Quantum Thermodynamics

Focus on heat-work conversion
Lecture 2

Liliana Arrachea - 2023

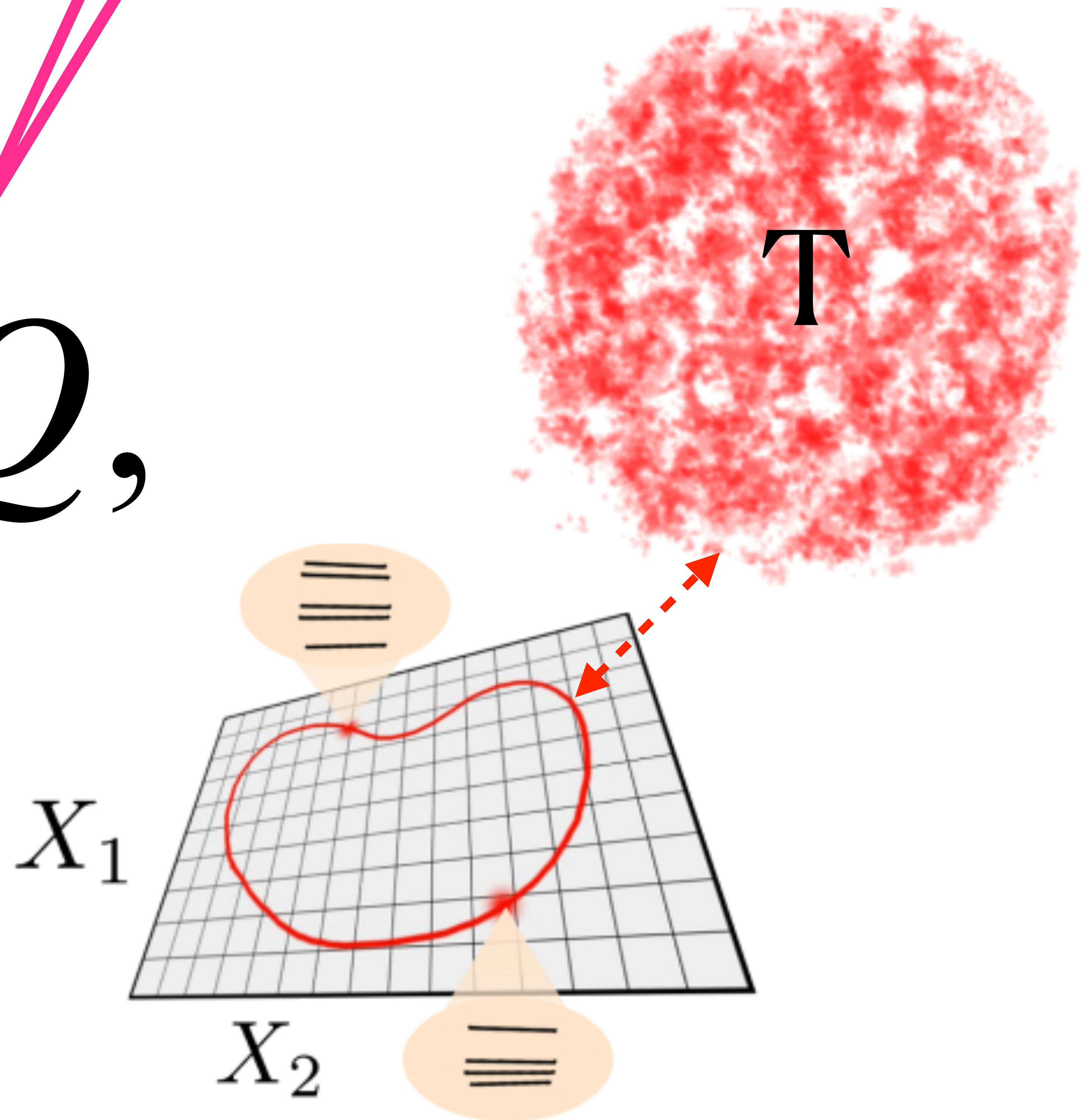
Heat and work
Quasistatic processes

Change in the internal energy

Work: controlled energetic change

Heat: energy exchange with a thermal bath

$$dE = \text{đ}W + \text{đ}Q,$$



Control parameters

$\overrightarrow{X}(t) = \big(X_1(t), \ldots, X_N(t)\big)$

Hamiltonian for the full system: $H(t) = H_B + H_S[\vec{X}(t)] + H_{\text{cont}}$

Bath (T) System Contact

- Weak coupling: $H_{\text{cont}} \to 0$
- Quasistatic: sequence of equilibrium processes described by the frozen Hamiltonian H_t
- State: $\rho_t = \rho_{t,S} \otimes \rho_B$

$$\rho_{t,S} = \frac{1}{Z_t} e^{-\beta H_{t,S}}, \qquad Z_t = \mathrm{Tr}[e^{-\beta H_{t,S}}], \qquad \rho_B = \frac{1}{Z_B} e^{-\beta H_B}, \qquad \beta = 1/(k_B T).$$

Internal energy: $E_t = \mathrm{Tr}\left[\rho_{t,S}\; H_S(\vec{X}_t)\right]$

Changes in the internal energy for changes $H_{t,S} \to H_{t+\delta t,S}$ **:**

$$dE = \mathrm{Tr}\left[\delta\rho_{t,S}\; H_S(\vec{X}_t)\right] + \mathrm{Tr}\left[\rho_{t,S}\; \frac{dH_S(t)}{dt}\right]\delta t.$$

Heat:

$$\delta Q_t = \mathrm{Tr}\left[\delta\rho_{t,S}\; H_S(\vec{X}_t)\right]$$

Work:

$$\delta W_t = \left[\rho_{t,S} \frac{dH_S(t)}{dt}\right]\delta t.$$

Changes in the entropy:

Assume no particle exchange

$$\rho_{t,S} = \frac{1}{Z_t} e^{-\beta H_{t,S}}, \qquad Z_t = \mathrm{Tr}[e^{-\beta H_{t,S}}], \qquad \rho_B = \frac{1}{Z_B} e^{-\beta H_B}, \qquad \beta = 1/(k_B T).$$

$$S_t = -\mathrm{Tr}\left[\rho_{t,S} \ln \rho_{t,S}\right] = -\mathrm{Tr}\left[\rho_{t,S}\left(-\beta H_{t,S} - \ln Z_{t,S}\right)\right]$$

$$\begin{aligned} dS_t &= -\mathrm{Tr}\left[d\rho_{t,S}\left(-\beta H_{t,S} - \ln Z_{t,S}\right)\right] - \mathrm{Tr}\left[\rho_{t,S}\left(-\beta dH_{t,S} - d(\ln Z_{t,S})\right)\right] \\ &= \beta \mathrm{Tr}\left[d\rho_{t,S} H_{t,S}\right] + \mathrm{Tr}\left[d\rho_{t,S}\right] \ln Z_{t,S} + \beta \mathrm{Tr}\left[\rho_{t,S} dH_{t,S}\right] + d(\ln Z_{t,S}) \\ &= \beta \mathrm{Tr}\left[d\rho_{t,S} H_{t,S}\right] + \beta \mathrm{Tr}\left[\rho_{t,S} dH_{t,S}\right] - \beta dF = \beta \mathrm{Tr}\left[d\rho_{t,S} H_{t,S}\right], \end{aligned}$$

$$\mathrm{Tr}\left[d\rho_{t,S}\right] = 0.$$

$$dS_t = \delta Q/T = k_B \beta \mathrm{Tr}\left[d\rho_{t,S} H_{t,S}\right]$$

In consistency with the previous definition

Heat and work
Finite-time processes

Fluxes and power

$$J_B^Q(t) = \frac{d\langle H_B \rangle}{dt} = -\frac{i}{\hbar}\langle [H_B, H] \rangle$$

Heat flux into the bath (assuming no particle exchange)

$$P(t) = -\langle \frac{\partial H_S}{\partial t} \rangle = \sum_\ell F_\ell \dot{X}_\ell$$

Power developed by the driving sources

Classical geometric forces of reaction: an exactly solvable model

By M. V. Berry and J. M. Robbins

H. H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, U.K.

Forces $F_\ell = -\langle \frac{\partial H_S}{\partial X_\ell} \rangle$

Strategy

Hamiltonian for the full system: $H(t) = H_B + H_S[\vec{X}(t)] + H_{\mathrm{cont}}$

Bath (T) System Contact

Force:

Non-equilibrium component

$$\overrightarrow{F}(t) = -\langle \partial_{\vec{X}} H \rangle(t) = -\langle \partial_{\vec{X}} H \rangle_t - \langle \partial_{\vec{X}} H \rangle_{\mathrm{ne}}(t)$$

Frozen component $\langle O \rangle_t = \mathrm{Tr}\left[\rho_t O\right]$ Equilibrium state with frozen parameters at t $\rho_t = e^{-\beta H_t}/Z_t$

Linear response approach

Linear response in amplitude

Kubo formalism Book: Bruus-Flensberg

Time-dependent Hamiltonian:

$$H(t) = H_0 + H'(t)\theta(t - t_0).$$

$t < t_0$

$$\langle A \rangle = \frac{1}{Z_0}\mathrm{Tr}\left[\rho A\right] = \frac{1}{Z_0}\sum_n \langle n|A|n\rangle e^{-\beta E_n}$$

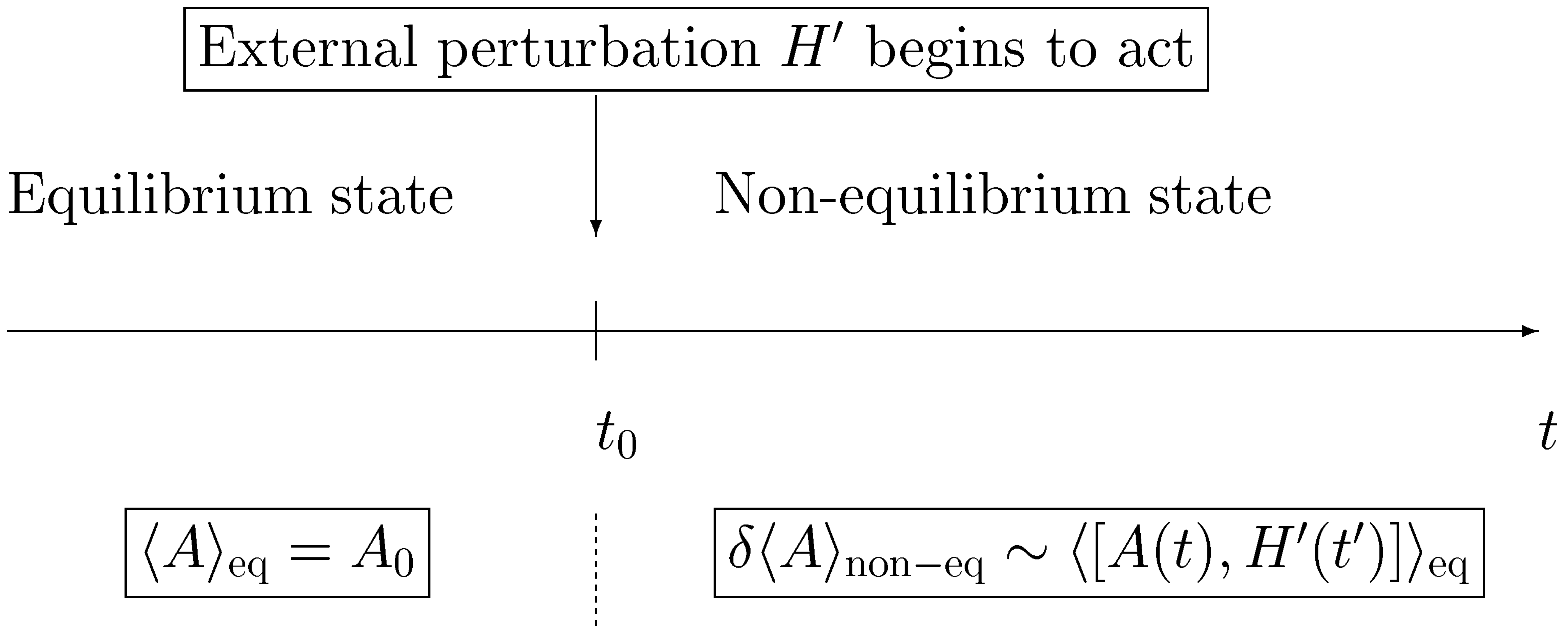
$$Z_0 = \mathrm{Tr}[\rho_0]$$

$$\rho_0 = e^{-\beta H_0} = \sum_n |n\rangle\langle n| e^{-\beta E_n},$$

$t > t_0$

$$\langle A(t) \rangle = \frac{1}{Z_0}\sum_n \langle n(t)|A|n(t)\rangle e^{-\beta E_n} = \frac{1}{Z_0}\mathrm{Tr}\left[\rho(t) A\right],$$

$$\rho(t) = \sum_n |n(t)\rangle\langle n(t)| e^{-\beta E_n}.$$



$t < t_0$: States distributed according to a Boltzman distribution

$e^{-\beta E_n^0} / Z_0$

$t > t_0$: States evolve preserving the same distribution.

Evolution of the states:

Schrödinger representation: $i\partial_t|n(t)\rangle = H(t)|n(t)\rangle.$

Interaction representation: $|n(t)\rangle = e^{-iH_0t}|\hat{n}(t)\rangle = e^{-iH_0t}\hat{U}(t,t_0)|\hat{n}(t_0)\rangle.$

Linear-response approximation:

$$\hat{U}(t,t_0) = T\exp\left\{-i\int_{t_0}^{t} dt' H'(t')\right\} \simeq 1 - i\int_{t_0}^{t} dt' \hat{H}(t')$$

Mean value of observables at linear order in $H'(t)$

$$\langle A(t)\rangle = \langle A\rangle_0 - i\int_{t_0}^{t} dt' \frac{1}{Z_0}\sum_n e^{-\beta E_n}\langle n(t_0)|\hat{A}(t)\hat{H}'(t') - \hat{H}'(t')\hat{A}(t)|n(t_0)\rangle$$

$$= \langle A\rangle_0 - i\int_{t_0}^{t} dt' \langle[\hat{A}(t),\hat{H}'(t')]\rangle_0.$$

$$\hat{O}(t) = e^{iH_0t}Oe^{-iH_0t}$$

Adiabatic linear response

Hamiltonian for the full system: $H(t) = H_B + H_S[\vec{X}(t)] + H_{\text{cont}}$

Bath (T) System Contact

Mean value of observables:

Non-equilibrium component

$$\langle O \rangle(t) = \langle O \rangle_t + \langle O \rangle_{\text{ne}}(t),$$

Frozen component

$$\langle O \rangle_t = \text{Tr}\left[\rho_t O\right]$$

Equilibrium state with frozen parameters at t

$$\rho_t = e^{-\beta H_t}/Z_t$$

Goal: evaluation of non-equilibrium component at $\mathcal{O}(\dot{\vec{X}})$

Kubo-like treatment

Ludovico, Battista, von Oppen, and Arrachea, Phys. Rev. B 93, 075136 (2016).

Closed systems: Weinberg, Bulov, D'Alessio, Polkonnikov, Phys Rep. 688 (2017)

- Frozen Hamiltonian: $H_t = H[\vec{X}]$, $\vec{X}$: Value of the parameters at time t
- Departures due to the variation of the parameters: δH treated as a perturbation
- δH expanded with respect to its value at t assuming slow variation of the parameters: $\delta H(t') \simeq -(t'-t)\vec{F}(\vec{X}) \cdot \dot{\vec{X}}(t)$
- Introduce the interaction picture treating $\delta H(t')$ as a time-dependent perturbation.
- The evolution operator is

$$U(t,t_0) \simeq T\exp\{-iH_t(t-t_0) - i\int_{t_0}^{t} dt'(t-t')\vec{F}(t')\cdot\dot{\vec{X}}(t)\}.$$

$$\vec{F}(t') \equiv e^{iH_t t'}\vec{F}(\vec{X})e^{-iH_t t'}$$

Expanding the evolution operator at linear order:

$$O(t) = \langle O \rangle_t - i \int_{t_0}^{t} dt'(t-t') \langle \left[O(t), F(t') \right] \rangle_t \cdot \dot{\vec{X}}(t)$$

$$t_0 \to -\infty$$

Adiabatic susceptibility

$$\Lambda_{O,j}(\vec{X}) = -i\theta(t-t') \int_{-\infty}^{t} dt'(t-t') \langle \left[O(t), F_j(t') \right] \rangle_t .$$

$$= \lim_{\omega \to 0} \frac{\mathrm{Im}\left[\chi_t^{O,j}(\omega)\right]}{\omega}$$

$$\langle \ldots \rangle_t = \mathrm{Tr}\left[\rho_t \ldots\right]$$

Kubo susceptibility

Comments

- Adiabatic linear response formalism is appropriate for time-dependent parameters varying in a time scale that is much larger than the time scale for the internal dynamics of the quantum system coupled to the bath.
- Linear-response coefficients can be expressed in terms of Kubo susceptibilities => micro reversibility => Onsager relations. Analytical and numerical methods for many-body systems in equilibrium can be used to evaluate them.

Adiabatic expansion:

$$\langle O\rangle(t) = \langle O\rangle_t + \sum_j \Lambda_{O,j}(\vec{X})\dot{X}_j\,,$$

Forces:

$$F_\ell(t) = F_{\ell,t} + \sum_{\ell'=1}^{N} \Lambda_{\ell,\ell'}(\vec{X})\dot{\vec{X}}_{\ell'}$$

Heat flux in bath:

$$J_B(t) = \cancelto{0}{J_{B,t}} + \sum_{\ell'=1}^{N} \Lambda_{B,\ell'}(\vec{X})\dot{\vec{X}}_{\ell'}$$

Conserved and dissipated work

Leading order

Substituting the adiabatic expansion for the force in the power:

$$P(t) = \sum_{\ell} F_{\ell,t}(\vec{X})\dot{X}_{\ell} + \sum_{\ell,j} \dot{X}_{\ell}\ \Lambda_{\ell,j}(\vec{X})\ \dot{X}_{j}$$

Conservative component

$$W_t = \sum_{\ell} \int_{t}^{t+\delta t} F_{\ell,t}\dot{X}_{\ell}dt = \int_{\vec{X}_t}^{\vec{X}_{t+\delta t}} \vec{F}(\vec{X}) \cdot d\vec{X} = \int_{\vec{X}_t}^{\vec{X}_{t+\delta t}} \nabla_{\vec{X}} E_t(\vec{X}) \cdot d\vec{X},$$

$W_t = 0$ in a cycle

Dissipated work

$$W^{(\mathrm{diss})} = \int_{t}^{t+\delta t} dt \sum_{\ell,j} \dot{X}_{\ell}\ \Lambda_{\ell,j}(\vec{X})\ \dot{X}_{j}.$$

Reversible heat exchange and entropy production

$$J_B^Q(t) = \cancelto{0}{J_{B,t}^Q} + \sum_j \Lambda_{B,j}(\vec{X})\dot{X}_j + T\dot{\Sigma}(t)$$

Reversible =0 in a cycle for a single bath:

$$Q_{\mathrm{rev}} = \sum_j \int_t^{t+\delta t} dt \Lambda_{B,j}(\vec{X})\dot{X}_j = \int_{\vec{X}_t}^{\vec{X}_{t+\delta t}} d\vec{X}\cdot\vec{\Lambda}(\vec{X}),$$

$$\vec{\Lambda}(\vec{X}) = \Big(\Lambda_{B,1}(\vec{X}),\ldots,\Lambda_{B,N}(\vec{X})\Big)$$

Entropy production:

$$T\Sigma = W^{(\mathrm{diss})} = \sum_{\ell,\ell'} \int_t^{t+\delta t} dt \dot{X}_\ell(t)\Lambda^S_{\ell,\ell'}(\vec{X})\dot{X}_{\ell'}(t), \quad \geq 0$$

Weak coupling to the bath

Master-equations approach

Master equation for the reduced density matrix corresponding to the frozen Hamiltonian:

$$\frac{d\rho_{t,\mathrm{S}}(t')}{dt'} = -i\left[\rho_{t,\mathrm{S}}, H_{S,t}(\vec{X})\right] + \mathcal{L}\left[\rho_{t,\mathrm{S}}\right] \to M(\vec{X})\vec{\rho}_{t,\mathrm{S}}$$

Dissipator: Coupling to reservoir

Stationary solution:

$$M(\vec{X})\vec{\rho}_{t,\mathrm{S}} = 0.$$

+ normalization

$$\rho_{t,\mathrm{S}} = 1/Z_t \sum_n e^{-\beta E_n} |n\rangle\langle n|$$

$$H_{S,t}|n\rangle = E_n|n\rangle$$

Solution of the master equation for $\rho_S(t)$ at $\mathcal{O}(\dot{\vec{X}})$:

$$\frac{d\rho_{\rm S}(t')}{dt'} = -\,i\left[\rho_{\rm S}(t'), H_S(\vec{X}(t'))\right] + \mathcal{L}\left[\rho_{\rm S}(t')\right]$$

$$\rho_{\rm S}(t') = \rho_{t,{\rm S}}(t') + \rho_{a,{\rm S}}(t') \qquad\qquad \rho_{a,{\rm S}}(t') \propto \dot{\vec{X}}$$

- For the rates: J. Splettstoesser, M. Governale, J. König, and R. Fazio, Phys. Rev. B **74**, 085305 (2006); R.-P. Riwar and J. Splettstoesser, Phys. Rev. B **82**, 205308 (2010).
- V. Cavina, A. Mari, and V. Giovannetti, Phys. Rev. Lett. 119, 050601 (2017) (Qubits)
- B. Bandhari et al, Phys. Rev. B 104, 035425 (2021)

Stationary solution of the frozen component:

$$M(\vec{X})\vec{\rho}_{t,\mathrm{S}} = 0. \quad + \text{Normalization}$$

$$\mathscr{M}(\overrightarrow{X})\overrightarrow{\rho}_{t,\mathrm{S}} = \vec{\gamma}$$

Adiabatic correction

$$\overrightarrow{\rho}_{\mathrm{S}}(t') \simeq \overrightarrow{\rho}_{t,\mathrm{S}}(t') + \overrightarrow{\rho}_{a,\mathrm{S}}(t') \qquad \overrightarrow{\rho}_{a,\mathrm{S}}(t') \propto \dot{\overrightarrow{X}}$$

Substituting in the master equation:

$$\sum_{\ell} \frac{\partial \overrightarrow{\rho}_{t,\mathrm{S}}}{\partial X_{\ell}} \dot{X}_{\ell} = \mathscr{M}(\overrightarrow{X})\overrightarrow{\rho}_{a,\mathrm{S}}$$

Steady state

$$\overrightarrow{\rho}_{a,\mathrm{S}} = \sum_{\ell} \overrightarrow{\lambda}_{\ell}(\overrightarrow{X})\dot{X}_{\ell} \qquad \overrightarrow{\lambda}_{\ell}(\overrightarrow{X}) = \mathscr{M}^{-1}(\overrightarrow{X})\frac{\partial \overrightarrow{\rho}_{t,\mathrm{S}}}{\partial X_{\ell}}$$

Evaluation of mean values: Forces and work

Conservative:

$$F_{\ell,t} = -\mathrm{Tr}[\rho_{t,\mathrm{S}} \frac{\partial H_S}{\partial X_\ell}], \longrightarrow W_t = \sum_\ell \int_t^{t+\delta t} F_{\ell,t} \dot{X}_\ell dt = \int_{\vec{X}_t}^{\vec{X}_{t+\delta t}} \nabla_{\vec{X}} E_t(\vec{X}) \cdot d\vec{X}$$

Non-conservative:

$$F_\ell^{\mathrm{non-cons}} = \mathrm{Tr}\left[\rho_{a,\mathrm{S}}\right] = \sum_{\ell'} \Lambda_{\ell,\ell'} \dot{X}_{\ell'}, \qquad \Lambda_{\ell,\ell'}(\vec{X}) = \mathrm{Tr}[\frac{\partial H_S}{\partial X_\ell} \lambda_{\ell'}(\vec{X})].$$

$$\longrightarrow W^{(\mathrm{diss})} = \int_t^{t+\delta t} dt \sum_{\ell,\ell'} \dot{X}_\ell \, \Lambda_{\ell,\ell'}(\vec{X}) \, \dot{X}_{\ell'}$$

Geometrical properties

Properties of the tensor $\Lambda_{\ell,\ell'}$

$$T\Sigma = W^{(\text{diss})} = \sum_{\ell,\ell'} \int_t^{t+\delta t} dt \dot{X}_\ell(t) \Lambda^S_{\ell,\ell'}(\vec{X}) \dot{X}_{\ell'}(t),$$

- Satisfies Onsager relations $\Lambda_{\ell,\ell'}(\overrightarrow{X}, B) = \pm \Lambda_{\ell',\ell}(\overrightarrow{X}, -B)$
- Only the symmetric component contributes to dissipation!
- Positive defined.
- Plays the role of a metric in the parameter space.

Thermodynamic length

F. Weinhold, Metric geometry of equilibrium thermodynamics Jour. Chem Phys. **63**,2479 (1975)

G. Ruppeiner, Thermodynamics: A Rimannian geometric model Phys. Rev. A **20**, 1608 (1979)

Salamon, P.; Nitzan, A.; Andresen, B.; Berry, R.S. Minimum entropy production and the optimization of heat engines. Phys. Rev. A 1980, 21, 2115–2129

D. A. Sivak and G. E. Crooks, Thermodynamic Metrics and Optimal Paths, Phys. Rev. Lett. **108**, 190602 (2012).

M. Scandi and M. Perarnau-Llobet, Thermodynamic length in open quantum systems, Quantum **3**, 197 (2019).

P. Abiuso and M. Perarnau- Llobet, Optimal cycles for low-dissipation heat engines, Phys. Rev. Lett. **124**, 110606 (2020)

Length of a curve connecting two points parametrized by t

$$\mathcal{L} = \int_{t}^{t+\delta t} dt \sqrt{\dot{\vec{X}}(t) \cdot \underline{\Lambda}^{S}(\vec{X}) \cdot \dot{\vec{X}}(t)},$$

Cauchy-Schwartz inequality: $\int_{t_0}^{t_1} dt f^2 \int_{t_1}^{t_2} dt g^2 \geq \left[\int_{t_1}^{t_2} dt f g dt\right]^2$

Considering $g = 1$

$$T\Sigma \geq \frac{\mathcal{L}^2}{\delta t}$$

Bound for dissipation!

$$T\Sigma \geq \frac{\mathcal{L}^2}{\delta t}$$

$$\mathcal{L} = \int_t^{t+\delta t} dt \sqrt{\dot{\vec{X}}(t) \cdot \underline{\Lambda}^S(\vec{X}) \cdot \dot{\vec{X}}(t)},$$

Depends on the trajectory

Geodesic: path minimizing the distance $\mathscr{L}$ between the two points in the parameter space:

Can be shown that corresponds to keep $\dot{\vec{X}} \cdot \underline{\Lambda}^S(\vec{X}) \cdot \dot{\vec{X}}$ constant.

$\Rightarrow$

Protocols leading to minimal dissipation are those for which the dissipation rate remains constant at each point of the trajectory

Example: dissipation of a driven qubit

Thermodynamic length in open quantum systems

Matteo Scandi and Martí Perarnau-Llobet

Quantum 3, 197 (2019)

$$H = r\cos\varphi\sin\theta\,\hat{\sigma}_x + r\sin\varphi\sin\theta\,\hat{\sigma}_y + r\cos\theta\,\hat{\sigma}_z,$$

Control parameters: $\overrightarrow{X} = \big(r, \theta, \varphi)\big)$

In the diagonal basis $\tilde{H} = \frac{1}{2} r\sigma^z = UHU^{-1}$

Linblad equation:

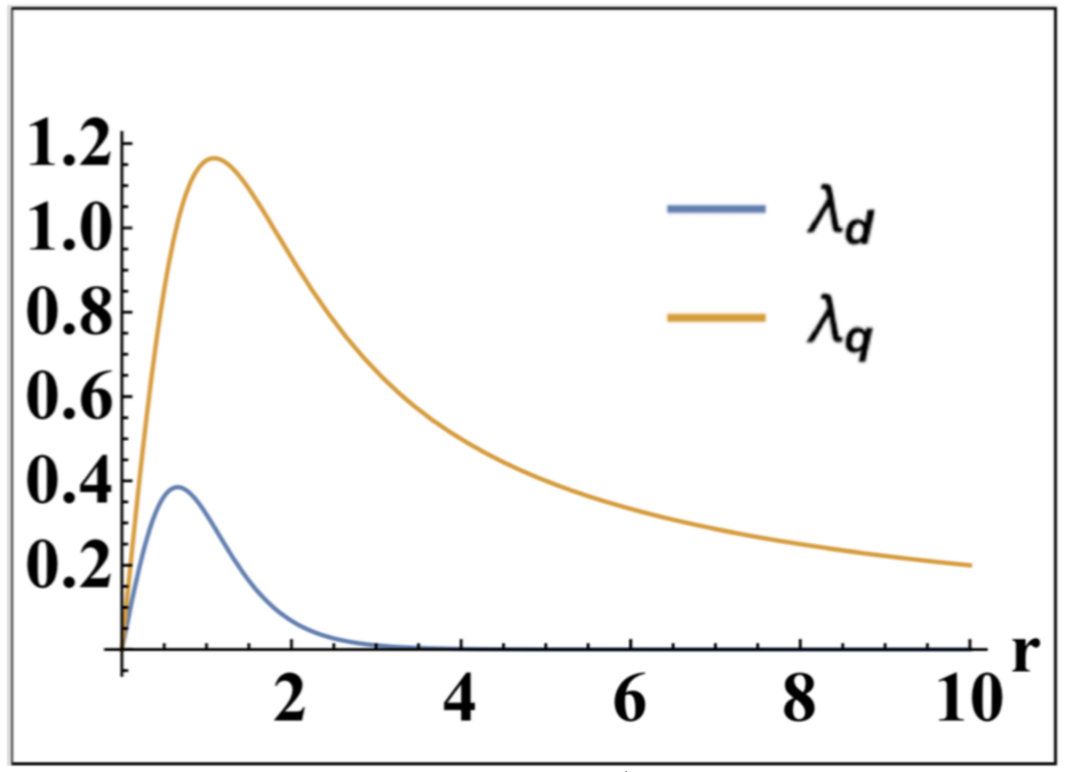
$$\dot{\rho}_t = \gamma_r(P_r+1)\left(\hat{\sigma}_-\rho_t\hat{\sigma}_+ - \frac{1}{2}\{\hat{\sigma}_+\hat{\sigma}_-, \rho_t\}\right) + \gamma_r P_r\left(\hat{\sigma}_+\rho_t\hat{\sigma}_- - \frac{1}{2}\{\hat{\sigma}_-\hat{\sigma}_+, \rho_t\}\right),$$

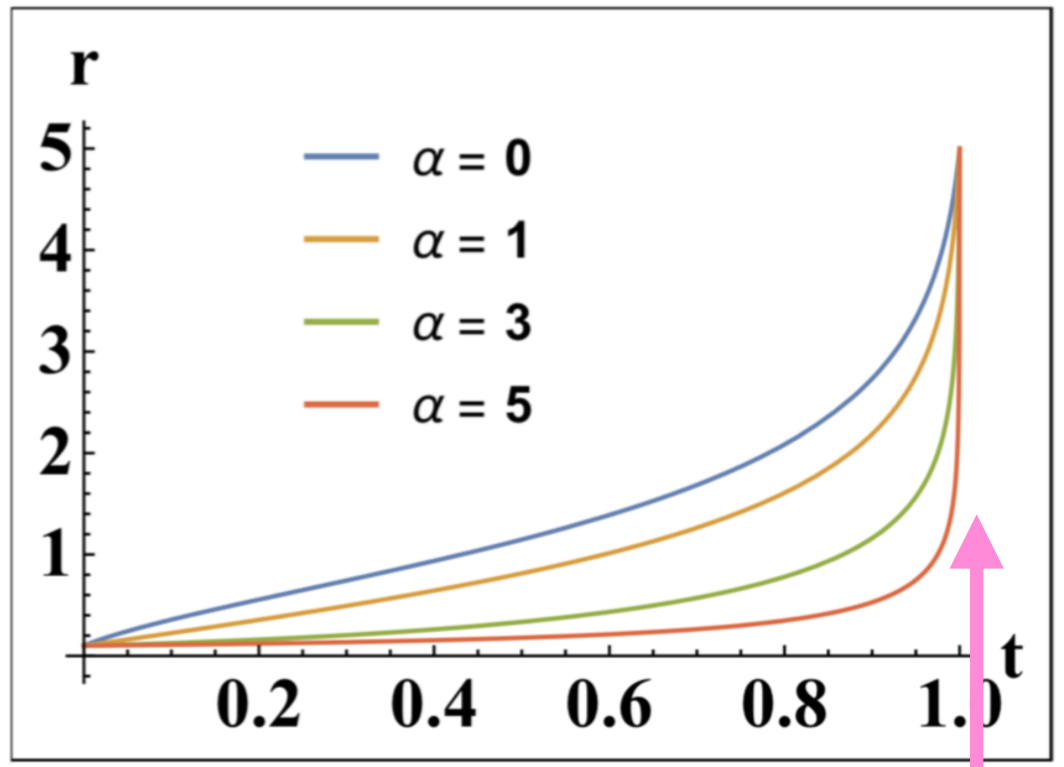
$\gamma_r = \tilde{\gamma}_0 r^\alpha$ $\qquad P_r = \dfrac{1}{e^{2\beta r}-1}.$ $\qquad \alpha >, =, < 1$ supraOhmic, Ohmic, subOhmic

Bloch representation: $\rho_t = \frac{1}{2}\left(\sigma^0 + \overrightarrow{\rho}_t \cdot \overrightarrow{\sigma}\right)$ $\longrightarrow$ Lindblad equation: $\mathscr{M}(\overrightarrow{X})\,\overrightarrow{\rho}_t = \vec{\gamma}(\overrightarrow{X})$

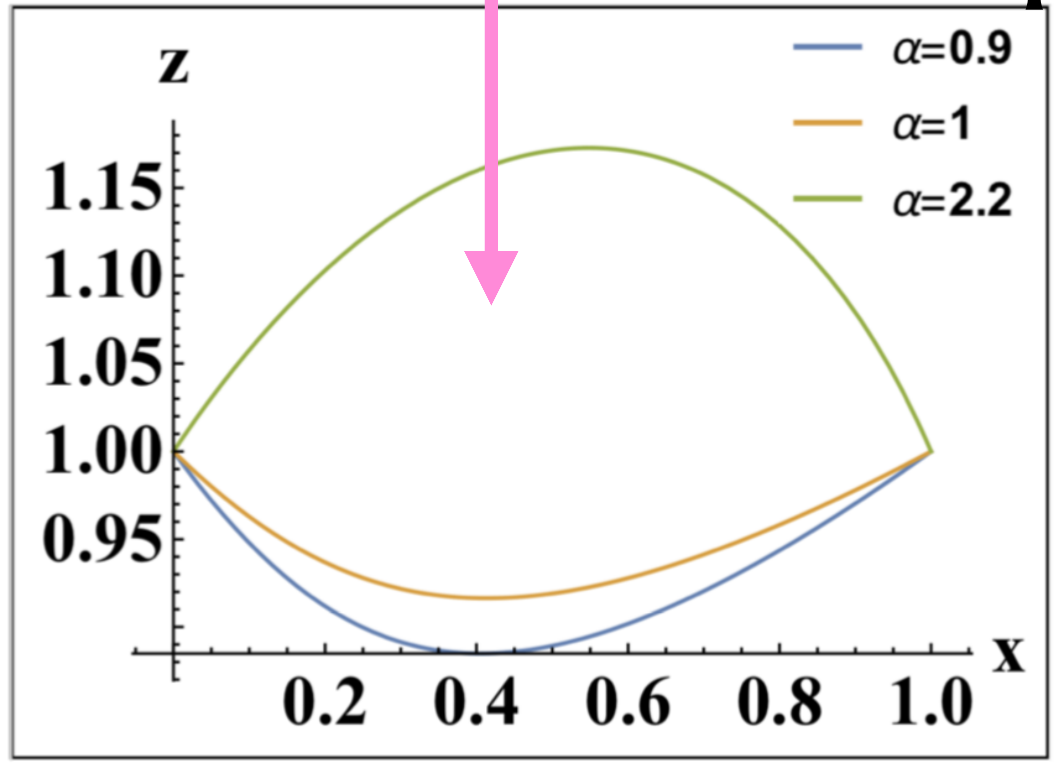
$$\tilde{\rho}_t = U\rho_t U^{-1}$$

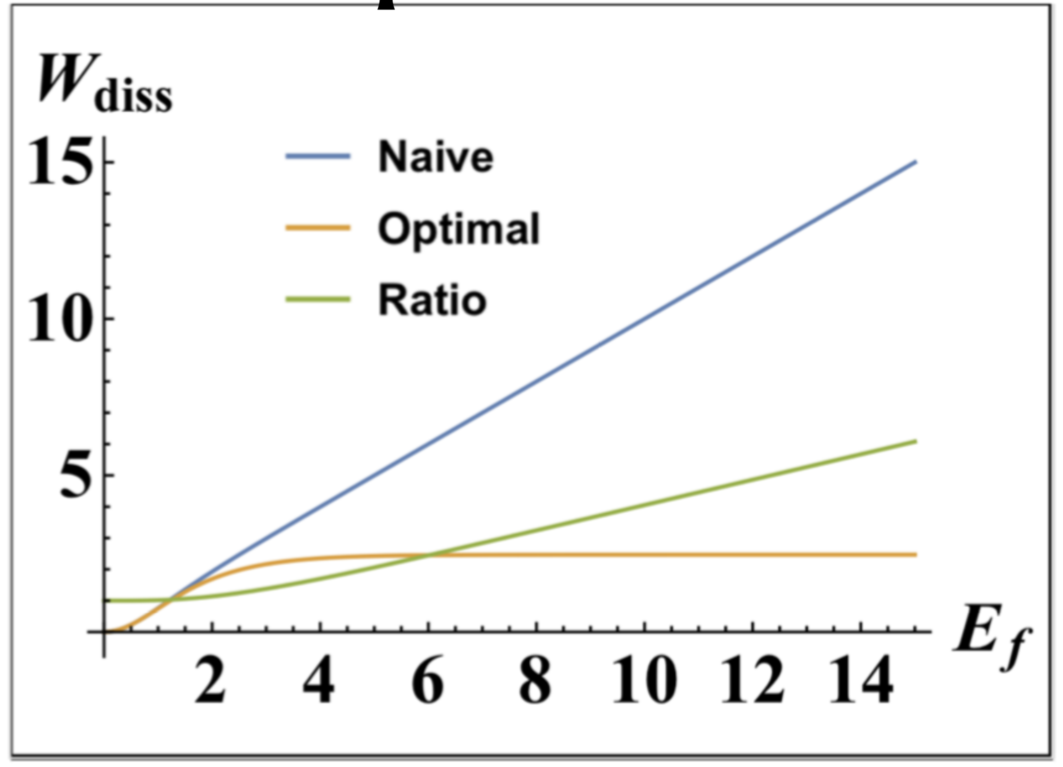
$$\overrightarrow{\rho}^a = \sum_{\ell} \mathscr{M}^{-1}(\overrightarrow{X})\frac{\partial \overrightarrow{\tilde{\rho}}_t}{\partial X_\ell}\dot{X}_\ell \longrightarrow \rho^a \longrightarrow W^{\mathrm{diss}} = \sum_{\ell} \mathrm{Tr}\left[\rho^a \frac{\partial H}{\partial X_\ell}\right]\dot{X}_\ell = \sum_{\ell\ell'} \dot{X}_\ell \Lambda^S_{\ell,\ell'} \dot{X}_{\ell'}$$





Geodesics: optimized protocols





Quantum

Classical

$$m^{\mathcal{L}} = \frac{1}{r^\alpha}\mathrm{diag}\left\{\lambda_d,\ \lambda_q\, r^2,\ \lambda_q\, r^2 \sin^2\theta\right\} \equiv \Lambda_{\ell,\ell'}(r,\theta,\varphi)$$

$$\lambda_d = \frac{\tanh(r)}{\cosh^2(r)}, \qquad \lambda_q = \frac{2\tanh^2(r)}{r}.$$

Power pumping

Power pumping = work-work conversion

$$\Lambda_{\ell,\ell'}(\overrightarrow{X}) = \Lambda^{S}_{\ell,\ell'}(\overrightarrow{X}) + \Lambda^{A}_{\ell,\ell'}(\overrightarrow{X})$$

Dissipation

Pumping

Campisi, Denisov, Hänggi, PRA 86, 032114 (2012)

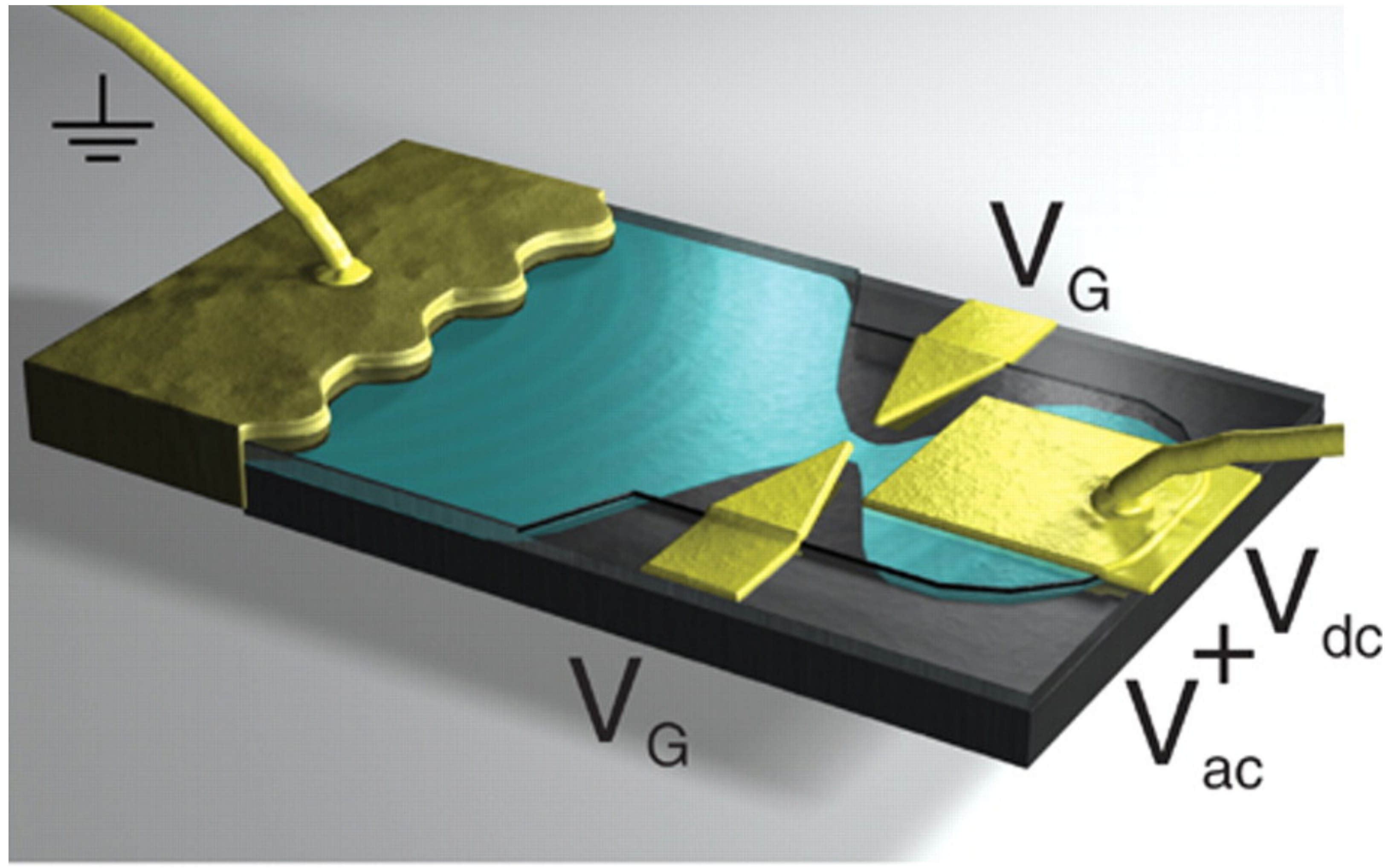
In the Floquet regime: Martin, Refael, Halperin, PRX.7.041008 (2017)

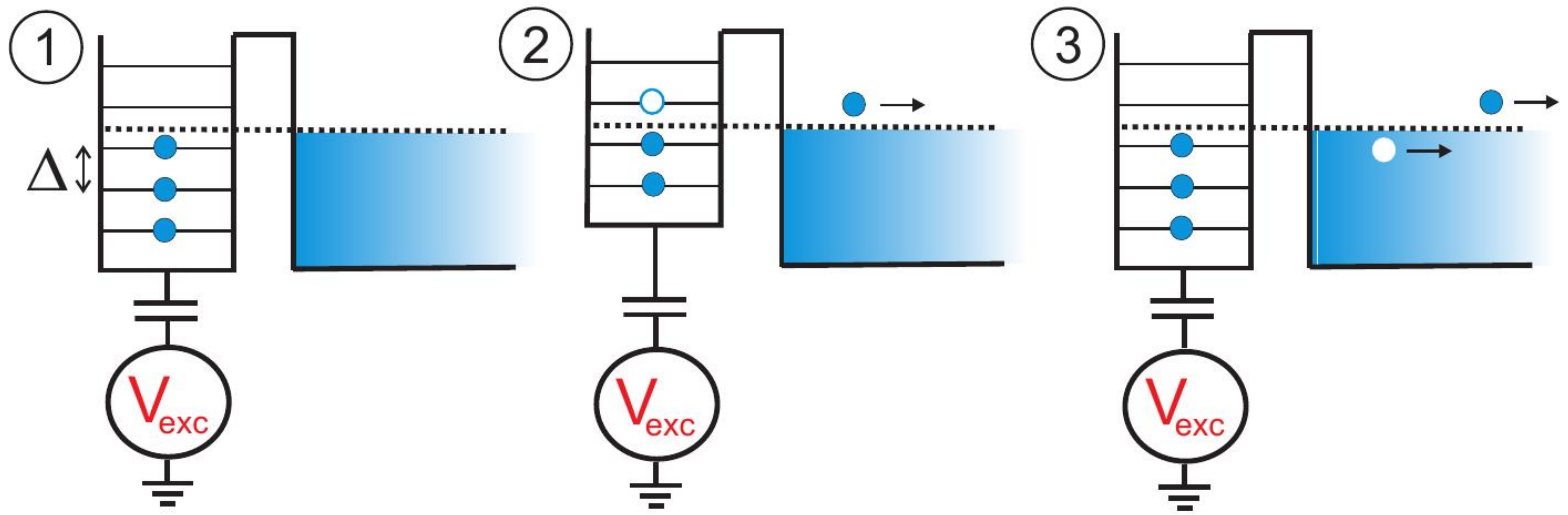
Power exchange between 2 forces $$P^{\text{pump})}_{1,2}(t) = \frac{1}{2}\Big(P_1(t) - P_2(t)\Big) = \dot{X}_1 \Lambda^{A}_{1,2}(\overrightarrow{X})\dot{X}_2$$

Dissipation in electron systems

Quantum capacitor

Büttiker, M.; Thomas, H.; Prêtre, A. *Phys. Lett. A* **1993**, *180*, 364–369. *Phys. Rev. B* **1996**, *54*, 8130–8143.
Gabelli, J.; Fève, G.; Berroir, J.M.; Plaçais, B.; Cavanna, A.; Etienne, B.; Jin, Y.; Glattli, D.C. *Science* **2006**, *313*, 499–502.
Gabelli, J.; Fève, G.; Berroir, J.M.; Plaçais, B. *Rep. Prog. Phys.* **2012**, *75*, 126504.





Linear circuit

$$\mathcal{I}(\omega) = \mathcal{A}(\omega) V_{\mathrm{g}}(\omega) + \mathcal{O}(V_{\mathrm{g}}^2)\,.$$

$$\mathcal{A}(\omega) = -i\omega C(1 + i\omega R_{\mathrm{q}} C) + \mathcal{O}(\omega^3)$$

$$\frac{1}{C} = \frac{1}{C_q} + \frac{1}{C_g}$$

Quantum

$$R_q = \frac{h}{2e^2}$$

Review

Phase-Coherent Dynamics of Quantum Devices with Local Interactions

Entropy **2020**, *22*, 847;

Michele Filippone [1,*], Arthur Marguerite [2], Karyn Le Hur [3], Gwendal Fève [4] and Christophe Mora [5]

Filippone, M.; Mora, C. *Phys. Rev. B* **2012**, *86*, 125311
Lee,M.;López,R.;Choi,M.S.;Jonckheere,T.;Martin,T. *Phys. Rev. B* **2011**, *83*, 20130

Hamiltonian for the capacitor:

$$\mathcal{H}_{\mathrm{c}} = -eV_{\mathrm{g}}(t)N + E_c N^2 .$$

$$V_{\mathrm{g}}(t) = V_{\mathrm{g}} + \varepsilon_\omega \cos(\omega\, t) .$$

Kubo linear response:

$$Q \;=\; e\,\langle N\rangle .$$

$$\mathcal{A}(\omega) = -i\omega \frac{Q(\omega)}{V_{\mathrm{g}}(\omega)} .$$

$$\chi_{\mathrm{c}}(t-t') = \frac{i}{\hbar}\theta(t-t')\,\langle[N(t),N(t')]\rangle_0 .$$

$$\mathcal{A}(\omega) = -i\omega e^2 \chi_{\mathrm{c}}(\omega) = -i\omega e^2\,\{\chi_{\mathrm{c}} + i\mathrm{Im}\,[\chi_{\mathrm{c}}(\omega)]\}$$

Quantum resistance

Korringa-Shiba relation

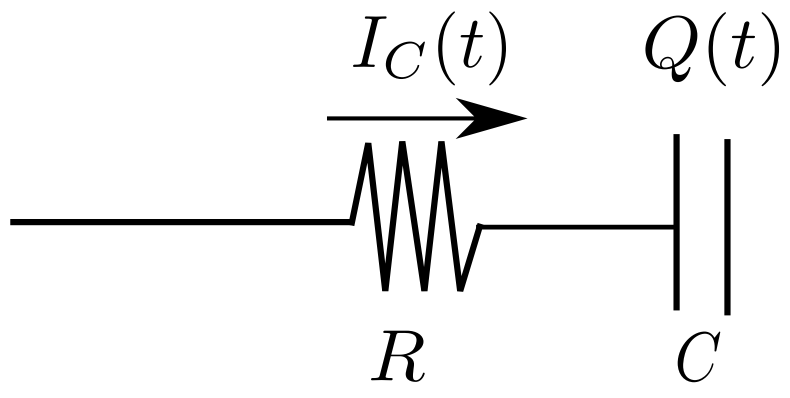
(Valid for Fermi liquids):

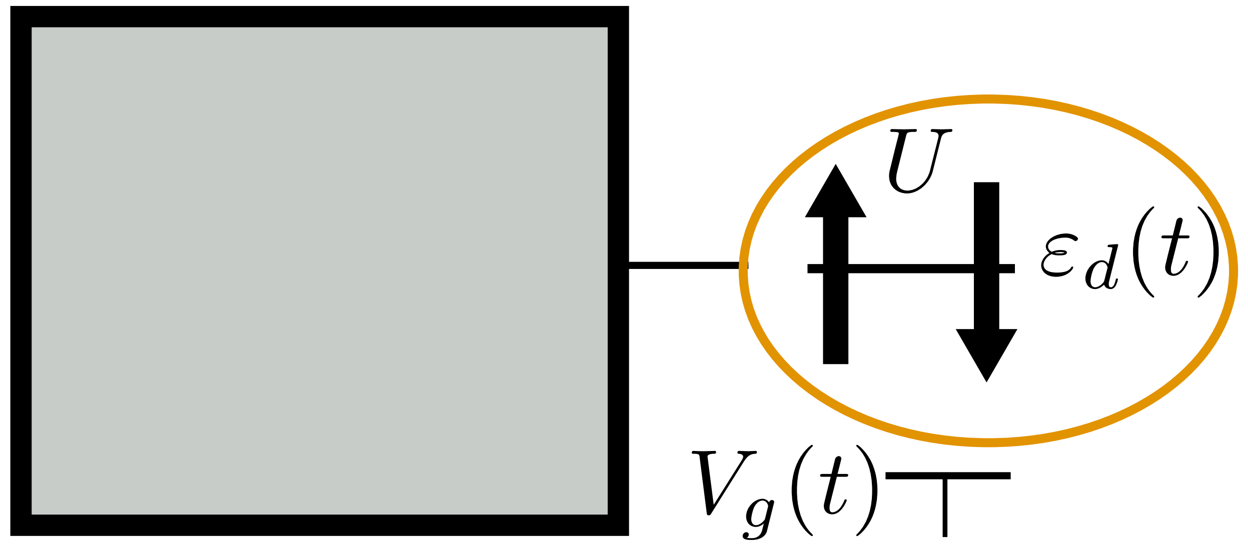
$$\mathrm{Im}\chi_{\mathrm{c}}(\omega)|_{\omega\to 0} = \hbar\pi\omega\chi_{\mathrm{c}}^2 .$$

$$R_q = \frac{h}{2e^2}$$

Slow driving:

J. Romero, P. Roura-Bas, A. A. Aligia Phys. Rev. B 2017, 95, 2017





Charge dynamics: $$e\dot{n}_d(t) = e\sum_\sigma \dot{n}_{d\sigma}(t) = \sum_\sigma I_{C,\sigma}(t),$$

Power: $$P(t) = e\sum_\sigma n_{d\sigma}(t)\dot{V}_g(t)$$

Adiabatic linear response: $$n_{d\sigma}(t) = n_{f\sigma}(t) + e\Lambda_\sigma(t)\dot{V}_g(t).$$

$$P_{\rm cons}(t) = e\sum_\sigma n_{f\sigma}(t)\dot{V}_g(t),$$

$$P_{\rm diss}(t) = e^2\sum_\sigma \Lambda_\sigma(t)[\dot{V}_g(t)]^2.$$

R_q

$$P_{\rm diss}(t) = \frac{h}{2e^2}\sum_\sigma [I_{C,\sigma}(t)]^2.$$

Korringa-Shiba law
Verified numerically

$$\lim_{\omega\to 0}\frac{{\rm Im}[\chi_t^c(\omega)]}{\hbar\omega} = -\frac{h}{2}\sum_\sigma [\chi_t^{\sigma\sigma}(0)]^2.$$

Joule law!

Outlook

- Dissipation in finite-time process at slow driving is a bilinear function of the velocities characterizing the change of the control parameters.
- Linear-response coefficients obey Onsager relations and define a tensor.
- The symmetric component of the tensor describes the dissipation and a metric in the parameter space.
- The antisymmetric part describes power pumping.

Example

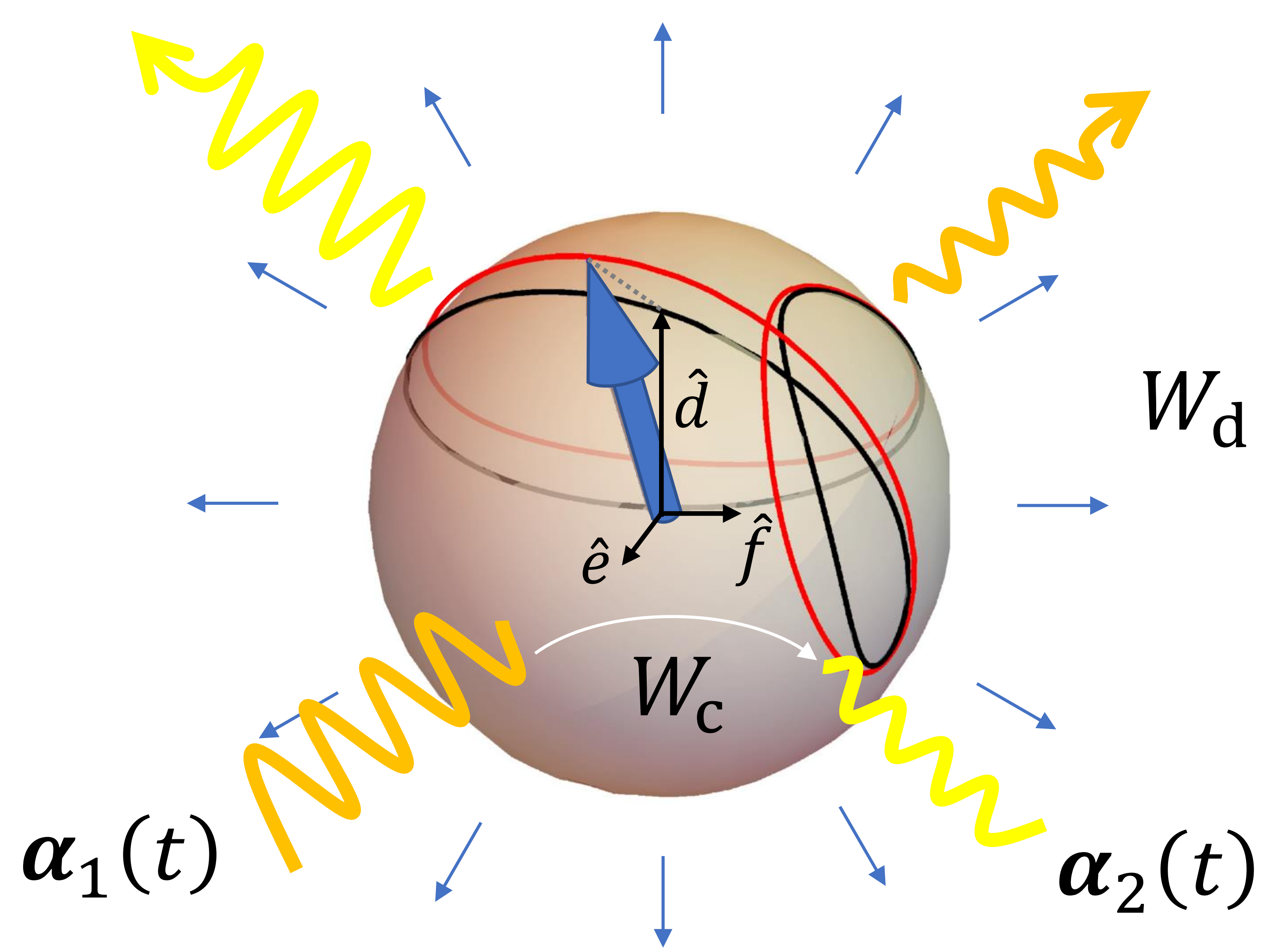
Quantum geometry and bounds on dissipation in slowly driven quantum systems

Iliya Esin,[1] Étienne Lantagne-Hurtubise,[1] Frederik Nathan,[1,2] and Gil Refael[1]

arXiv: 2306.17220

$$H(t) = h_0(t) + \mathbf{d}(t) \cdot \boldsymbol{\sigma}$$

Two-frequency driving: $\phi_1(t) = \omega_1 t$, $\phi_2(t) = \omega_2 t$



Net work conversion:

$$\overline{W}_\mathrm{c} = \frac{\omega_1\omega_2}{T}\int_0^T dt \left[\frac{\tau_2^2\Delta^2}{1+\tau_2^2\Delta^2}\right]\Omega_{12},$$

Berry curvature: $\Omega_{12} = \frac{1}{2}\mathbf{d}\cdot\left(\partial_{\phi_1}\hat{\mathbf{d}} \times \partial_{\phi_2}\hat{\mathbf{d}}\right)$