERS 14 January 1992

Europhys. Lett., **17** (3), pp. 249-254 (1992) *published in December 1991*

Single-Electron Pump Based on Charging Effects.

H. POTHIER, P. LAFARGE, C. URBINA, D. ESTEVE and M. H. DEVORET

Service de Physique de l'Etat Condensé, Centre d'Etudes de Saclay
91191 Gif-sur-Yvette, France

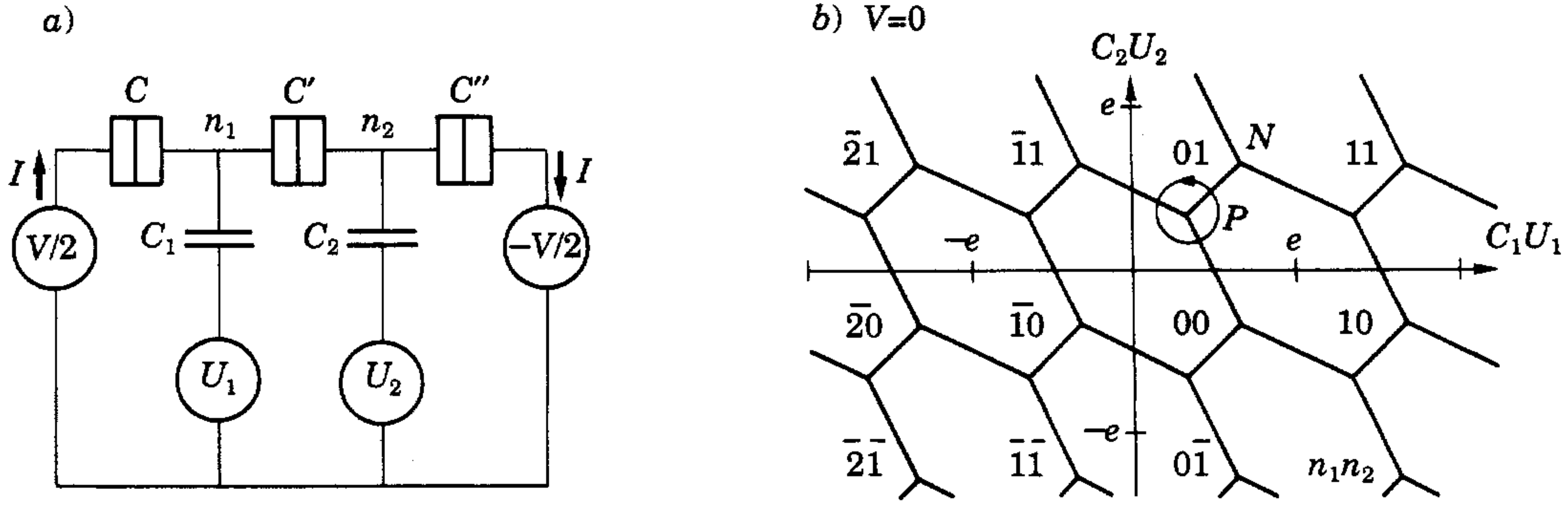


Fig. 1. – Principle of reversible transfer of a single electron using a «pump» controlled by two gate voltages U_1 and U_2. *a)* Circuit schematic: the nanoscale junctions constituting the pump are represented by double-box symbols. *b)* Stable configuration diagram for $V=0$ and $C=C'=C''$. One turn around a triple point such as P or N, obtained by modulating the gate voltages by two phase-shifted signals, induces one electron to go around the circuit.

PHYSICAL REVIEW B VOLUME 58, NUMBER 16 15 OCTOBER 1998-II

Scattering approach to parametric pumping

P. W. Brouwer
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

Scattering matrix

Pumped current

Similar to a Berry curvature!

$$\delta I = \frac{\omega e \sin\phi\, \delta X_1 \delta X_2}{2\pi} \sum_{\alpha \in 1} \sum_{\beta} \operatorname{Im} \frac{\partial S^*_{\alpha\beta}}{\partial X_1} \frac{\partial S_{\alpha\beta}}{\partial X_2}.$$

At least two parameters are needed!

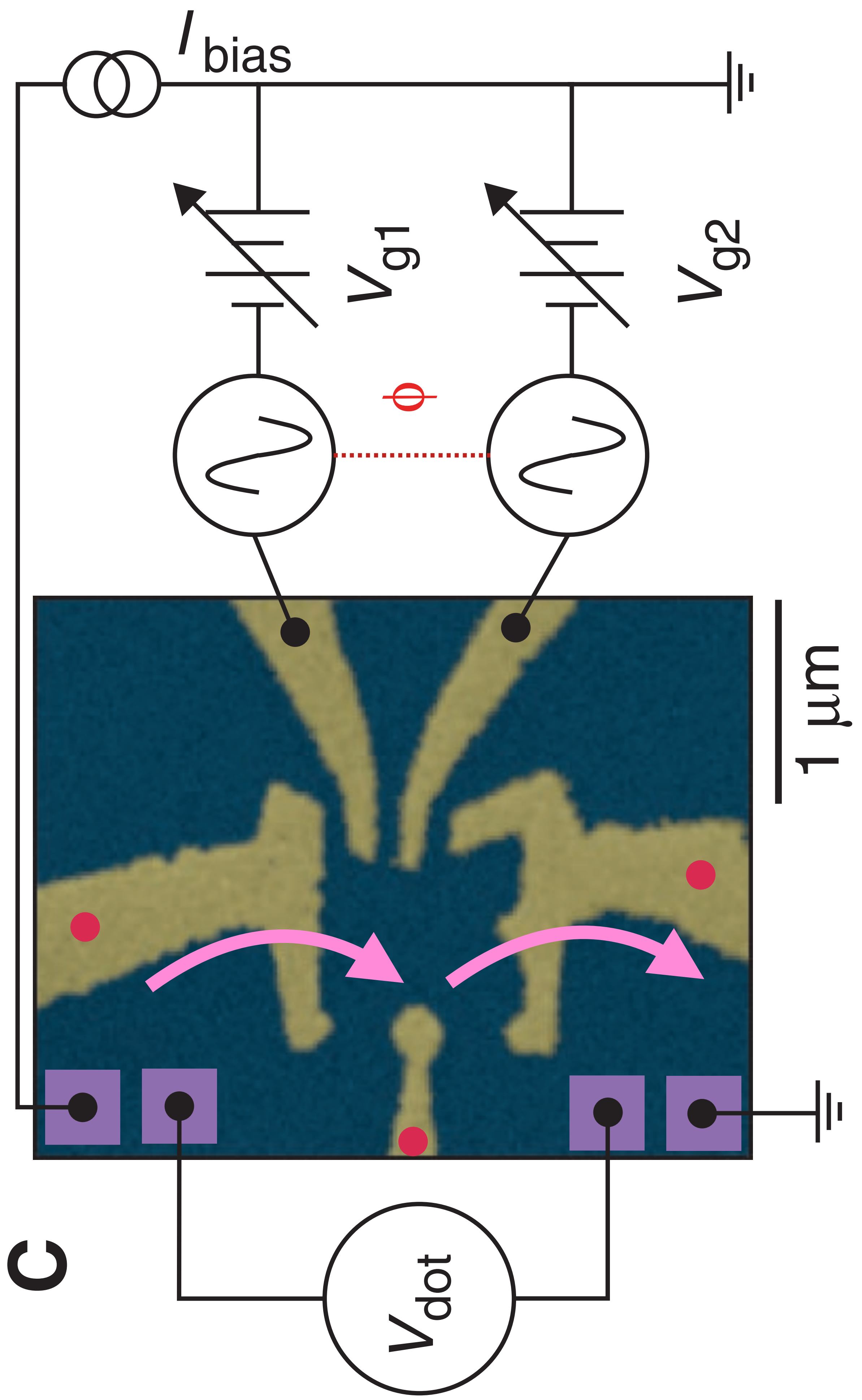
D. J. Thouless, Quantization of particle transport, Phys. Rev. B **27**, 6083 (1983).

J. E. Avron, A. Elgart, G. M. Graf, and L. Sadun, Optimal Quantum Pumps, Phys. Rev. Lett. **87**, 236601 (2001).

www.sciencemag.org SCIENCE VOL 283 19 MARCH 1999

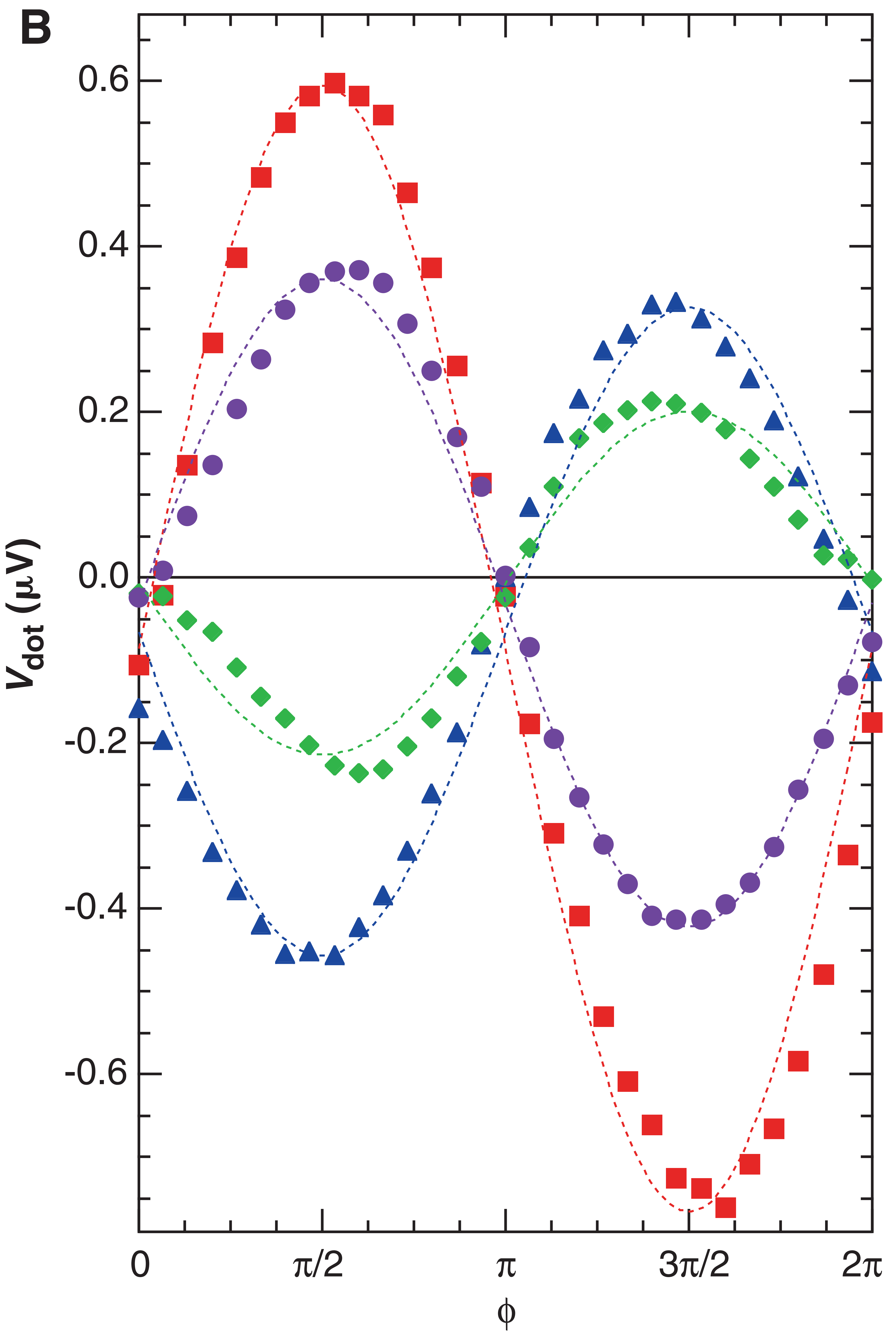
An Adiabatic Quantum Electron Pump

M. Switkes,[1] C. M. Marcus,[1*] K. Campman,[2] A. C. Gossard[2]



$$V_{g1}(t) = V_0 \cos(\omega t)$$

$$V_{g2}(t) = V_0 \cos(\omega t + \phi)$$

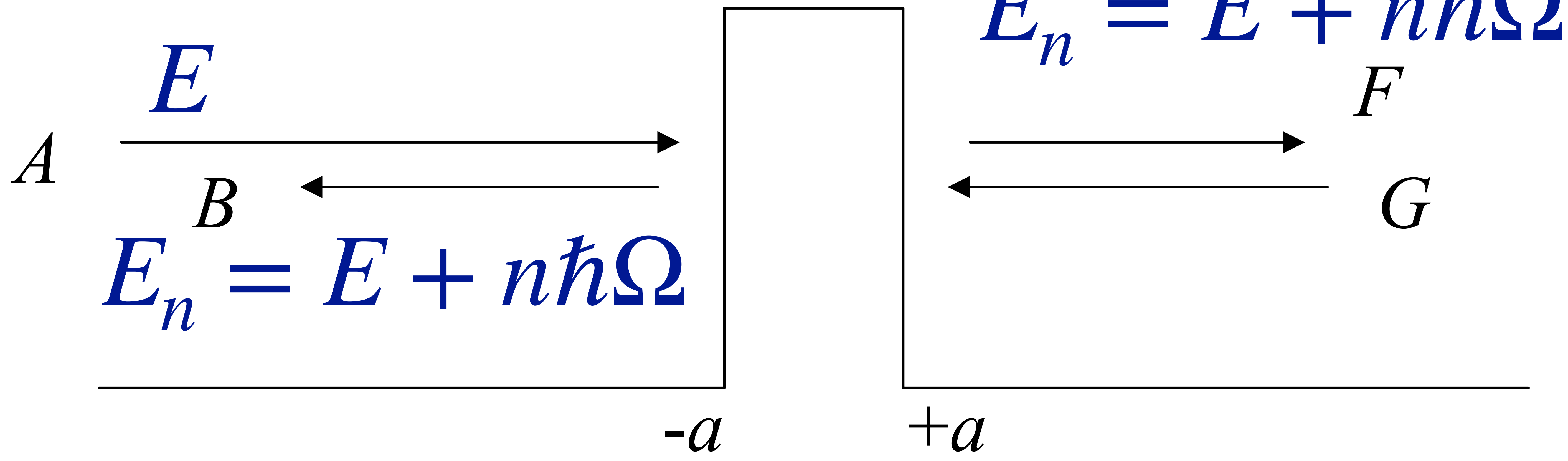


PHYSICAL REVIEW B **66**, 205320 (2002)

Floquet scattering theory of quantum pumps

M. Moskalets[1] and M. Büttiker[2]

$$\vec{X}(t) = \vec{X}(t+\tau), \ \ \tau = \frac{2\pi}{\Omega}$$



$$E_n = E + n\hbar\Omega$$

$$E_n = E + n\hbar\Omega$$

Floquet scattering matrix

$$S_{F,\alpha\beta}(E,E_n)$$

Adiabatic description (small Ω)

$$S_{\alpha\beta,t}(E) + S_{\alpha\beta}^{(\mathrm{ad})}(E), \qquad S_{\alpha\beta}^{(\mathrm{ad})}(E) \propto \Omega$$

PHYSICAL REVIEW B **74**, 245322 (2006)

Relation between scattering-matrix and Keldysh formalisms for quantum transport driven by time-periodic fields

Liliana Arrachea[1,2] and Michael Moskalets[3]

Cours/Lecture Series

1992 – 1993 ACADEMIC TRAINING PROGRAMME

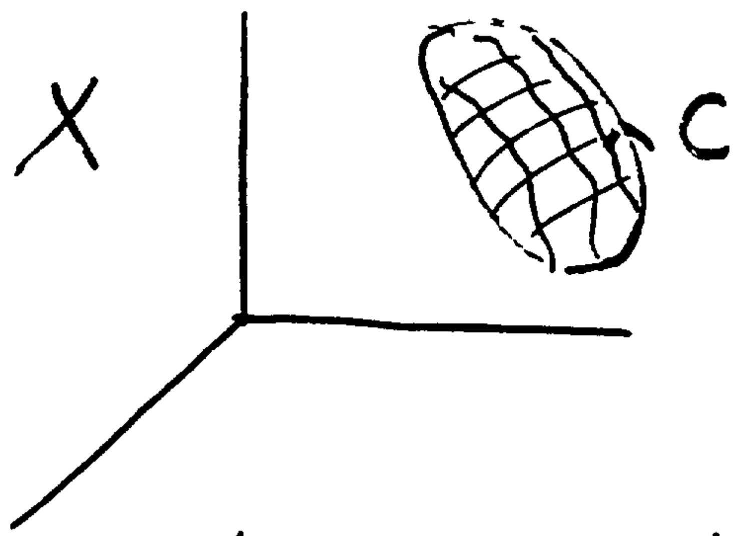
LECTURE SERIES

SPEAKER : Michael BERRY / University of Bristol
TITLE : Geometric Phases
TIME : 8, 9, 10 February, 11.00 to 12.00 hrs
PLACE : Auditorium

ABSTRACT

An elementary account will be given of the mathematical phenomenon of "global change without local change", i.e. anholonomy, applied to phases in quantum mechanics and angles in classical mechanics. Phases and angles in photons, electrons, and molecules will be discussed, with a historical emphasis. More advanced topics may include a detailed illustration of the vector and scalar gauge forces associated with the geometric reaction of a light system or a heavy one (modern Born-Oppenheimer theory) and high-order adiabatic corrections to the geometric phase and its relation to recent developments in asymptotics.

geometric phase : independent of T, dependent only on n and the geometry of C ; system's answer to :



"where have you been?"

$\gamma_n(C)$ is phase anholonomy

As with other circuit-dependent quantities in physics,

$\gamma_n(C)$ = flux of something through C

(e.g. emf = flux of rate of change of magnetic field)

Here (from Schrödinger equation)

$$\gamma_n(C) = -\iint_{\partial S = C} \mathrm{Im}\langle dn|\wedge|dn\rangle$$ abstractly (2 form)

$$= -\iint_{\partial S = C} dX_1 dX_2 \iiint d\underset{\sim}{r}\, \mathrm{Im}\left[\frac{\partial \psi_n^*(\underset{\sim}{r};X)}{\partial X_1}\frac{\partial \psi_n(\underset{\sim}{r};X)}{\partial X_2} - \frac{\partial \psi_n^*(\underset{\sim}{r};X)}{\partial X_2}\frac{\partial \psi_n(\underset{\sim}{r};X)}{\partial X_1}\right]$$

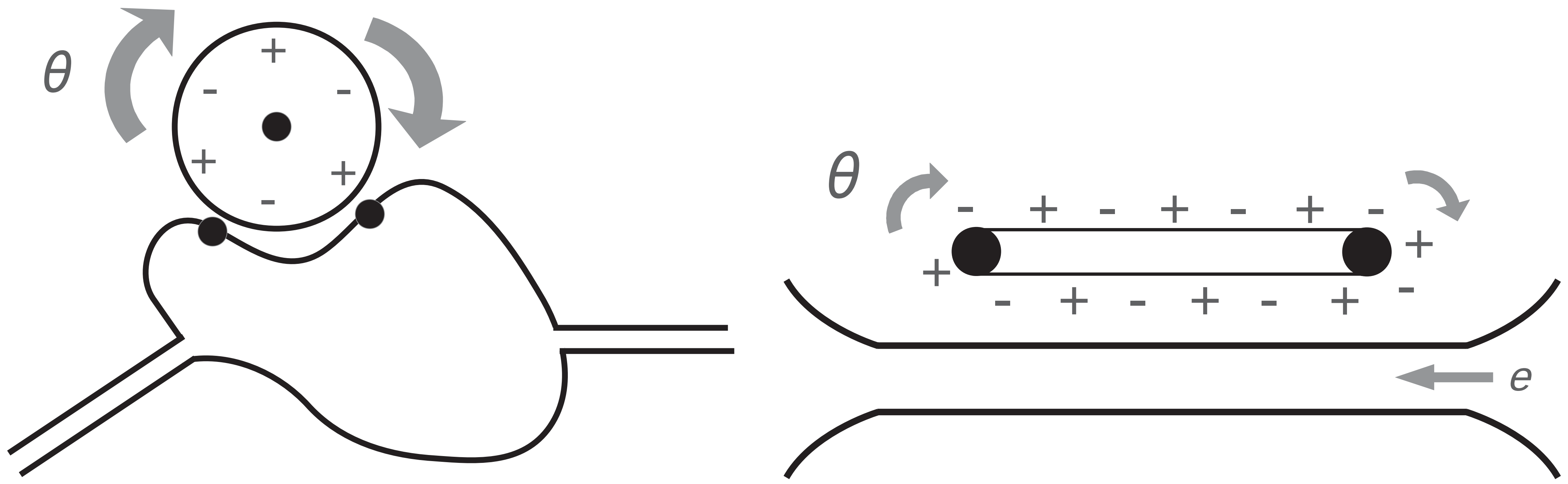
coordinates spanning C ($dX_1 dX_2$) — concretely

Motors

Selected for a Viewpoint in *Physics*
PRL **111,** 060802 (2013) PHYSICAL REVIEW LETTERS week ending 9 AUGUST 2013

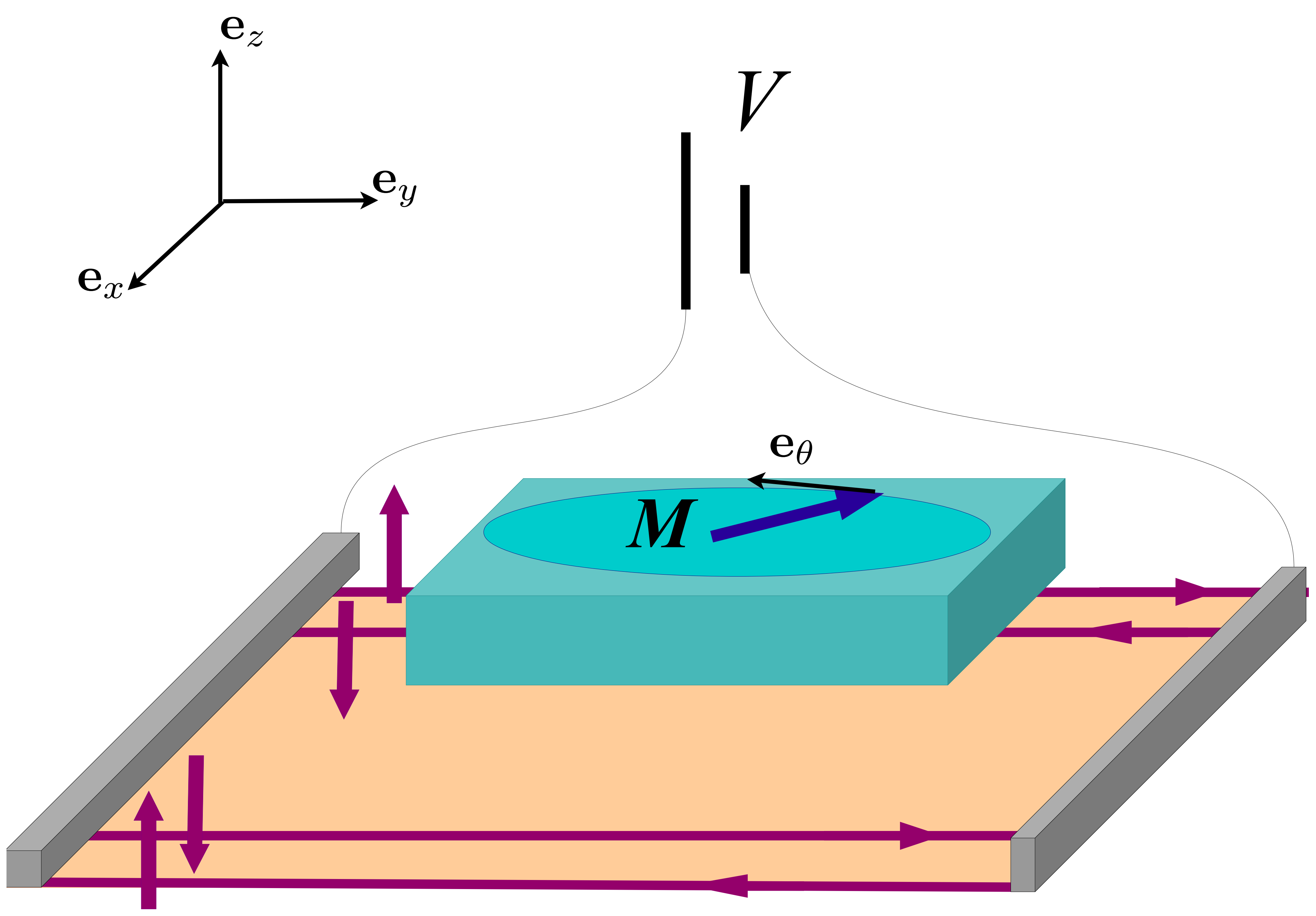
Adiabatic Quantum Motors

Raúl Bustos-Marún,[1,2] Gil Refael,[3,1] and Felix von Oppen[1,3]



(a) (b)

Topological version



LA and F von Oppen, Physica E 74 (2015)

Conversion between dc electrical power and power by time-dependent driving forces

Finite-time thermodynamic cycles in continuous coupling to the baths

PHYSICAL REVIEW B **102**, 155407 (2020)

Editors' Suggestion   Featured in Physics

Geometric properties of adiabatic quantum thermal machines

Bibek Bhandari,[1] Pablo Terrén Alonso,[2] Fabio Taddei,[3] Felix von Oppen,[4] Rosario Fazio,[5,6] and Liliana Arrachea[2]

[1] *NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, I-56126 Pisa, Italy*

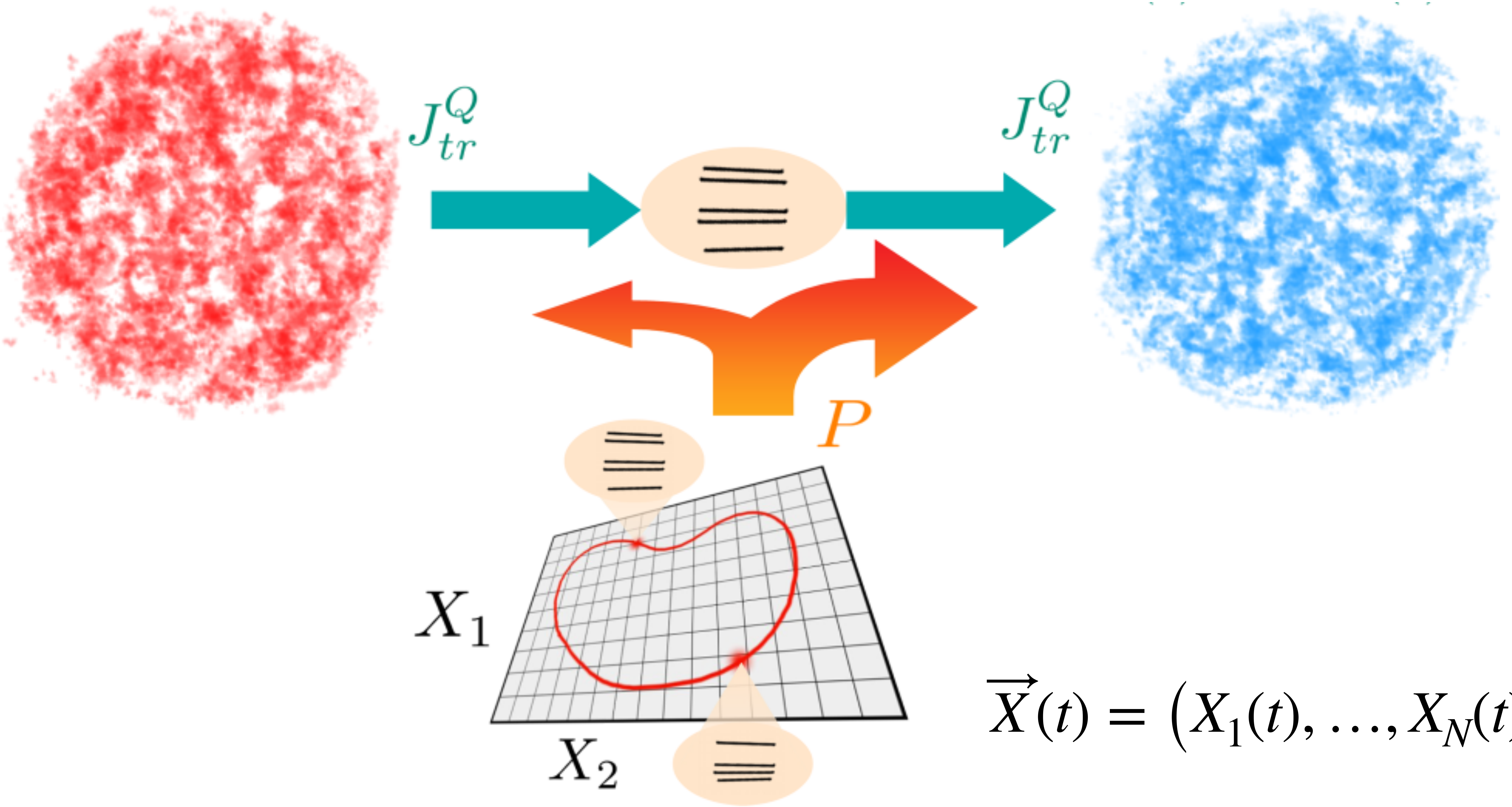
[2] *International Center for Advanced Studies, Escuela de Ciencia y Tecnología and ICIFI, Universidad Nacional de San Martín, Avenida 25 de Mayo y Francia, 1650 Buenos Aires, Argentina*

[3] *NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, I-56126 Pisa, Italy*

[4] *Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany*

[5] *Abdus Salam ICTP, Strada Costiera 11, I-34151 Trieste, Italy*

[6] *Dipartimento di Fisica, Università di Napoli "Federico II," Monte S. Angelo, I-80126 Napoli, Italy*

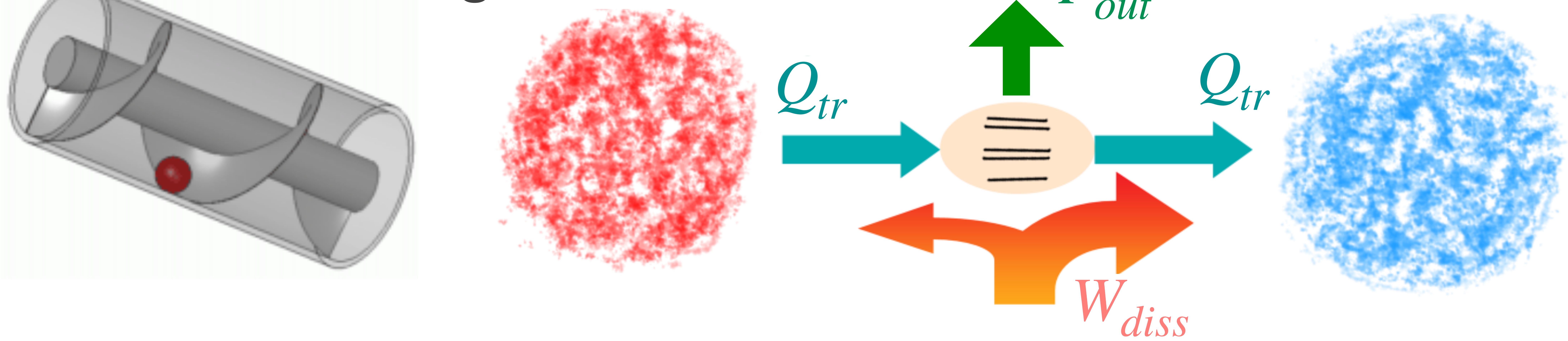


$$\overrightarrow{X}(t) = \big(X_1(t), \ldots, X_N(t)\big)$$

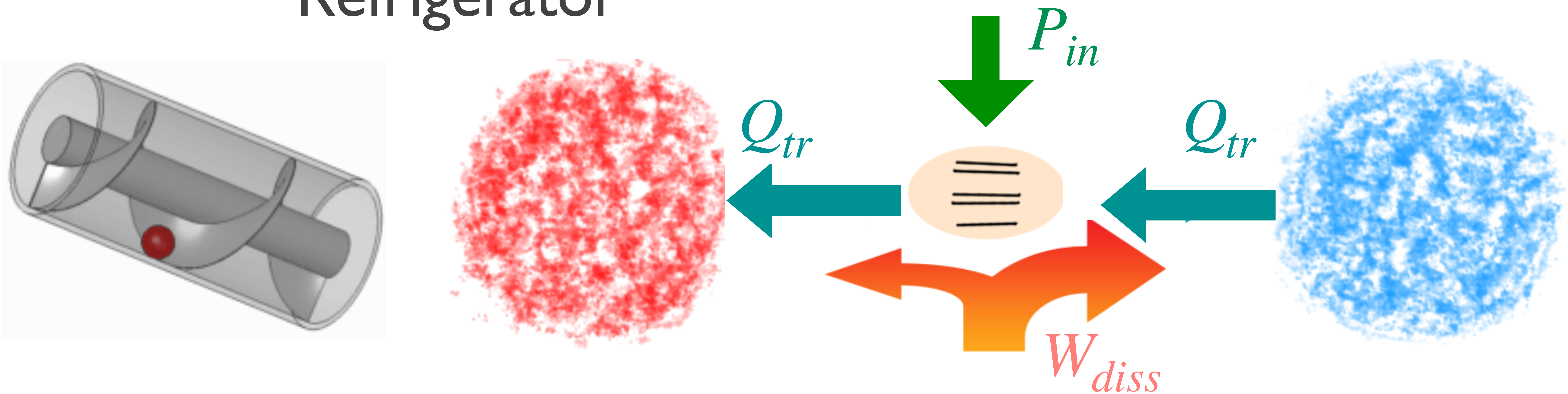
N+1 velocities: $\dot{\mathbf{X}}(t) = \{\dot{\vec{X}}(t), \Delta T(t)/T\}.$

Operational modes

Heat engine



Refrigerator



Heat fluxes and power

Heat flux into L reservoir

Heat flux into R reservoir

Power developed by external forces

$$J^Q_{\mathrm{L}} + J^Q_{\mathrm{R}} = P,$$

Dissipation

$$J^Q_{\mathrm{tr,R}} = -J^Q_{\mathrm{tr,L}} \equiv J^Q_{\mathrm{tr}},$$

Transport component

Operational Regime

- Adiabatic: Time scale of the driving much larger than any other internal time scale of the small quantum system, including the relaxation time with the reservoirs (Small $\dot{\vec{X}}$)
- Small temperature difference between the two reservoirs. (Small $\Delta T/T$)

Ideal scenario to combine adiabatic linear response in $\dot{\vec{X}}$ with linear response in $\Delta T/T$

Hamiltonian

$\mathcal{H}_{\text{baths}} = \mathcal{H}_{\text{R}} + \mathcal{H}_{\text{L}}$

Contacts between system and baths

$$\mathcal{H}(t) = \mathcal{H}_S(t) + \mathcal{H}_{\text{baths}} + \mathcal{H}_c + \mathcal{H}_{\text{th}}(t).$$

$$\mathcal{H}_S(t) \equiv \mathcal{H}_S[\vec{X}(t)]$$

$\vec{X}(t) = \{X_\ell(t)\}$

$\ell = 1, \ldots, N$

Luttinger Hamiltonian to represent thermal bias

Luttinger approach

J. M. Luttinger, Theory of thermal transport coefficients, Phys.Rev. **135**, A1505 (1964).

G. Tatara, Thermal Vector Potential Theory of Transport Induced by a Temperature Gradient, Phys. Rev. Lett. **114**, 196601 (2015).

$$\mathcal{H}_{\rm th}(t) = -\sum_{\alpha=L,R} \mathcal{J}_\alpha^E(t)\xi_\alpha(t),$$

Thermal vector potential

$$\dot{\xi}_\alpha(t) = \delta T_\alpha(t)/T.$$

Leads to proper results for heat and thermoelectric current in linear response (Kubo) formalism upon carefully treating the “diamagnetic” term.

Energy Flux and force operators

$\frac{\Omega}{2\pi}$: period of the cycle

$$\mathcal{J}_\alpha^E = \dot{\mathcal{H}}_\alpha = -i[\mathcal{H}_\alpha, \mathcal{H}]/\hbar. \quad \longrightarrow \quad J_\alpha^Q = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt \langle \mathcal{J}_\alpha^E(t)\rangle.$$

$$\mathcal{F}_\ell = -\frac{\partial \mathcal{H}}{\partial X_\ell}, \quad \text{with } \ell = 1, \ldots, N \quad \longrightarrow \quad \boldsymbol{\mathcal{F}} = \left(\vec{\mathcal{F}}, \mathcal{J}_R^E\right).$$

Energy flux operator into the cold reservoir

Adiabatic linear response

$$\langle \mathcal{O} \rangle(t) = \langle \mathcal{O} \rangle_t + \sum_{\ell=1}^{N} \chi_t^{\text{ad}}[\mathcal{O}, \mathcal{F}_\ell]\dot{X}_\ell(t) + \sum_{\alpha=L,R} \chi_t^{\text{ad}}[\mathcal{O}, \mathcal{J}_\alpha^E]\dot{\xi}_\alpha(t).$$

$$\rho_t = \sum_m p_m |m\rangle\langle m|,$$

$$p_m = e^{-\beta\varepsilon_m}/Z_t$$

$$\mathcal{H}_t|m\rangle = \varepsilon_m|m\rangle$$

$$\mathcal{H}_t = \mathcal{H}_S(t) + \mathcal{H}_{\text{baths}} + \mathcal{H}_c$$

$$\chi_t^{\text{ad}}[\mathcal{O}_1, \mathcal{O}_2] = -\frac{i}{\hbar}\int_{-\infty}^{t} dt'(t-t')\langle[\mathcal{O}_1(t), \mathcal{O}_2(t')]\rangle_t.$$

Recall: Adiabatic susceptibilities are evaluated wrt the equilibrium frozen Hamiltonian

Adiabatic forces

$$F_\ell(t) = F_{\ell,t} + \sum_{\ell'=1}^{N} \Lambda_{\ell,\ell'}(\overrightarrow{X})\dot{X}_{\ell'} + \Lambda_{\ell,N+1}(\overrightarrow{X})\frac{\Delta T}{T}$$

$$J^Q(t) = \sum_{\ell'=1}^{N} \Lambda_{N+1,\ell'}(\overrightarrow{X})\dot{X}_{\ell'} + \Lambda_{N+1,N+1}(\overrightarrow{X})\frac{\Delta T}{T}$$

Thermal geometric tensor: $\Lambda_{\mu,\nu}(\vec{X})$ $\mu, \nu = 1, \ldots, N+1$

Onsager relations

$$\Lambda_{\mu\nu}(\overrightarrow{X}, B) = \pm\, \Lambda_{\nu\mu}(\overrightarrow{X}, -B)$$

In many examples

$$\Lambda_{\ell,\ell'}(\overrightarrow{X}) = \Lambda_{\ell',\ell}(\overrightarrow{X}), \qquad \ell, \ell' = 1,\ldots,N$$

$$\Lambda_{\ell,N+1}(\overrightarrow{X}) = -\, \Lambda_{N+1,\ell}(\overrightarrow{X})$$

Notation

$$\Lambda_{\ell,\ell'}(\overrightarrow{X}),\ \ \ell,\ell' = 1,\ldots,N, \longrightarrow \underline{\Lambda}(\overrightarrow{X})$$

$$\overrightarrow{\Lambda}(\overrightarrow{X}) = \left(\Lambda_{N+1,1}(\overrightarrow{X}),\ldots,\Lambda_{N+1,N}(\overrightarrow{X})\right)$$

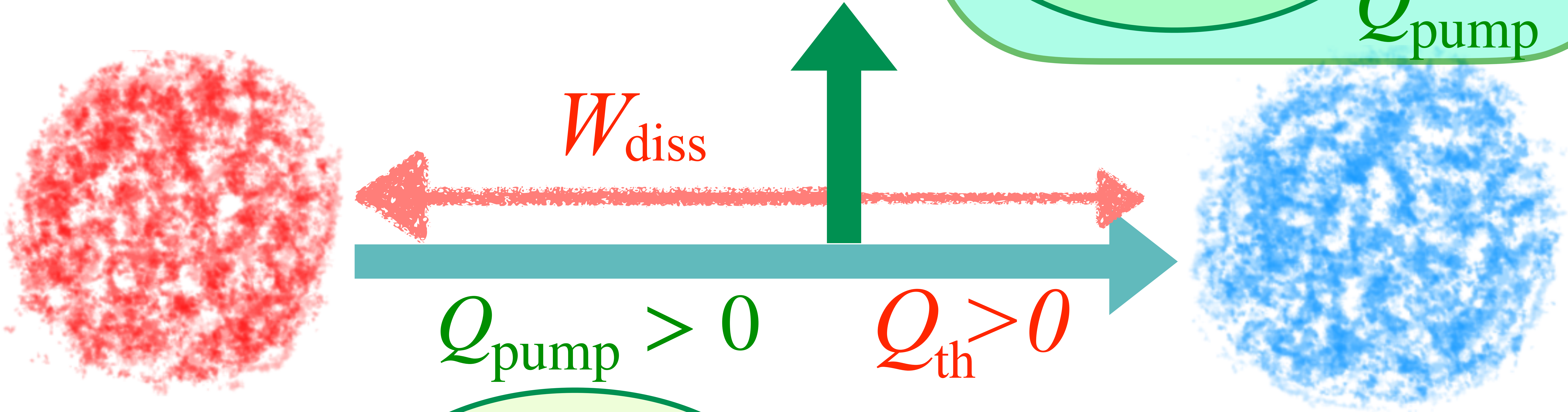
$$\kappa = \Lambda_{N+1,N+1}(\overrightarrow{X})$$

Heat engine

Heat-work conversion

$$W = -\int_0^\tau dt\, \dot{\vec{X}} \cdot \overline{\Lambda}^S \cdot \dot{\vec{X}} + \oint \vec{\Lambda} \cdot \vec{dX} \frac{\Delta T}{T}$$

Q_{pump}



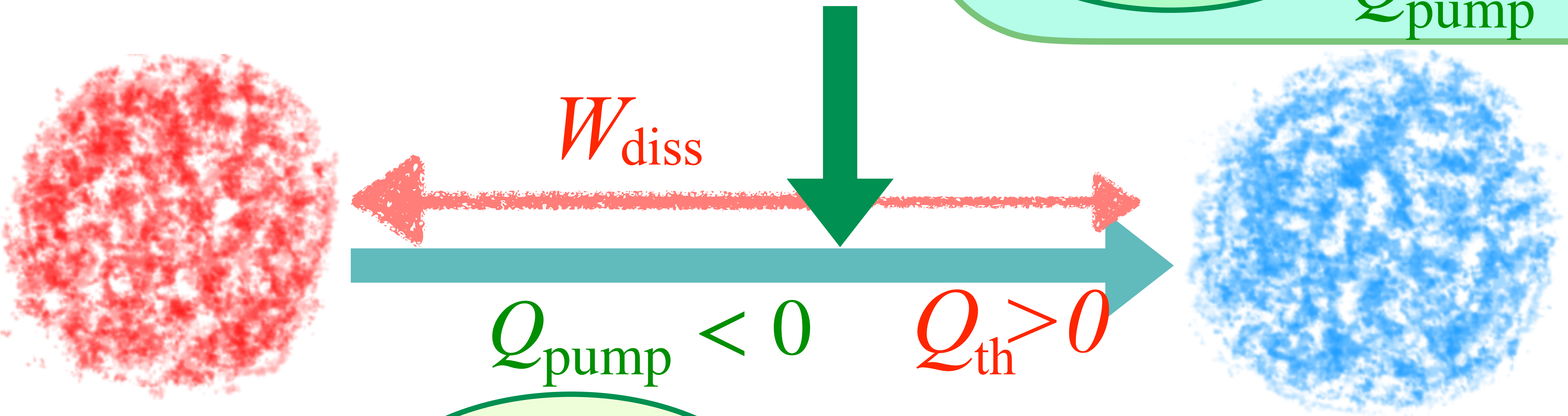
$$Q_{\mathrm{tr}} = \oint \vec{\Lambda} \cdot \vec{dX} + \int_0^\tau dt \kappa \frac{\Delta T}{T}$$

Refrigerator

Heat-work conversion

$$W = -\int_0^\tau dt\,\vec{\dot{X}} \cdot \overline{\Lambda}^S \cdot \vec{\dot{X}} + \oint \vec{\Lambda} \cdot \vec{dX}\,\frac{\Delta T}{T}$$

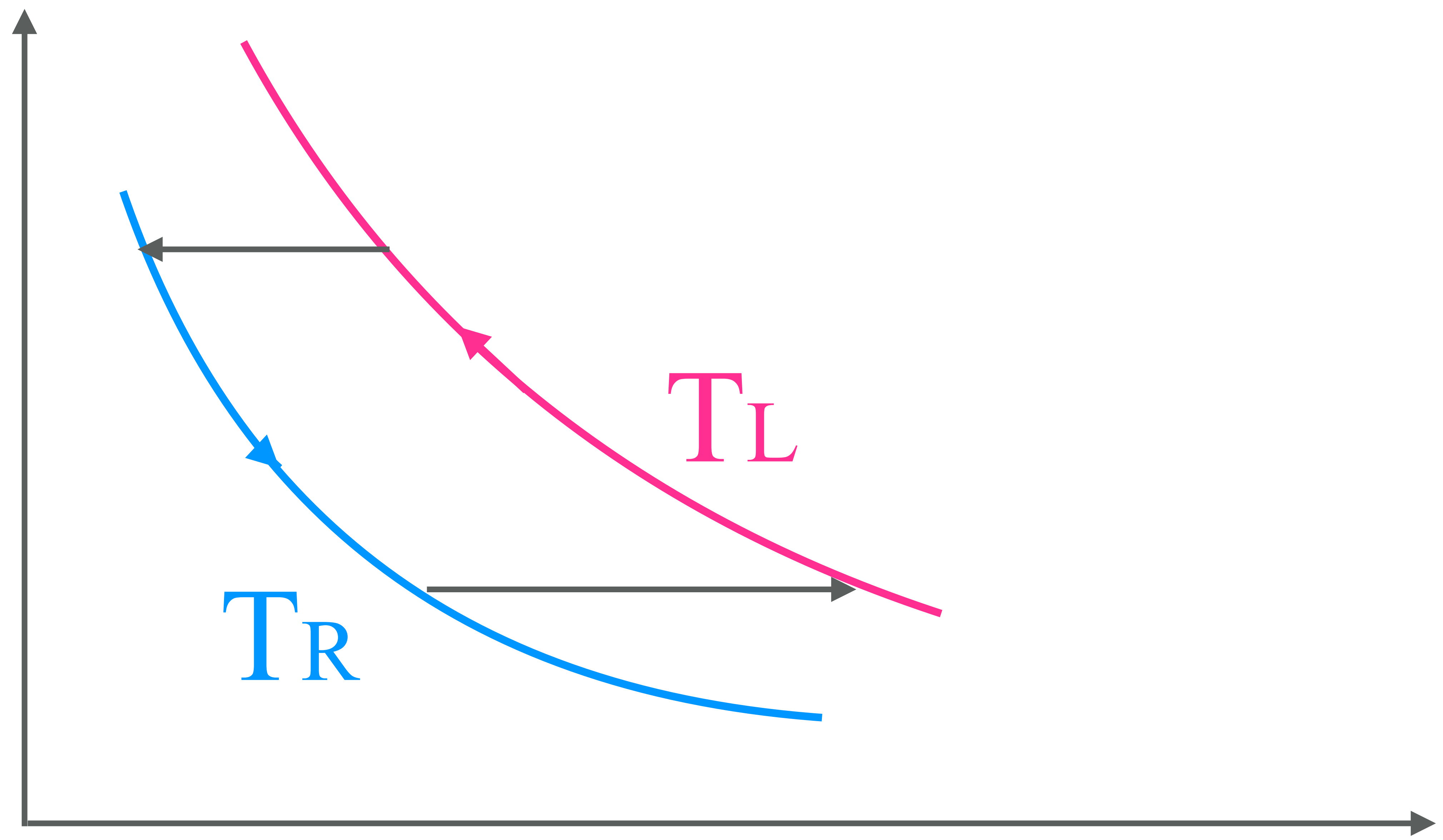
Q_{pump}



$$Q_{\text{tr}} = \oint \vec{\Lambda} \cdot \vec{dX} + \int_0^\tau dt\,\kappa\,\frac{\Delta T}{T}$$

Highlight (I)

$$W_{\text{out/in}} = -Q_{\text{pump}}\frac{\Delta T}{T}$$



Heat-work conversion term is the counterpart of work produced in usual Carnot cycle
(limit of zero dissipation)

Highlight (II)

$$W_{\mathrm{out/in}} = -Q_{\mathrm{pump}}\frac{\Delta T}{T}$$

$$\vec{\Lambda}(\vec{X}) = \left(\Lambda_{N+1,1}(\vec{X}), \ldots, \Lambda_{N+1,N}(\vec{X})\right)$$ ~ Berry connection

~ Berry phase

$$Q_{\mathrm{pump}} = \oint \vec{\Lambda}(\vec{X}) \cdot d\vec{X}$$

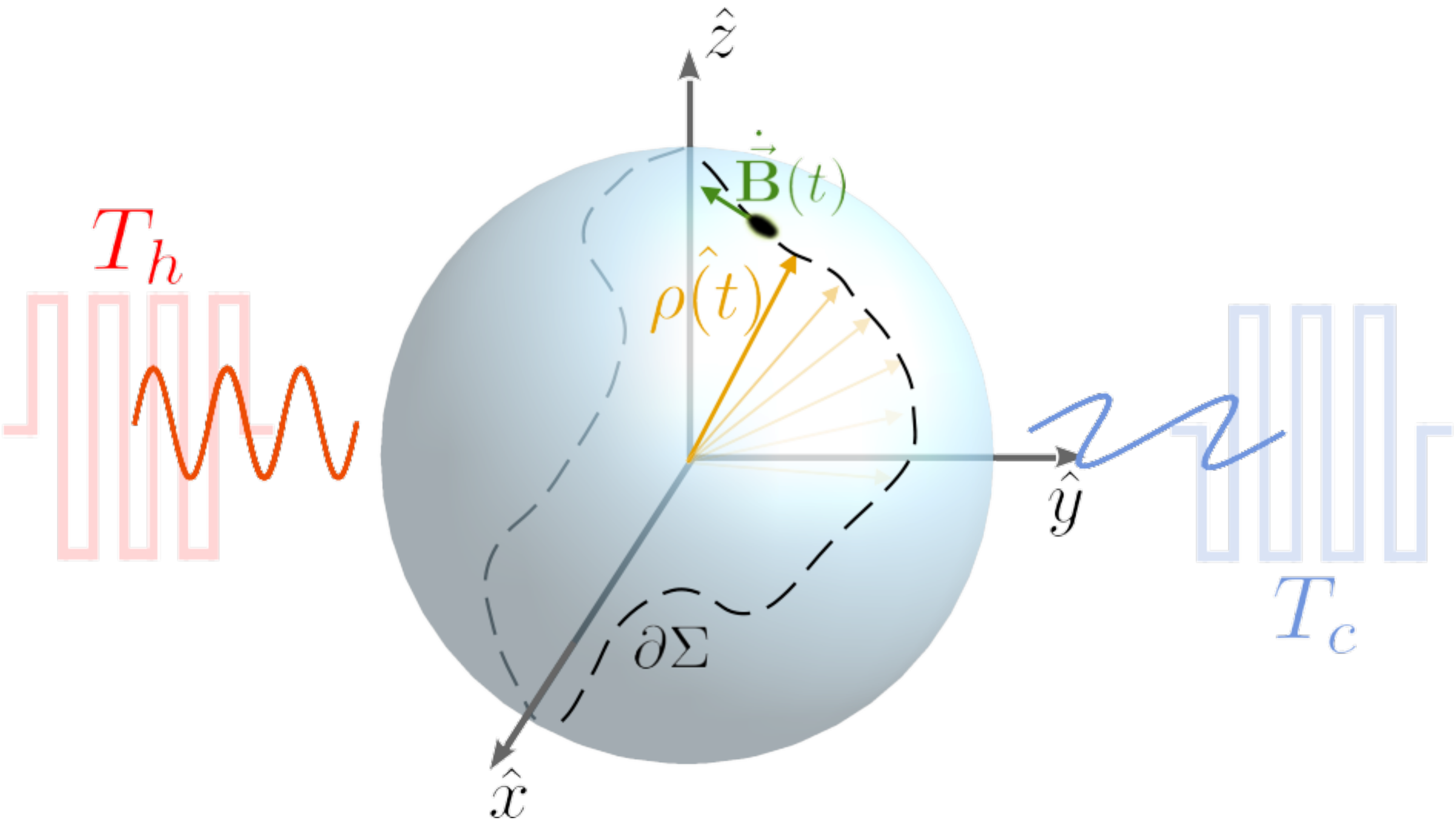
At least two control parameters are necessary to have a non-vanishing value!

Driven q-bit

$$\mathcal{H}_S(t) = \vec{B}(t) \cdot \hat{\vec{\sigma}}.$$

$$\vec{B}(t) = (B_x(t), 0, B_z(t)),$$

$$\mathcal{H}_\alpha = \sum_k \varepsilon_{k\alpha} b^\dagger_{k\alpha} b_{k\alpha}, \quad \alpha = L, R,$$



$$\mathcal{H}_{c,\alpha} = \sum_k V_{k\alpha} \hat{\tau}_\alpha (b_{k\alpha} + b^\dagger_{k\alpha}),$$

$\hat{\tau}_L = \hat{\sigma}_x$ and $\hat{\tau}_R = \hat{\sigma}_z$.

Solution with master equations: weak coupling

Calculated with frozen Hamiltonian

Adiabatic correction

Reduced density matrix: $\hat{\rho}_{\mathrm{S}}(t) = \rho^{(f)}(t) + \rho^{(a)}(t)$

Frozen: $$\frac{d\rho^{(f)}}{dt} = -\frac{i}{\hbar}\left[H_{\mathrm{S}}, \hat{\rho}^{(f)}\right] + \sum_{\alpha} \mathcal{L}_{\alpha}\left[\hat{\rho}^{(f)}\right]$$

Adiabatic: $$\frac{d\rho^{(f)}}{d\vec{X}} \cdot \dot{\vec{X}}(t) = \sum_{\alpha} \mathcal{L}_{\alpha}\left[\hat{\rho}^{(a)}\right]$$

Energy current: $$J_{\alpha}^{f/a}(t) = \mathrm{Tr}\left[H_{\mathrm{S}}(\vec{X})\mathcal{L}_{\alpha}\left[\hat{\rho}^{(f/a)}\right]\right]$$

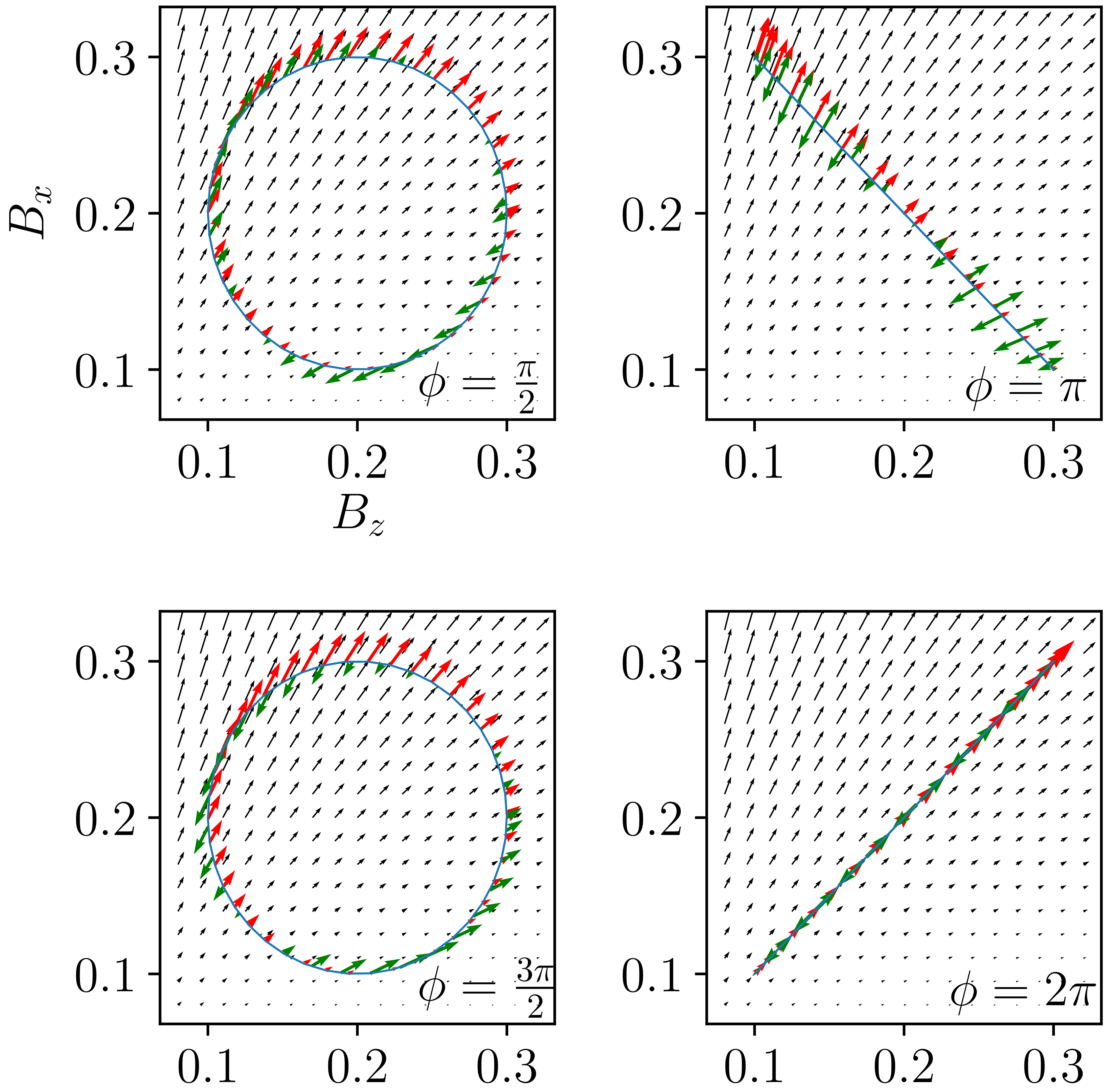
Results

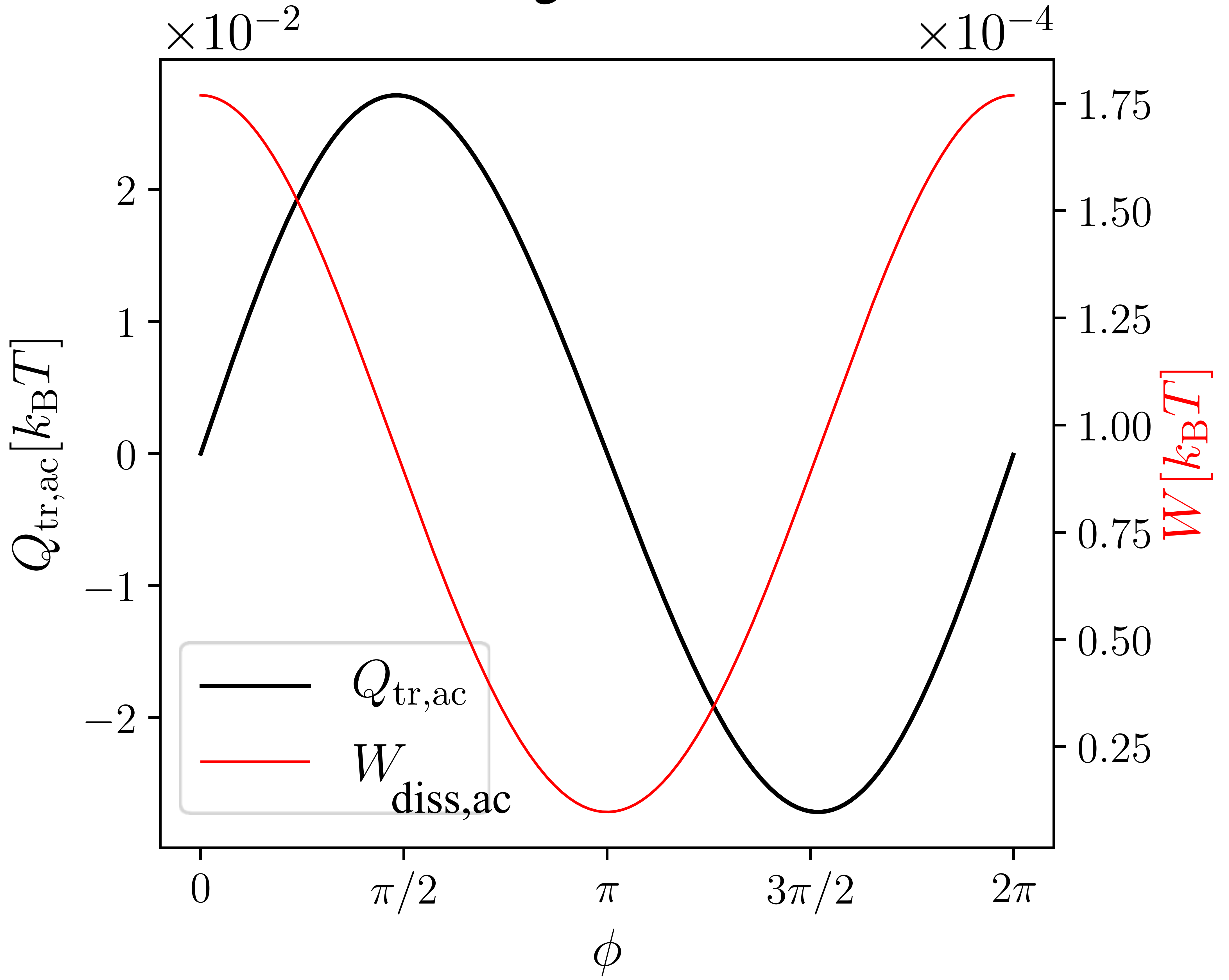
$$\overrightarrow{X} = (B_x(t), B_z(t))$$

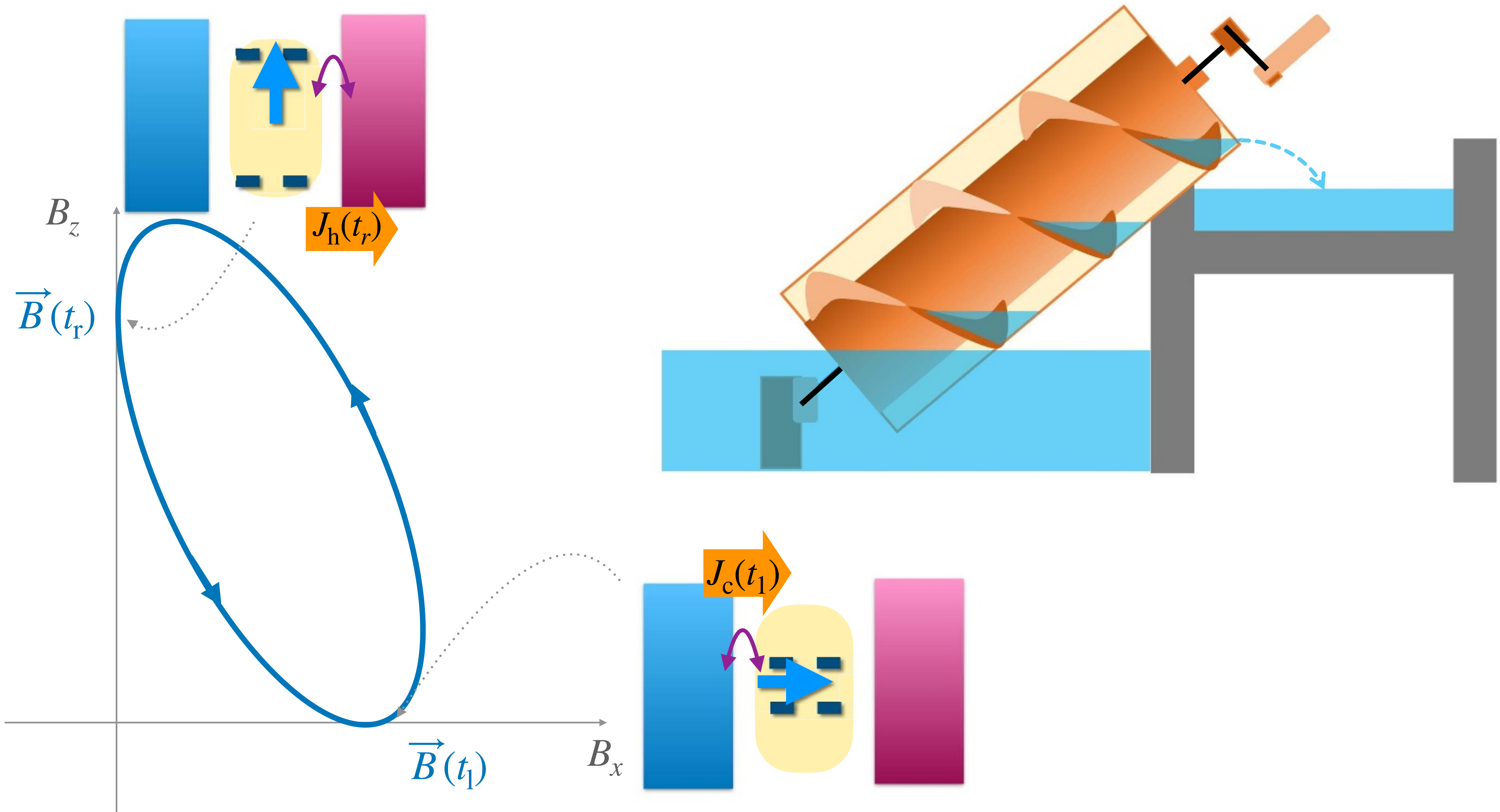
$$B_x(t) = B_{x,0} + B_{x,1}\cos(\Omega t + \phi),$$
$$B_z(t) = B_{z,0} + B_{z,1}\cos(\Omega t).$$

$$\overrightarrow{\Lambda}(\overrightarrow{X}) = \left(\Lambda_{N+1,1}(\overrightarrow{X}), \ldots, \Lambda_{N+1,N}(\overrightarrow{X})\right)$$

$$Q_{\text{pump}} = \oint \overrightarrow{\Lambda}(\overrightarrow{X}) \cdot d\overrightarrow{X}$$






B_z
$\vec{B}(t_r)$
$J_h(t_r)$
$\vec{B}(t_l)$
B_x
$J_c(t_l)$

Geometric optimization of non-equilibrium adiabatic thermal machines and implementation in a qubit system

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Heat engine

$$\overrightarrow{X}(t) \equiv \overrightarrow{B}(t)$$

$$t = \tau\theta$$

$$\vec{B}(t) = \vec{B}(\theta\tau)\,, \quad \theta \in [0,1]$$

Heat-work conversion   Dissipation

$$W = \frac{\Delta T}{T} A - \frac{L^2}{\tau}$$

$$Q = A + \frac{\Delta T}{T}\tau\langle\kappa\rangle.$$

Pumping   Thermal transport

"Berry" Curvature

$$A = \int_{\partial\Sigma} \vec{\Lambda} \cdot d\vec{B} = \int_{\Sigma} (\vec{\nabla}_B \wedge \vec{\Lambda}) \cdot d\vec{\Sigma}$$

Thermodynamic length

$$L^2 \geq \left(\int_0^1 d\theta \sqrt{\dot{\vec{B}} \cdot \underline{\Lambda} \cdot \dot{\vec{B}}} \right)^2 \equiv \mathcal{L}^2$$

Heat conductance $\langle\kappa\rangle = \int_0^1 d\theta\, \kappa$

Power and efficiency of the heat engine

$$P = \frac{W}{\tau} = \frac{\Delta T}{T} \frac{A(1 - \frac{\tau_D}{\tau})}{\tau} ,$$

$$\eta = \frac{W}{Q} = \eta_C \frac{1 - \frac{\tau_D}{\tau}}{1 + \frac{\tau}{\tau_k}} ,$$

$$\tau_D = \frac{T}{\Delta T} \frac{L^2}{A} , \quad \tau_\kappa = \frac{T}{\Delta T} \frac{A}{\langle \kappa \rangle}$$

Maximal Power and efficiency

Optimizing with respect to the duration of the cycle:

$$P_{\max} = \frac{1}{4}\frac{(\Delta T)^2}{T^2}\frac{A^2}{L^2}, \qquad \eta_{P_{\max}} = \frac{\eta_C}{2}\frac{x-1}{x+1} \qquad x = 1 + \frac{A^2}{L^2\langle\kappa\rangle}.$$

Optimizing the power reduces to an isoperimetric problem:

The task of finding the shape that maximizes the ratio between area and perimeter in a space with non-trivial metric.

Isoperimetric problem
= Cheeger problem

Interesting open problem in geometry!

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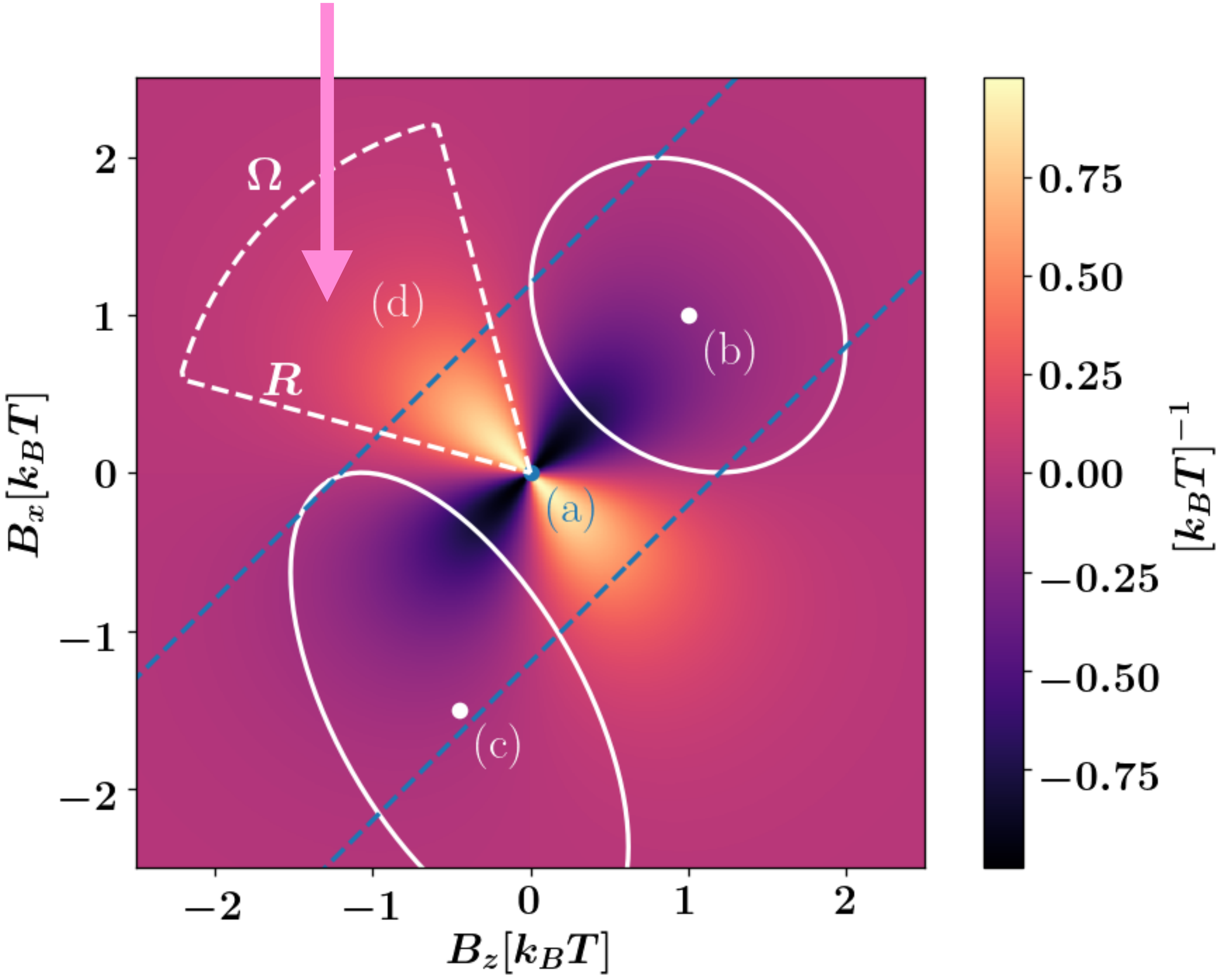
Results

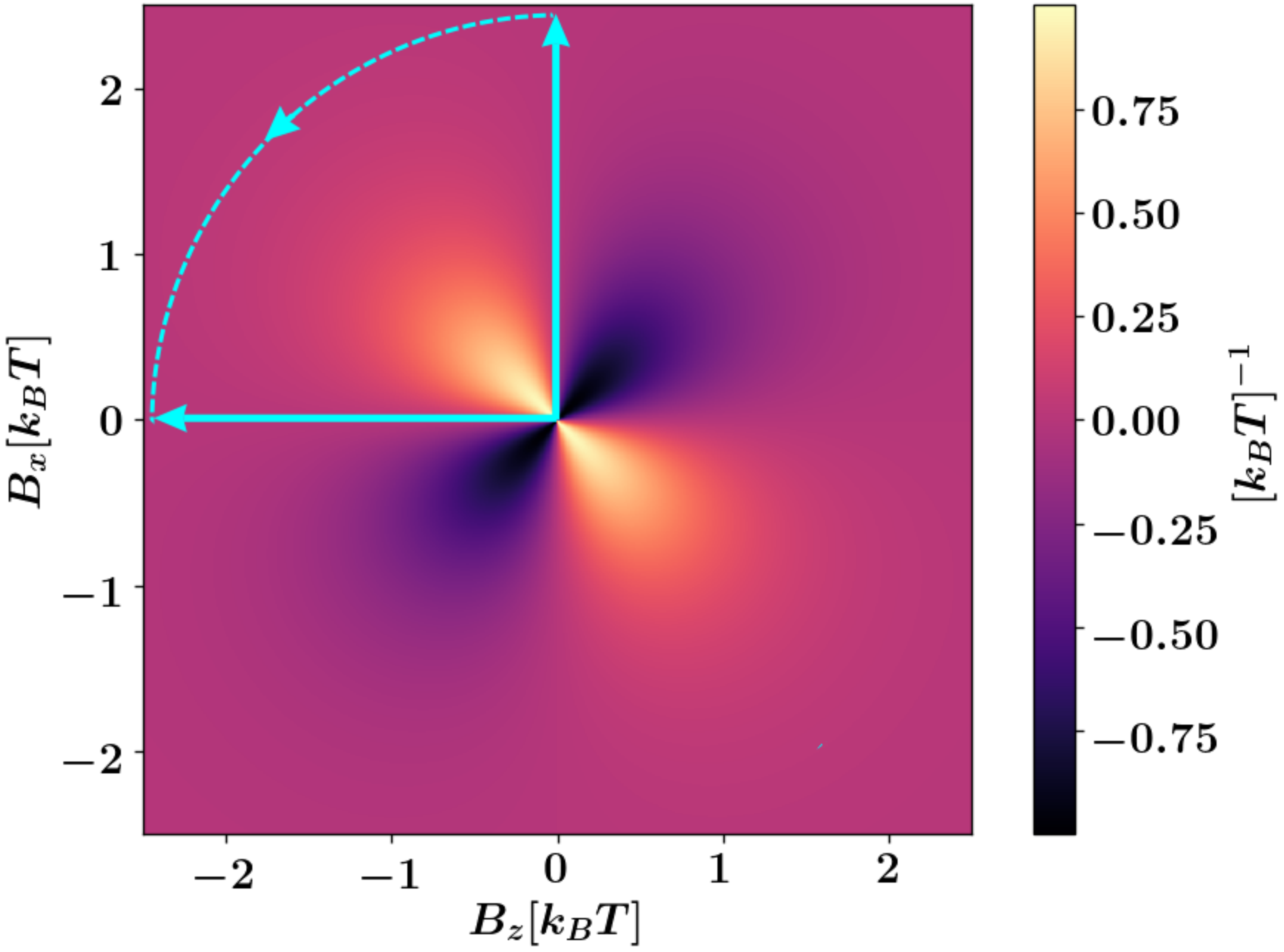
The Berry-type curvature $\left[\vec{\nabla}_B \wedge \vec{\Lambda}(B)\right]_y$.

This protocol saturates the bound A= $k_B T \log(2)$

Landauer bound in the quasistatic limit

Optimal A



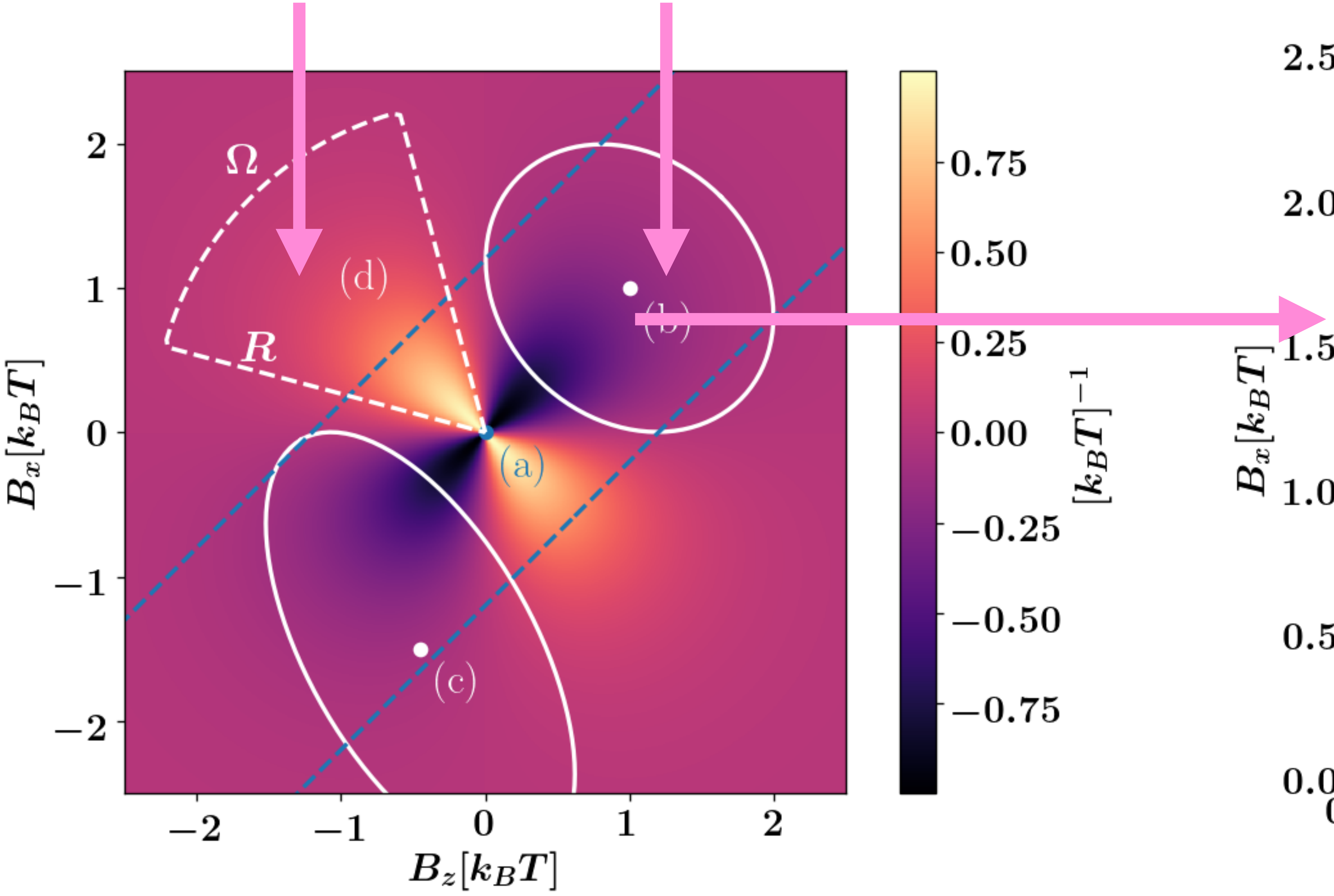


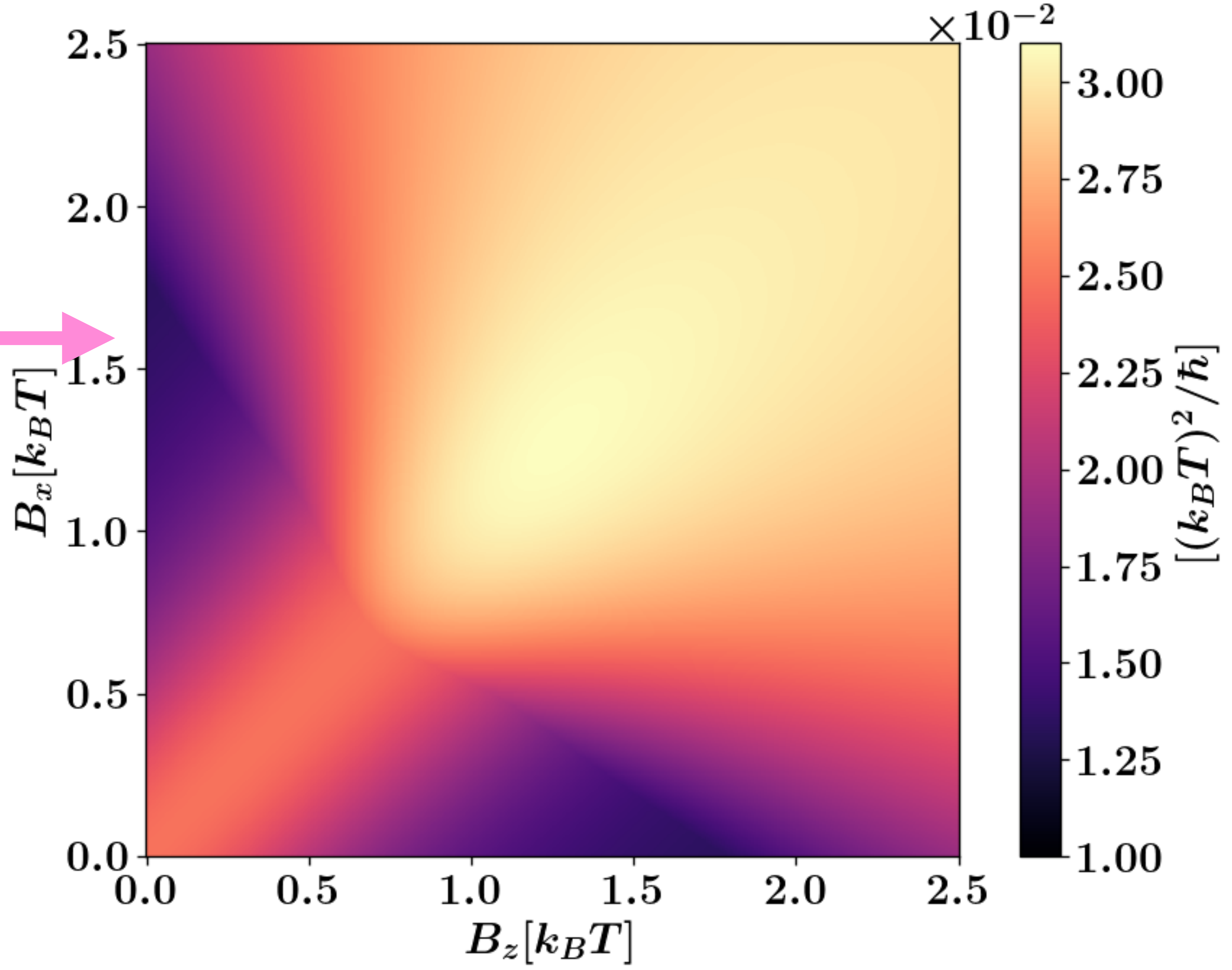
Results:

$\max A^2/\mathcal{L}^2$

Optimal A

Optimal A/L





Open problems

Connection with topology

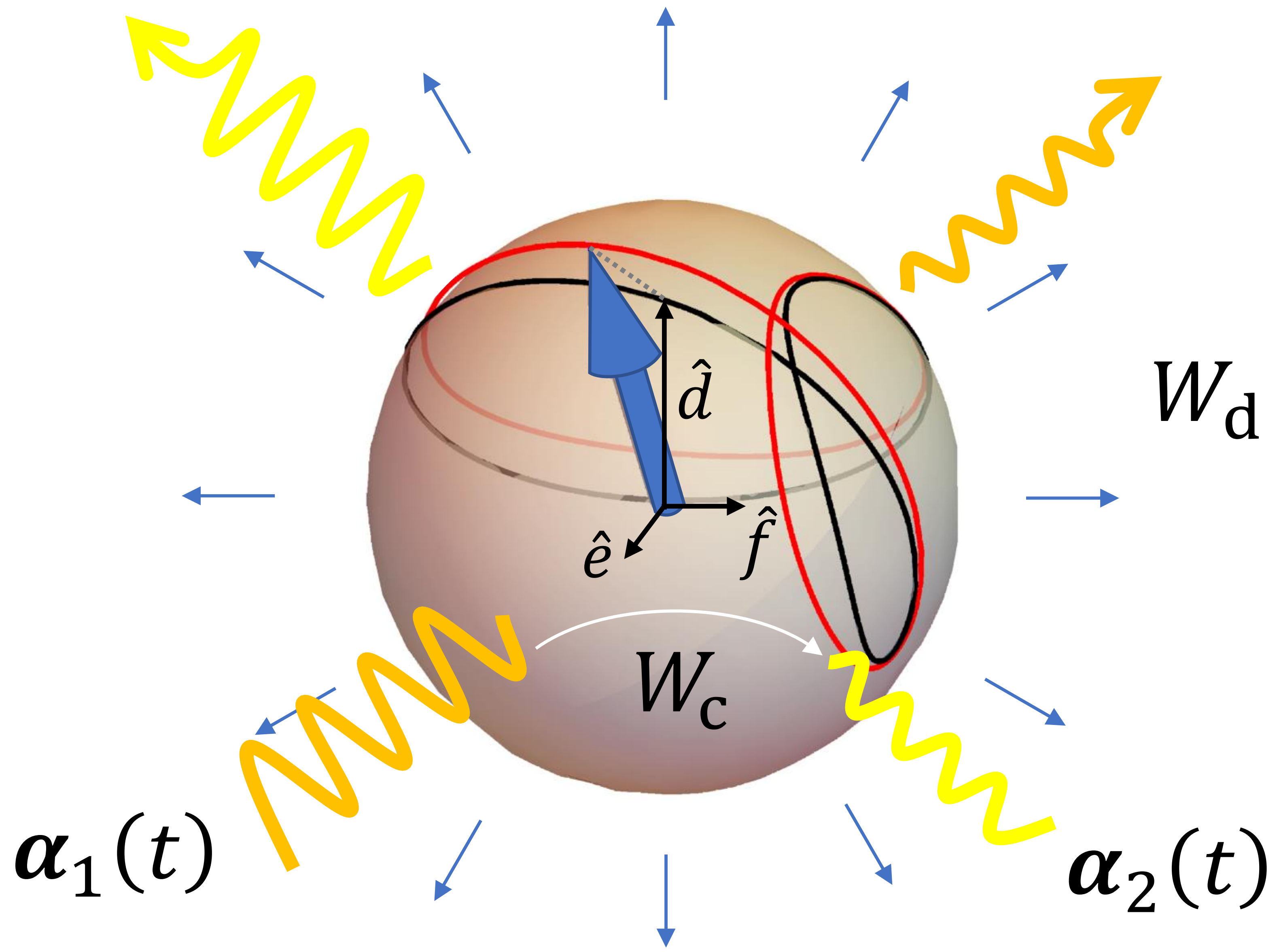
Quantum geometry and bounds on dissipation in slowly driven quantum systems

Iliya Esin,[1] Étienne Lantagne-Hurtubise,[1] Frederik Nathan,[1,2] and Gil Refael[1]

arXiv: 2306.17220

$$H(t) = h_0(t) + \mathbf{d}(t) \cdot \boldsymbol{\sigma}$$

Two-frequency driving: $\phi_1(t) = \omega_1 t$, $\phi_2(t) = \omega_2 t$



Net work conversion:

$$\overline{W}_\mathrm{c} = \frac{\omega_1 \omega_2}{T} \int_0^T dt \left[\frac{\tau_2^2 \Delta^2}{1 + \tau_2^2 \Delta^2} \right] \Omega_{12},$$

Berry curvature: $\Omega_{12} = \frac{1}{2} \mathbf{d} \cdot \left(\partial_{\phi_1} \hat{\mathbf{d}} \times \partial_{\phi_2} \hat{\mathbf{d}} \right)$

Spin-boson problem out of equilibrium

Corrected: Publisher correction

ARTICLE

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Probing the strongly driven spin-boson model in a superconducting quantum circuit

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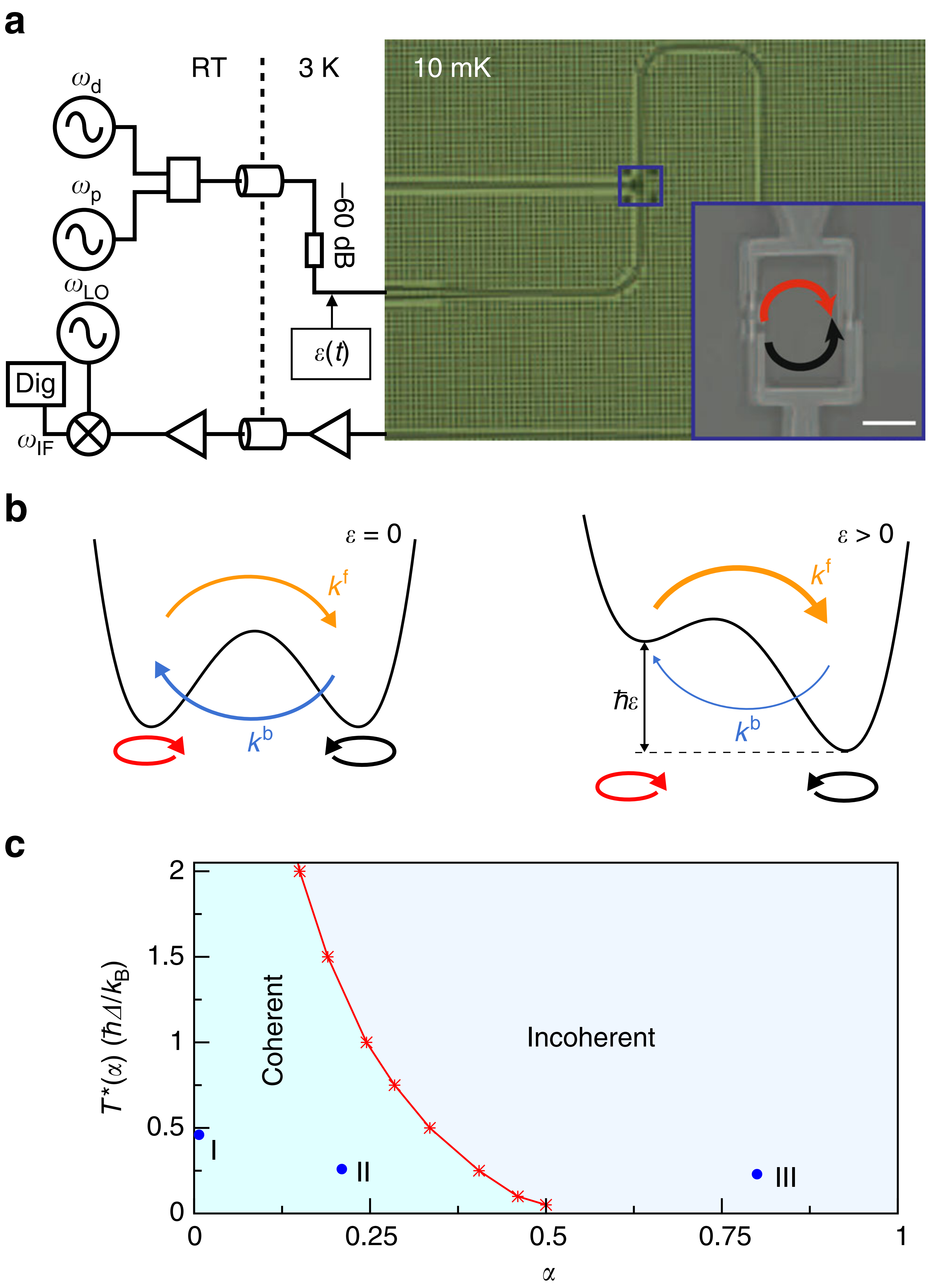

Take-home message

- Quantum systems operating under periodic driving in contact to two thermal reservoirs at different temperatures may operate as thermal machines: engines or refrigerators if and only if there exist a heat-work conversion mechanism.
- For slow driving such mechanism is associated to pumping and it is described by a geometric quantity akin to a Berry phase.
- Useful for optimizing protocols in combination with the thermodynamic length.

Collaborators

CONICET

- PhD Students:, Juan Herrera Mateos, Gabriel Rodriguez Ruiz, Gerónimo Caselli
- Ex students/postdocs: Daniel Gresta, Pablo Terren Alonso, Leonel Gruñeiro, Gianmichele Blasi, Bibek Bhandari, Florencia Ludovico, Nastaran Dashi, Paolo Abiuso, Francesca Battista, Javier Romero
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- Pablo Roura-Bas, Armando Aligia, Carlos Balseiro (Bariloche)
- Eduardo Fradkin (Urbana-Illinois), Alfredo Levy Yeyati (Madrid), David Sanchez and Rosa Lopez (Balears)
- Marti Perarnau-Llobet (Genève), Janine Splettstoesser (Chalmers)
- Alesandro Braggio, Fabio Taddei, Matteo Carrega (Pisa)
- Michael Moskalets (Kharkiv), Rosario Fazio (Trieste), Felix von Oppen (Berlin)

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Thank you!

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Review

Energy dynamics, heat production and heat–work conversion with qubits: toward the development of quantum machines

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Xul Solar, Argentina, 1937-1963

Optimal driving for a given trajectory

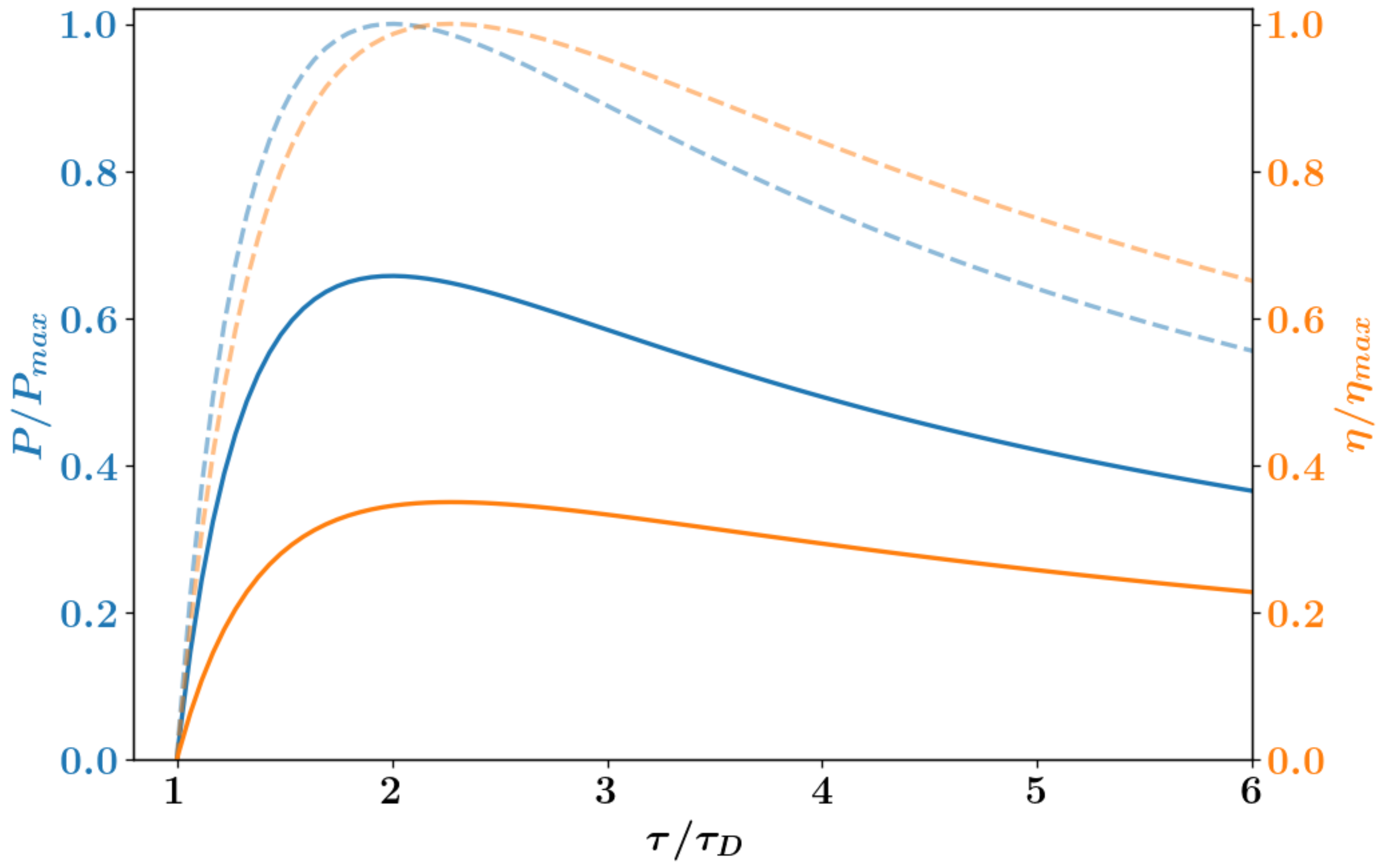

FIG. 8. Power (blue) and efficiency (orange) for curve (b) of Fig. 6 as a function of the cycle duration τ. Solid lines: circulating around the curve at constant angular velocity. Dashed lines: Using the optimal velocities given by Eqs. (13) (for power) and (38) (for efficiency).

Adiabatic forces

$$\langle \mathcal{F} \rangle(t) = \langle \mathcal{F} \rangle_t + \underline{\underline{\mathbf{\Lambda}}}(\vec{X}) \cdot \dot{\mathbf{X}}.$$

$$\Lambda_{\mu,\nu}(\vec{X}) = \begin{cases} \chi_t^{\text{ad}}[\mathcal{F}_\mu, \mathcal{F}_\nu] & \mu \leqslant N \\ \sum_{\alpha=L,R} \chi_t^{\text{ad}}\left[\mathcal{J}_\alpha^E, \mathcal{F}_\nu\right] & \mu = N+1 \end{cases}$$

$$\dot{\vec{X}}(t)$$

$$\dot{X}_{N+1}(t) = \Delta T(t)/T$$

$$\chi_t^{\text{ad}}\left[\mathcal{F}_l, \mathcal{J}_\alpha^E\right] = -\chi_t^{\text{ad}}\left[\mathcal{F}_l, \mathcal{J}_{\bar{\alpha}}^E\right],$$

$$\chi_t^{\text{ad}}\left[\mathcal{J}_\alpha^E, \mathcal{F}_l\right] = -\chi_t^{\text{ad}}\left[\mathcal{J}_{\bar{\alpha}}^E, \mathcal{F}_l\right],$$