DQMP Department of Quantum Matter Physics

Tomonaga-Luttinger liquids (TLL)

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MaNEP SWITZERLAND

Plan of the lectures

- Equilibrium, basic notions of Tomonaga-Luttinger liquid
- More on TLL (topology, boundaries, etc.)
  Effect of perturbations (Lattice and disorder)
- Beyond TLLs:
  From 1D to 3D; Ladders; Dimensional crossover

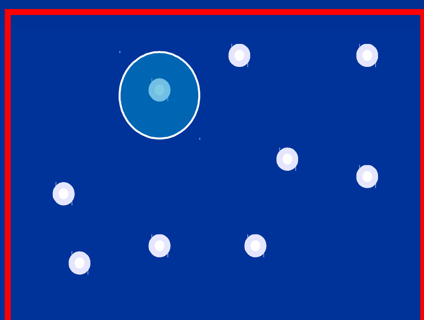
Why one dimension ?

Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

One dimension is specially interesting

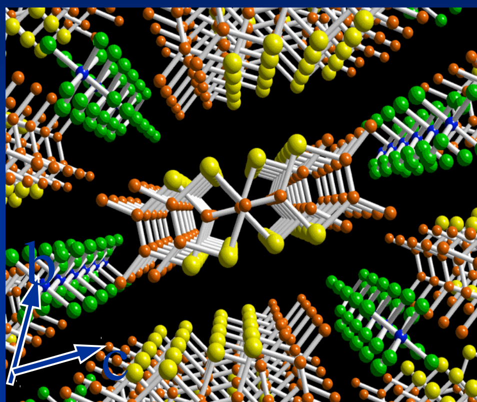
- No individual excitation can exist (only collective ones)

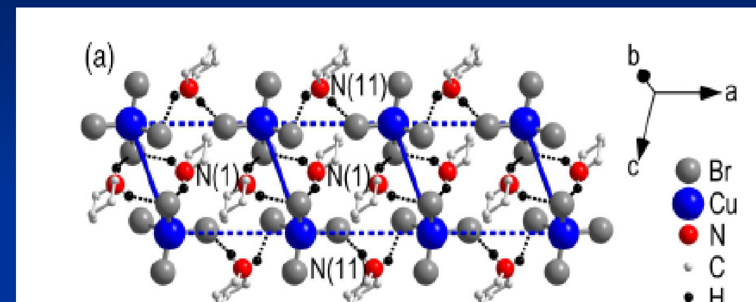


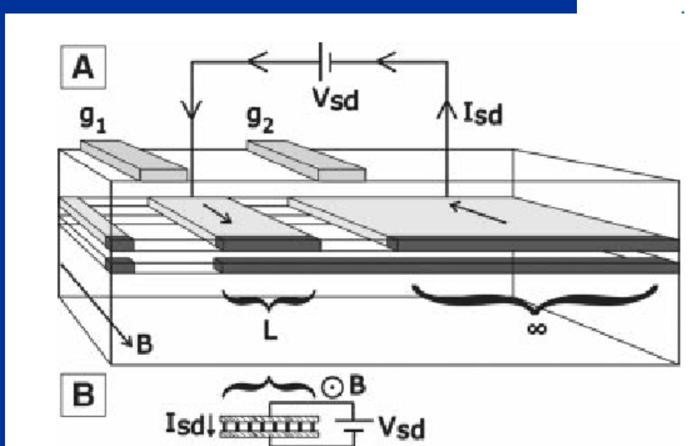
- Strong quantum fluctuations

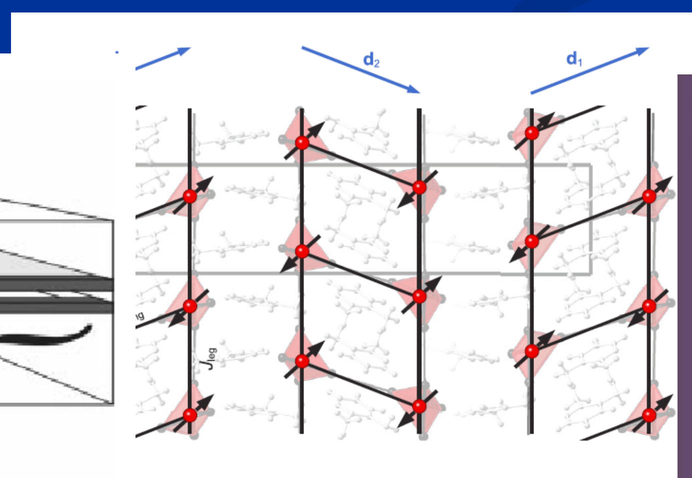
Difficult to order

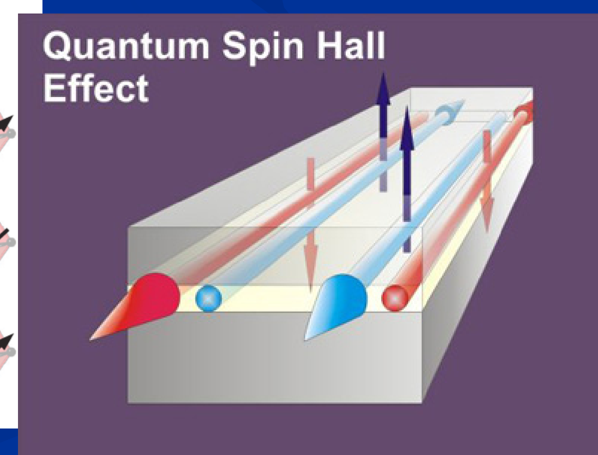
Many CM or cold atoms Systems









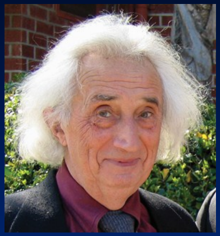


Drastic evolution of the 1d world

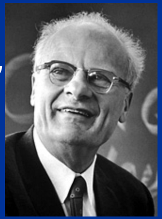
- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

How to treat ?

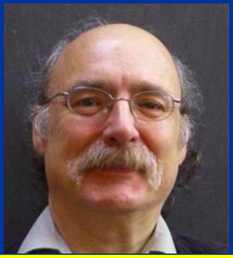
- ``Standard'' many body theory

- Exact Solutions (Bethe ansat

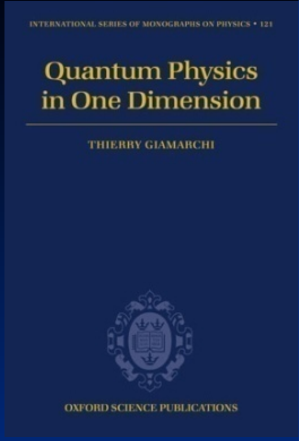
- Field theories (bosonization, CFT)

- Numerics (DMRG, MC,

References



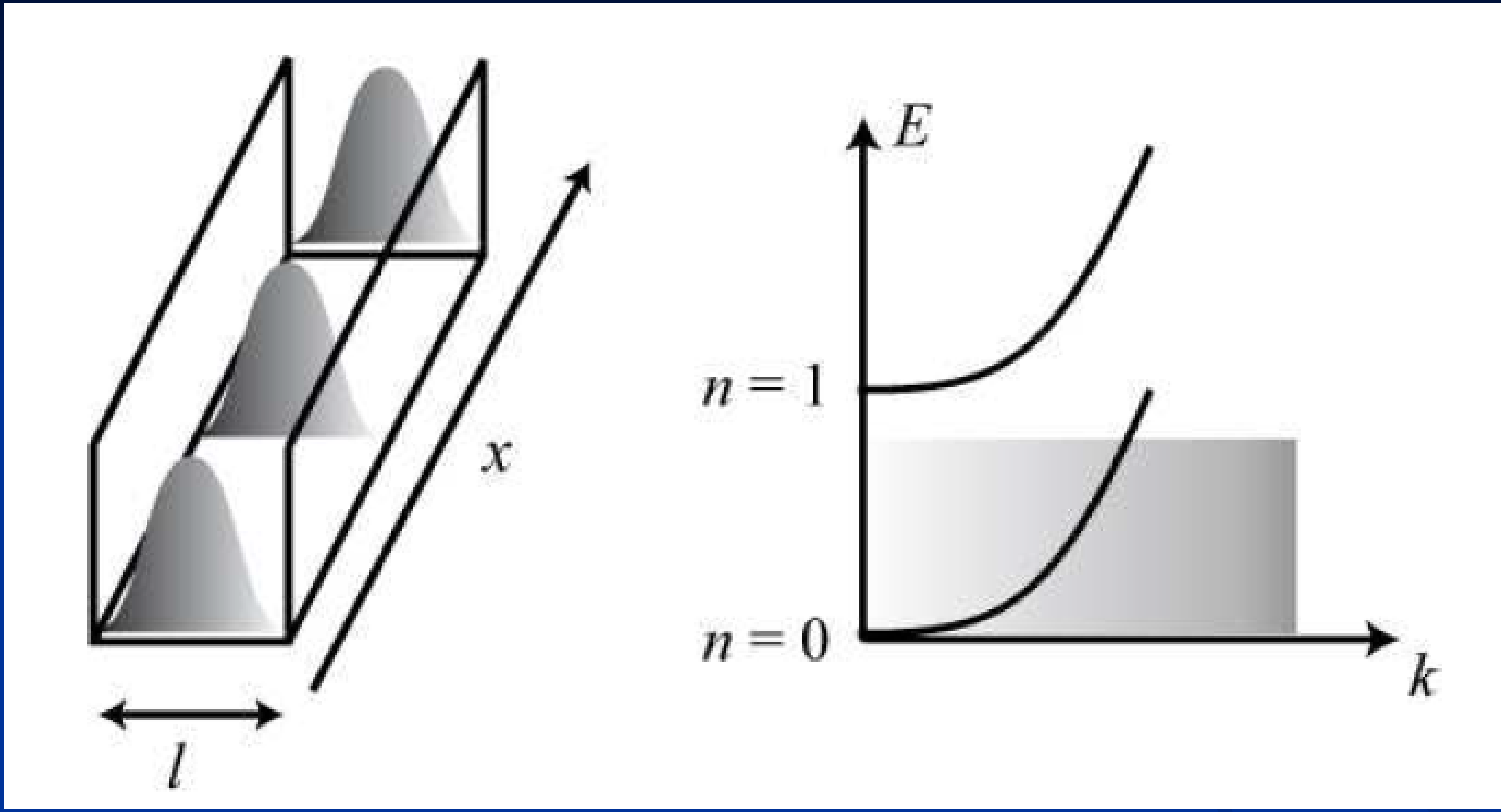
TG, Quantum physics in one dimension, Oxford (2004)

TG in ``Understanding Q. Phase Transitions'', Ed. L. Carr, CRC Press (2010)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

And now we start….



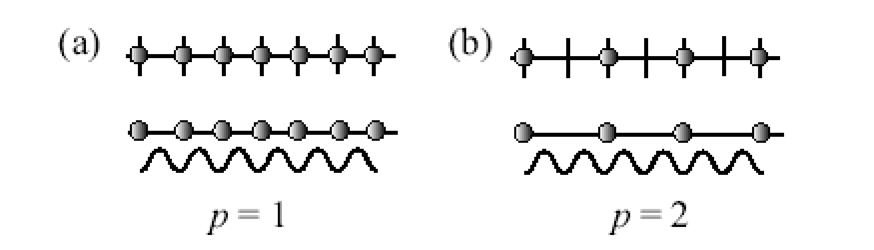
x
l
E
n = 1
n = 0
k

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Lattice:



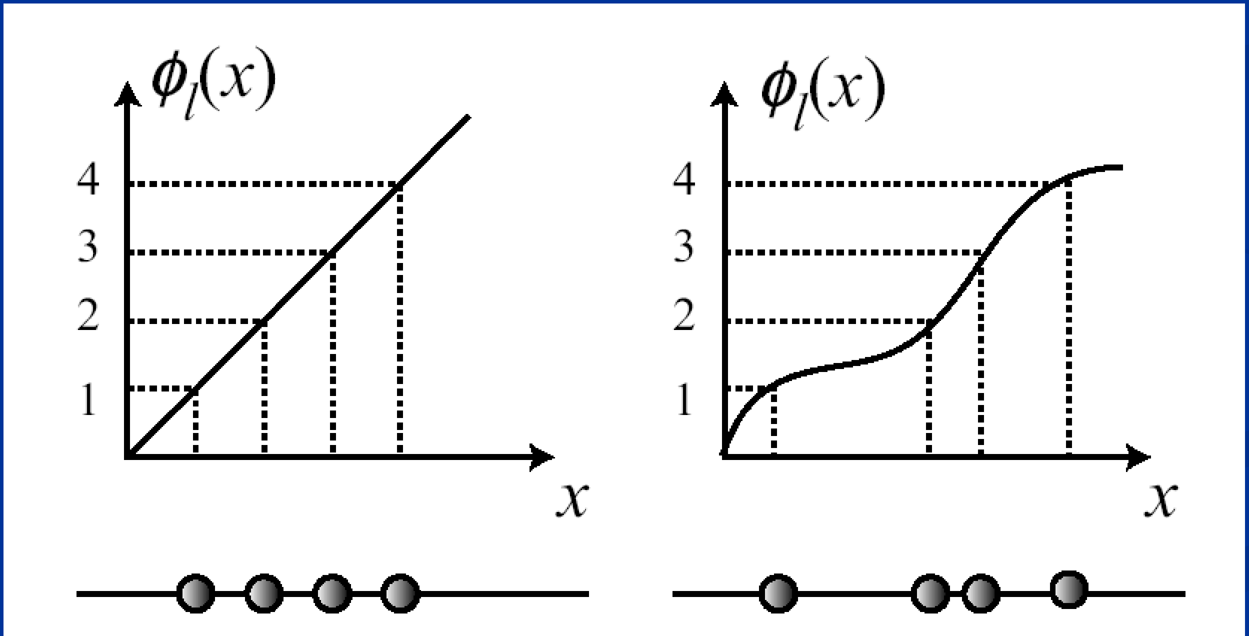
$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

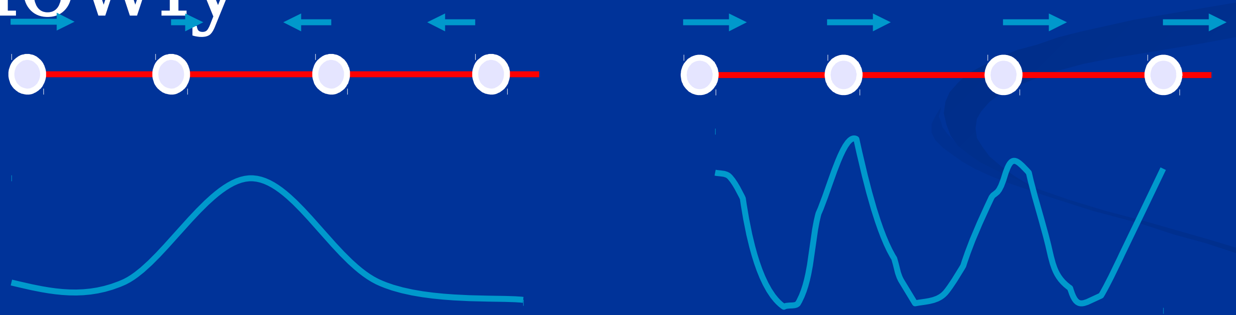
1D: unique way of labelling



$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

φ(x) varies slowly



CDW

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

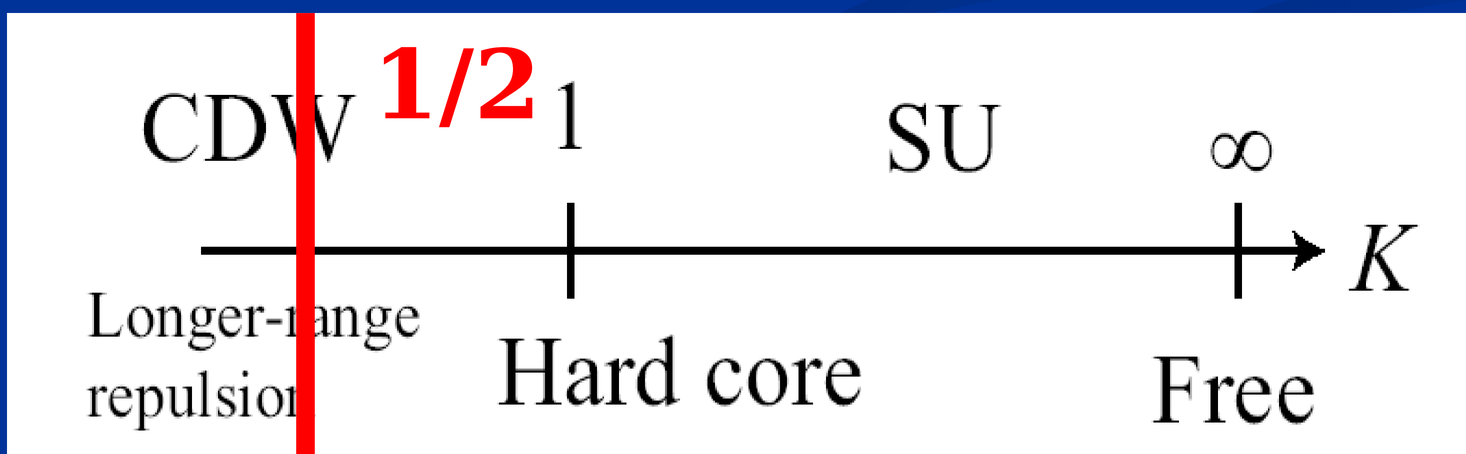
Quantum fluctuations

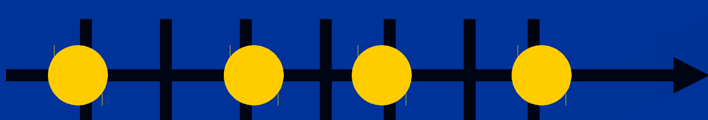
$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

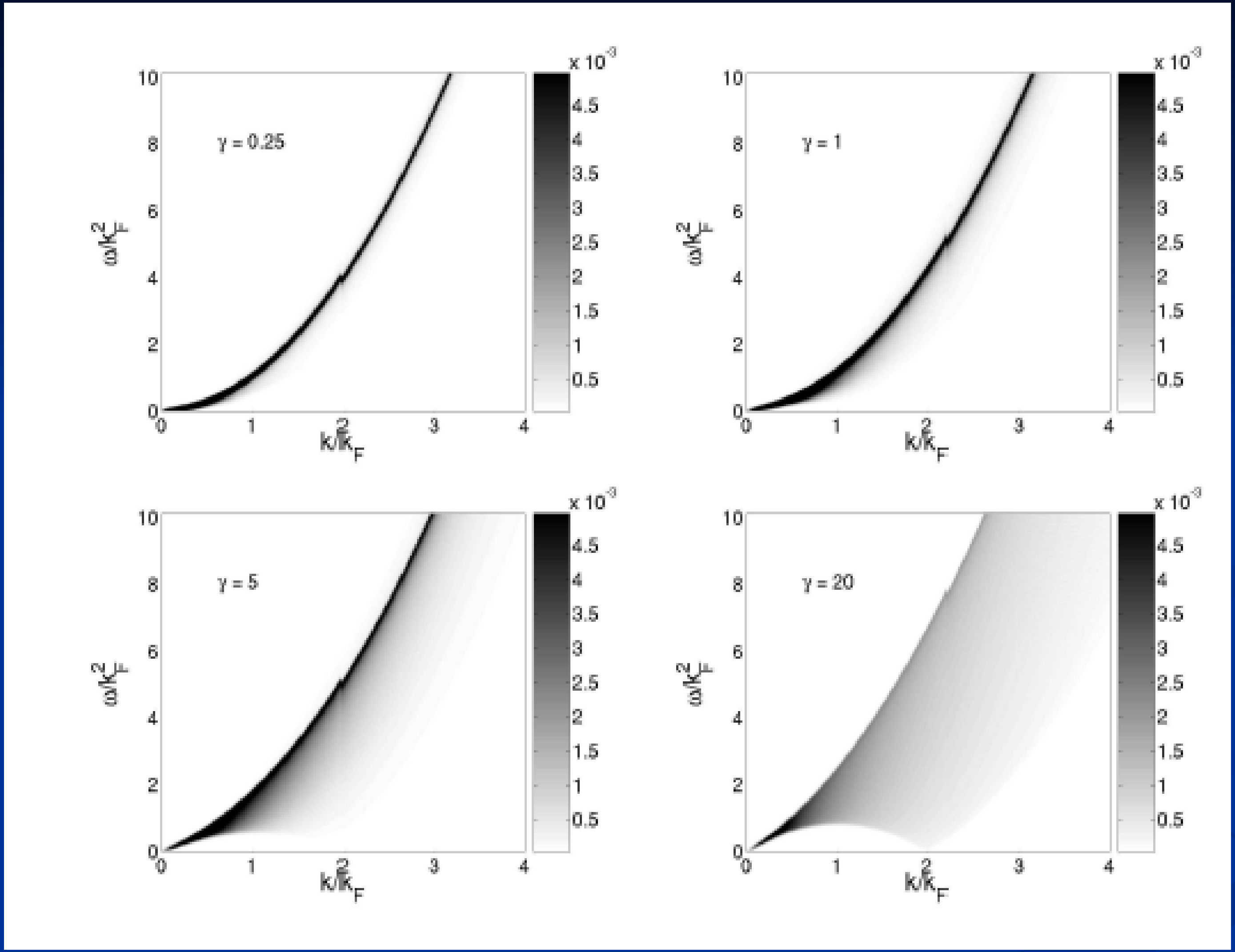
Correlations

$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$



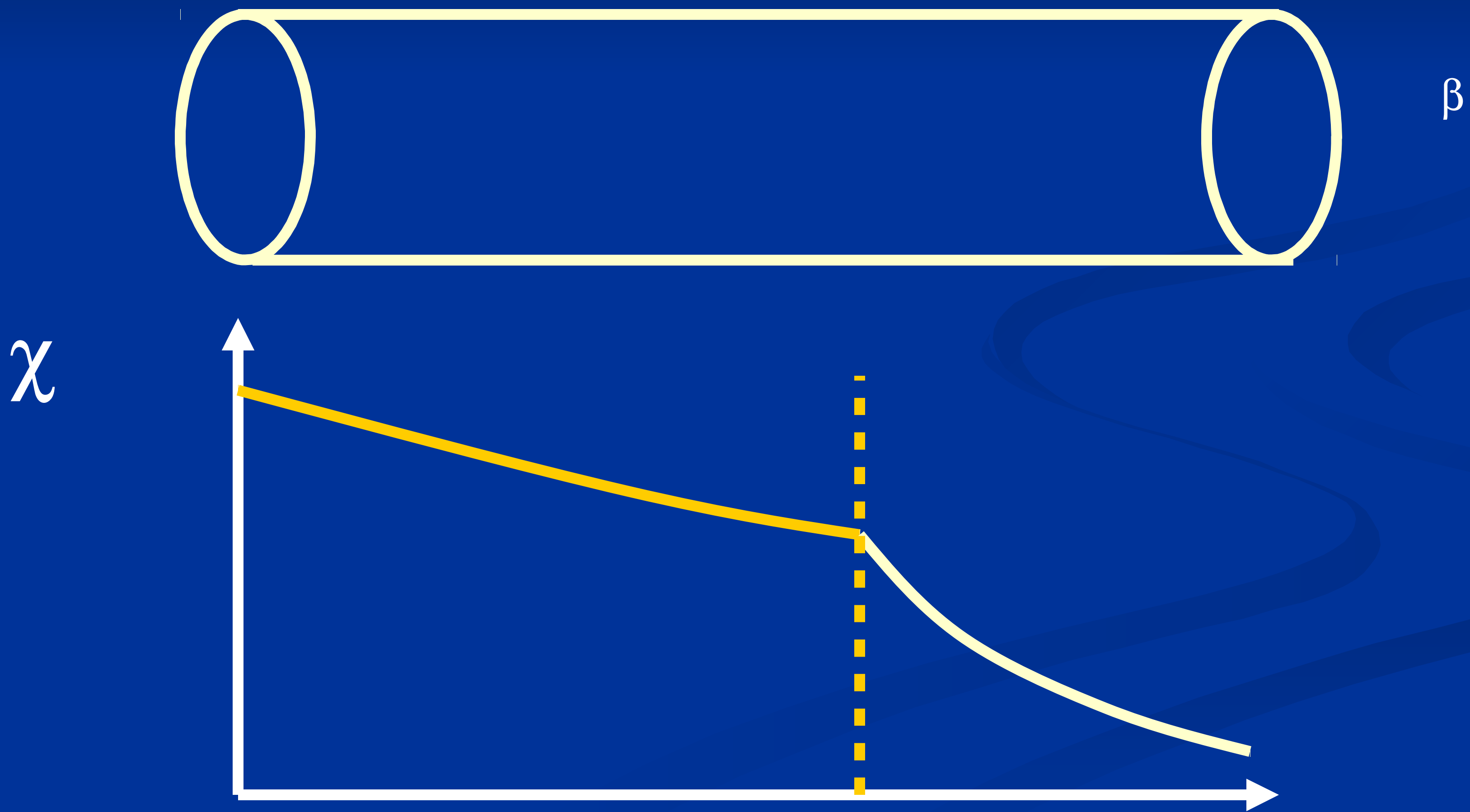




S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory



Other 1D systems

Spins

Use boson or fermions mapping

Powerlaw correlation functions

$$\langle S^z(x,0)S^z(0,0)\rangle = C_1\frac{1}{x^2} + C_2(-1)^x\left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x,0)S^-(0,0)\rangle = C_3\left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

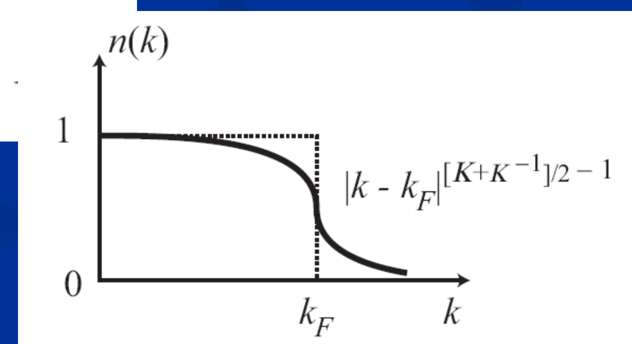
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

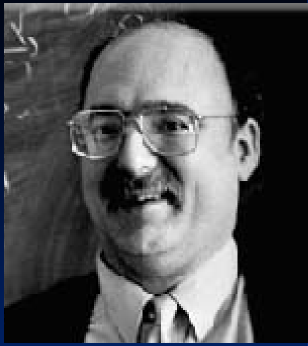
$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$)

$$\langle \rho(x,\tau)\rho(0,0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(x^2 + y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{2K} + \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{8K}$$



TLL concept



- How much is perturbative ?

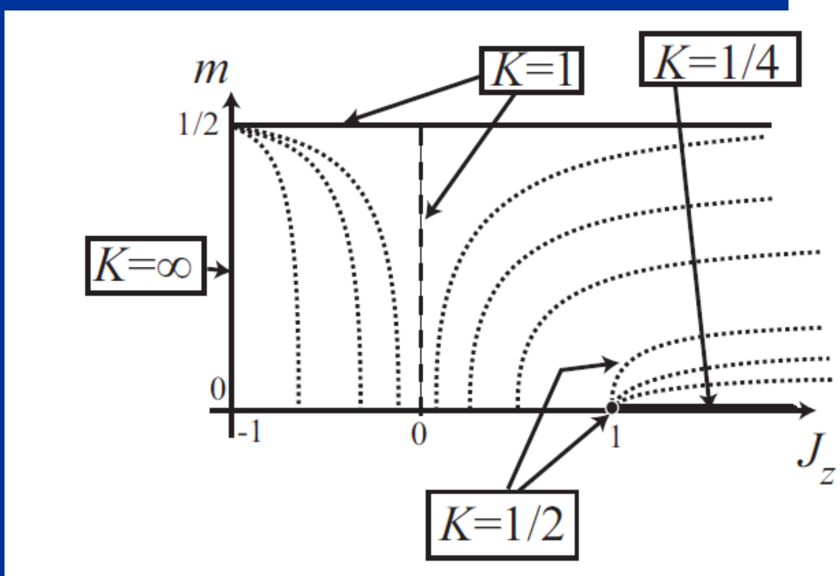
- Nothing (Haldane):
  provided the correct u,K are used
- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

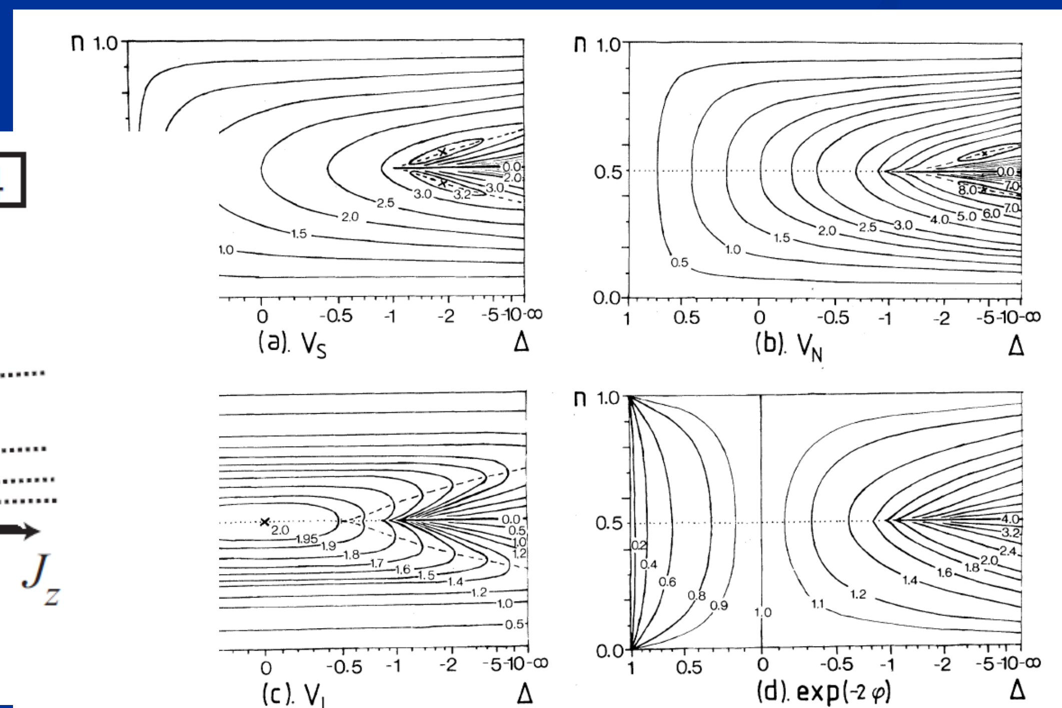
Luttinger parameters

General Relation of Correlation Exponents and Spectral Properties of One-Dimensional Fermi Systems: Application to the Anisotropic $S = \frac{1}{2}$ Heisenberg Chain

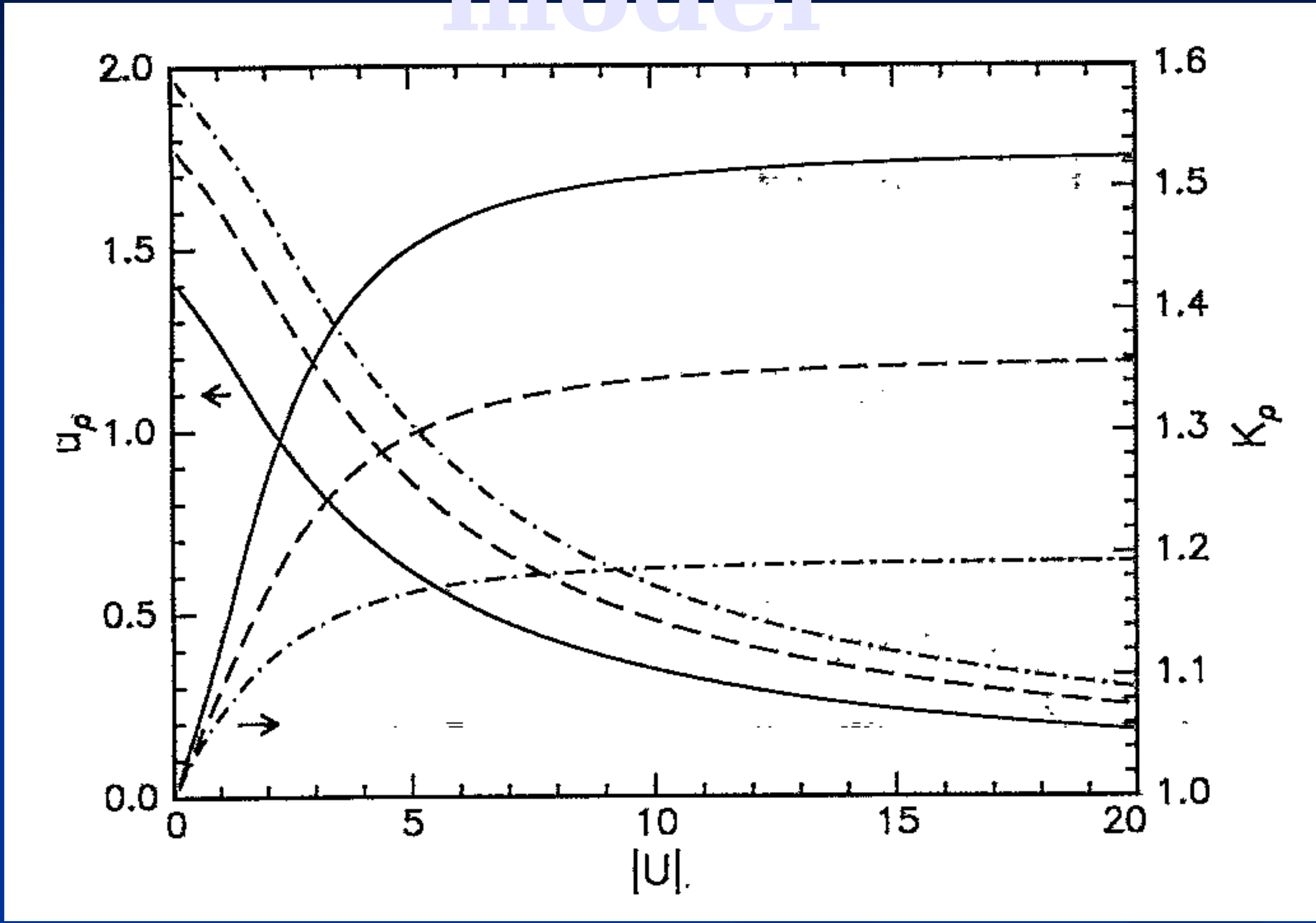
F. D. M. Haldane

Haldane, F. D. M. (1980). *Phys. Rev. Lett.*, **45**, 1358.





Attractive Hubbard model

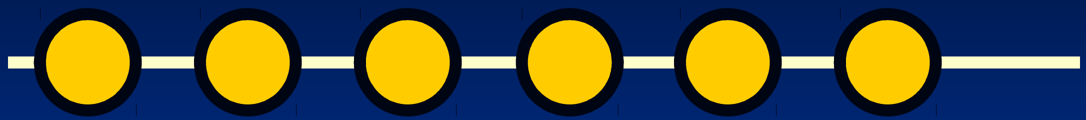


TG + B. S. Shastry PRB 51 10915 (1995)

``Quantitative'' theory possible

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

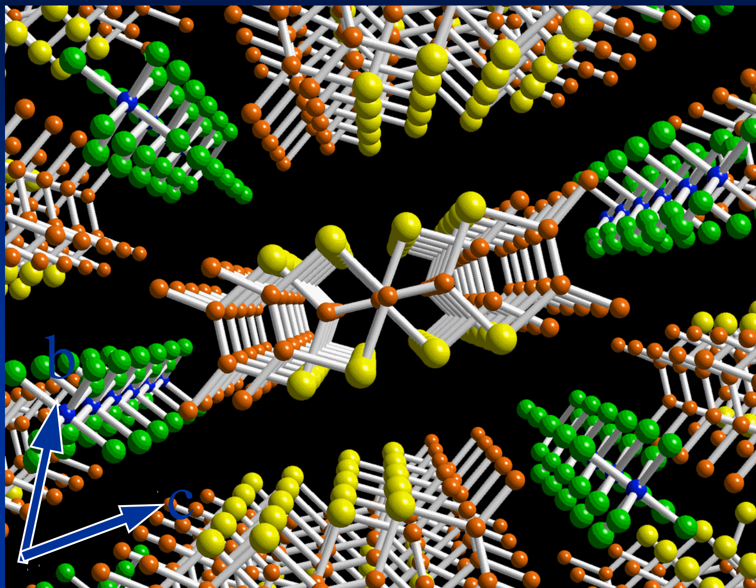
Free fermions:

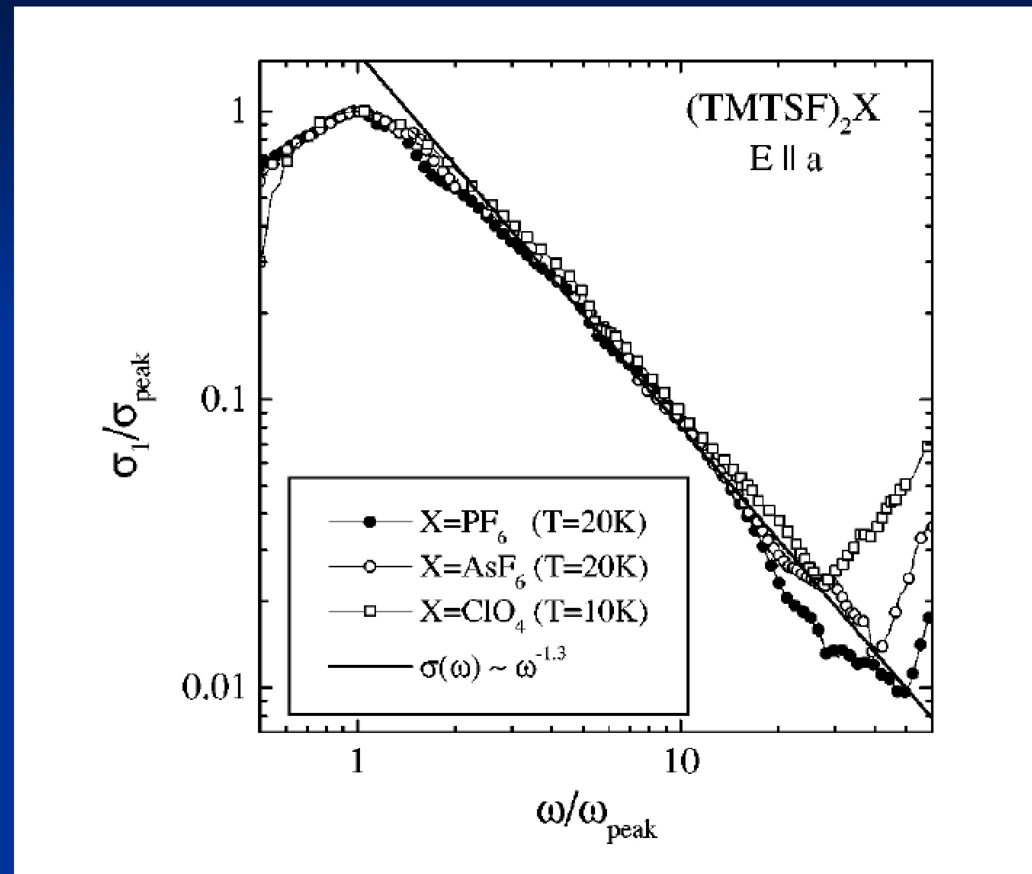
K=1

Note:

Tests of Luttinger liquids

Organic conductors





$$\sigma(\omega) \sim \omega^{-\nu}$$

TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!

Cold gases

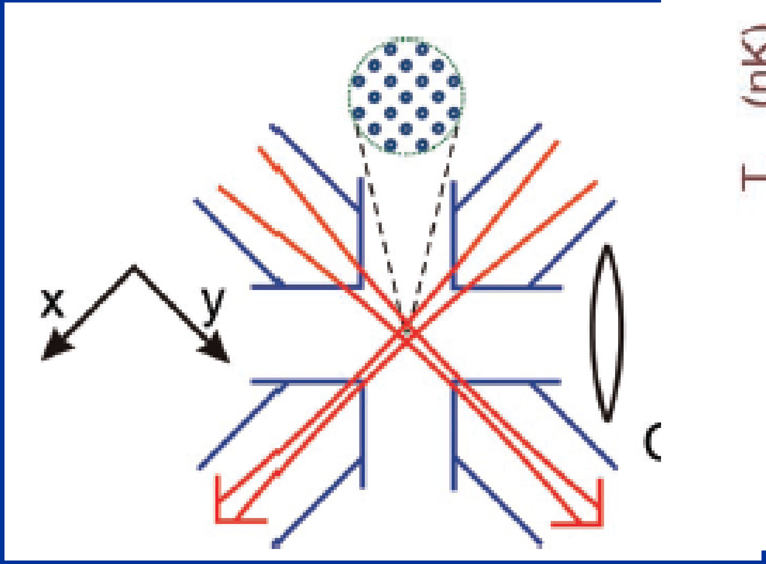
Bosons (continuum)

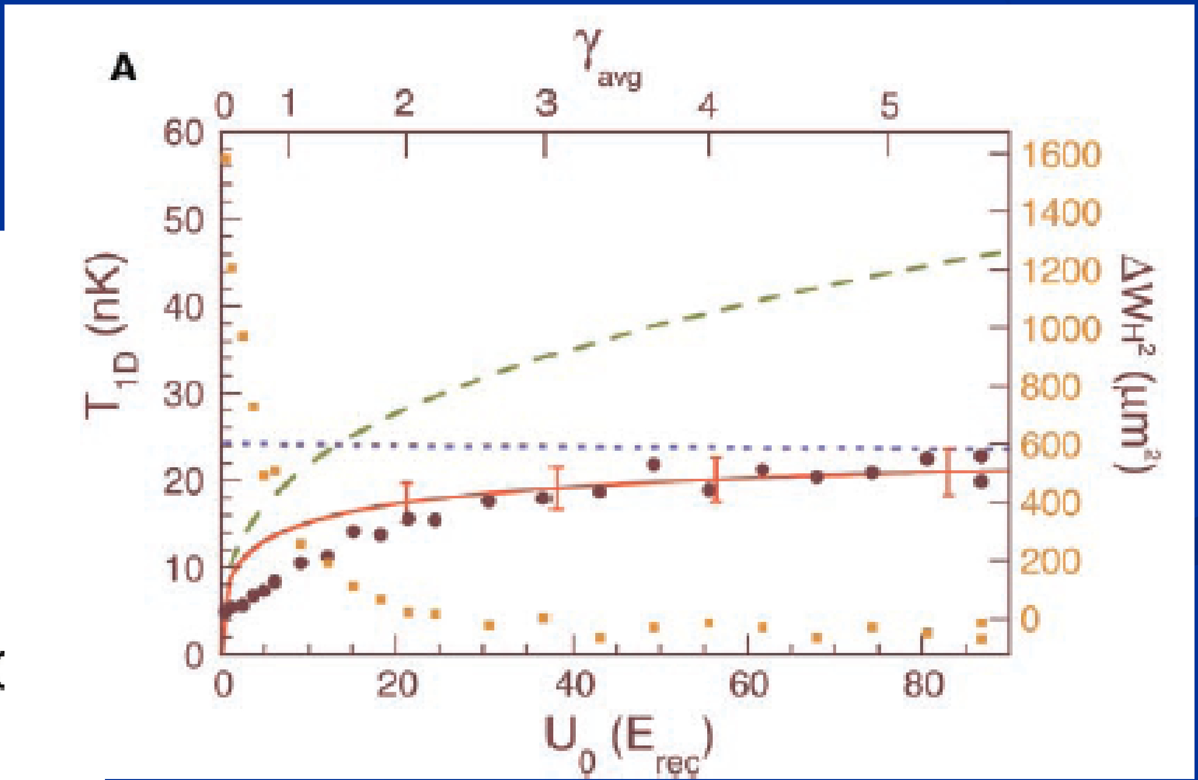
Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004

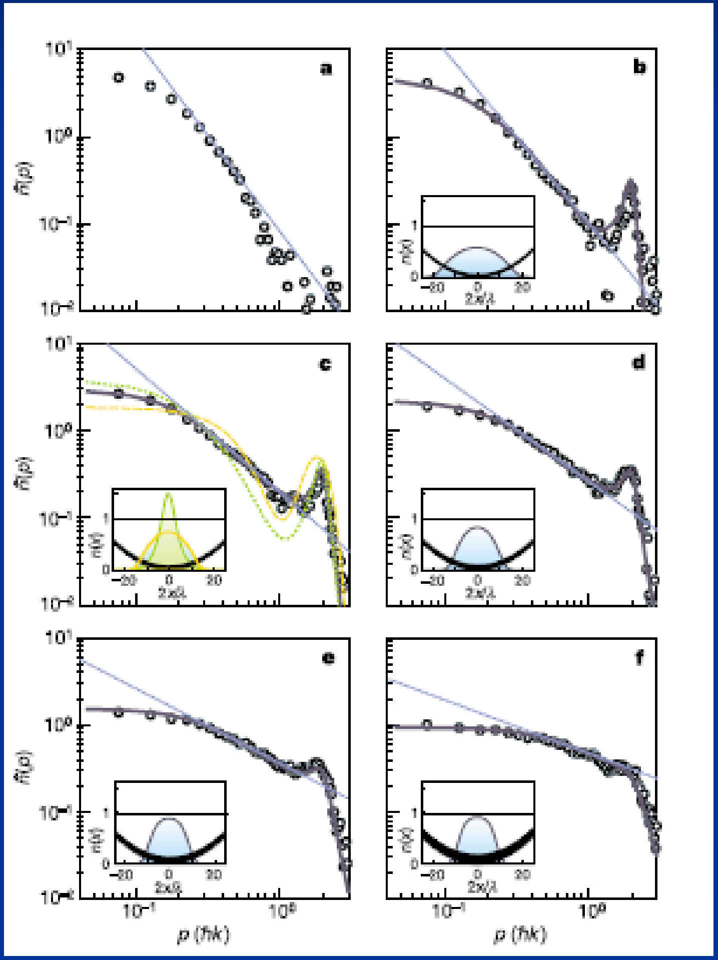
1125

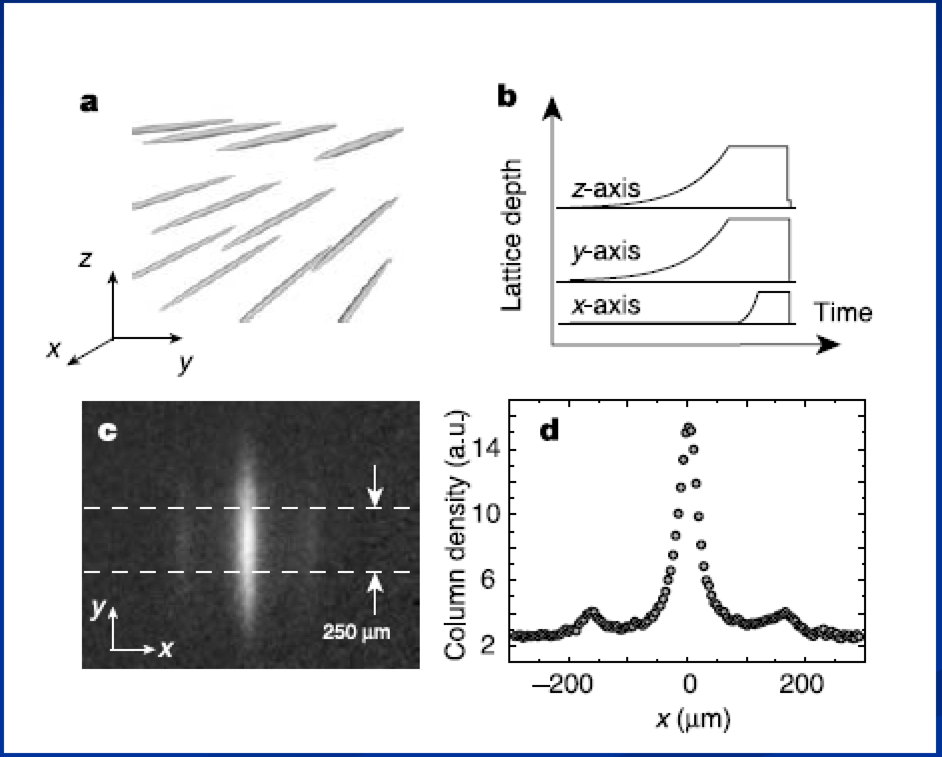




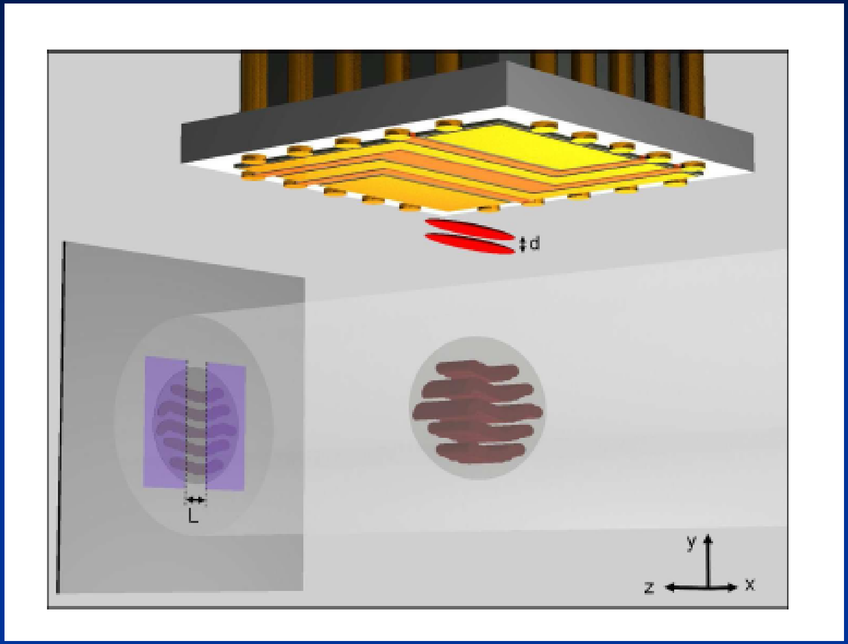
Optical lattices (dilute

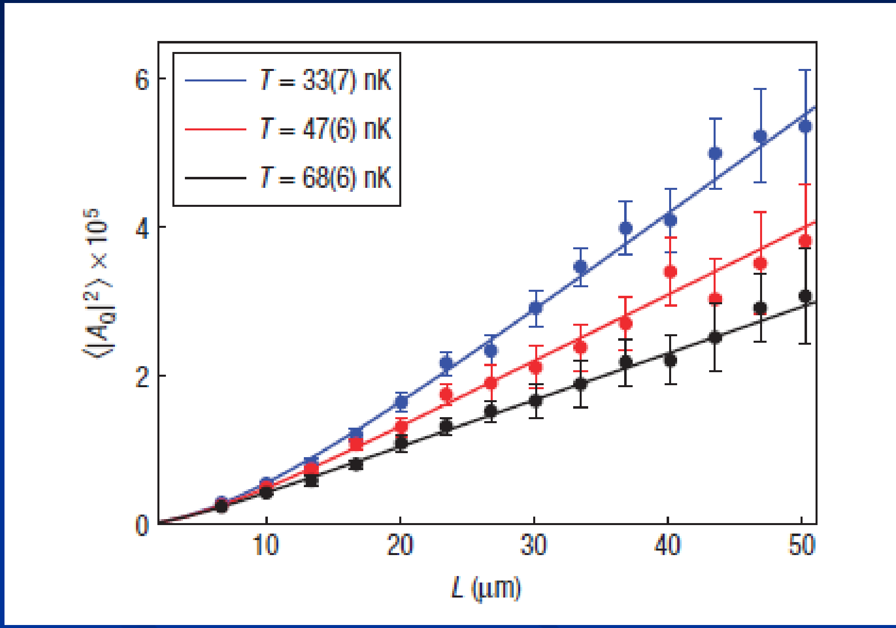
B. Paredes et al., Nature 429 277 (2004)



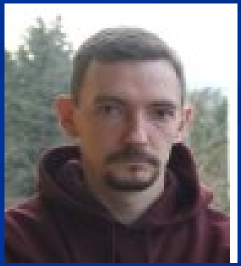


Atom chips





S. Hofferberth et al. Nat. Phys 4 489 (2008)



K large (42)

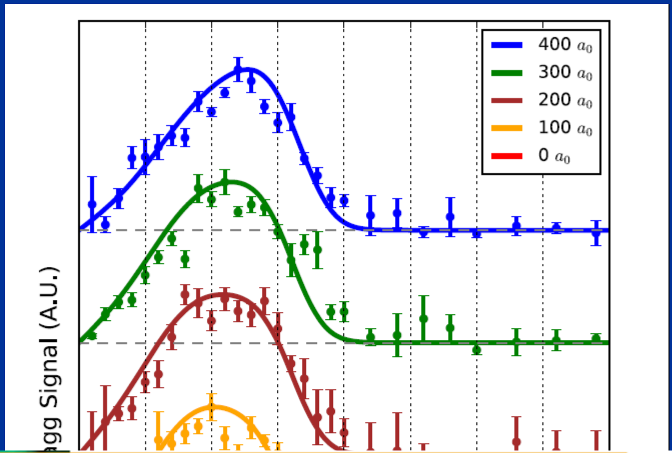
Charge velocity

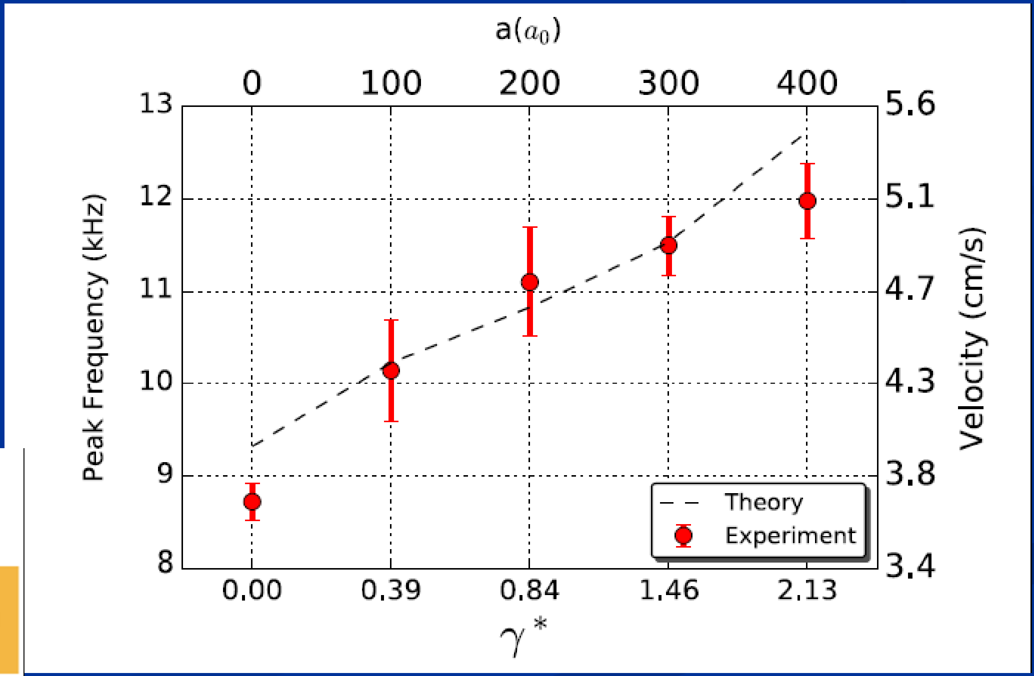
PHYSICAL REVIEW LETTERS **121,** 103001 (2018)

Editors' Suggestion

Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,[1] P. Grišins,[2] Y. T. Chang,[1] Z. H. Zhao,[1] C. Y. Shih,[1] T. Giamarchi,[2] and R. G. Hulet[1]



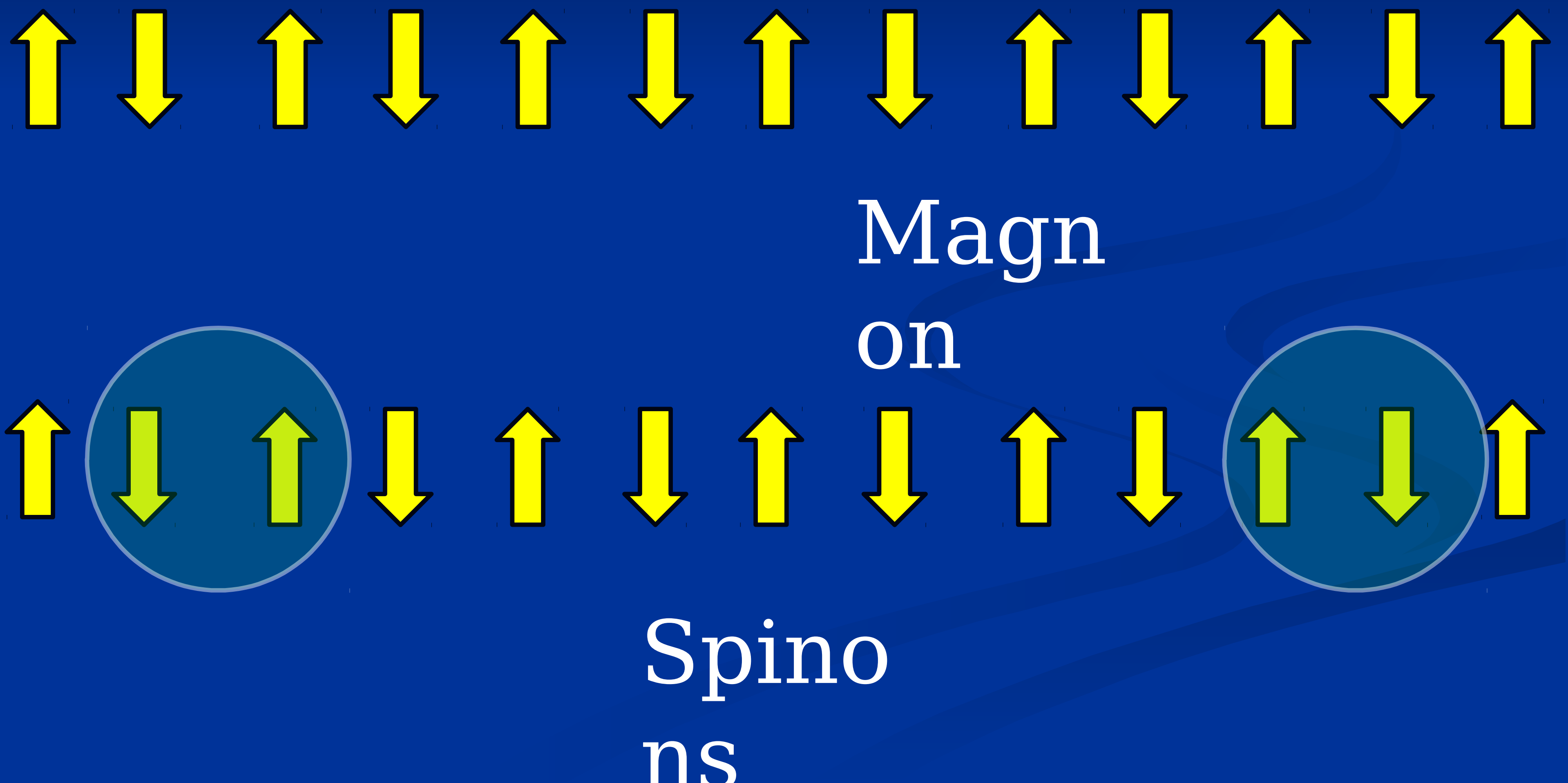


ULTRACOLD ATOMS CONFIRM 55-YEAR-OLD PHYSICS THEORY

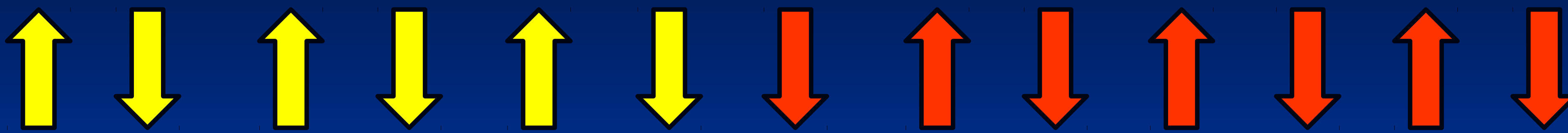
https://www.futurity.org/one-dimensional-electrons-physics-1858622/

Other important 1D properties
Fractionalization of excitations

Fractionalization of excitations



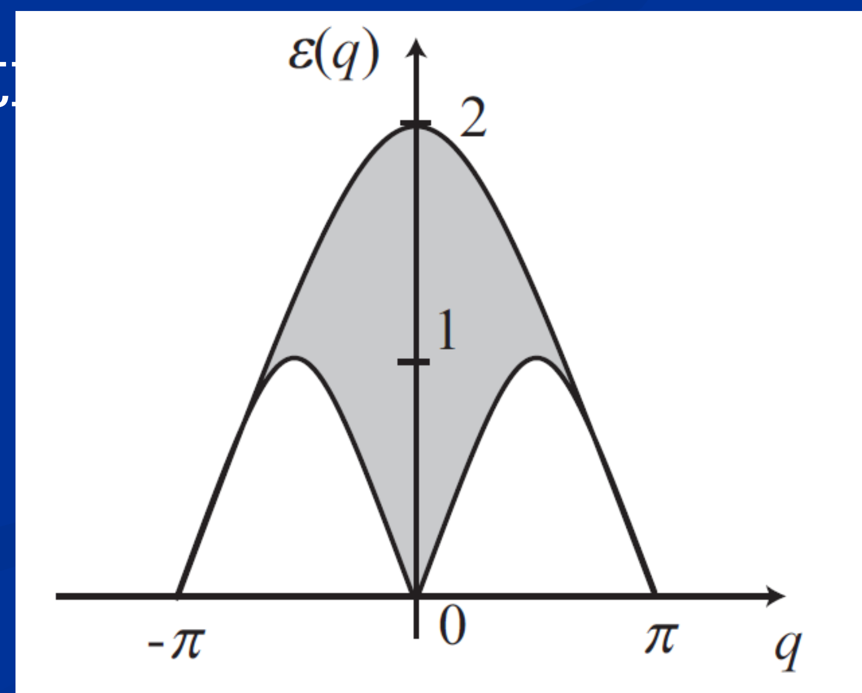
Magnons and spinons: 1 = ½ + ½



- Hidden (topological) order parameters
- Continuum of excitat

$E(k) = \cos(k_1) + \cos(k_2)$

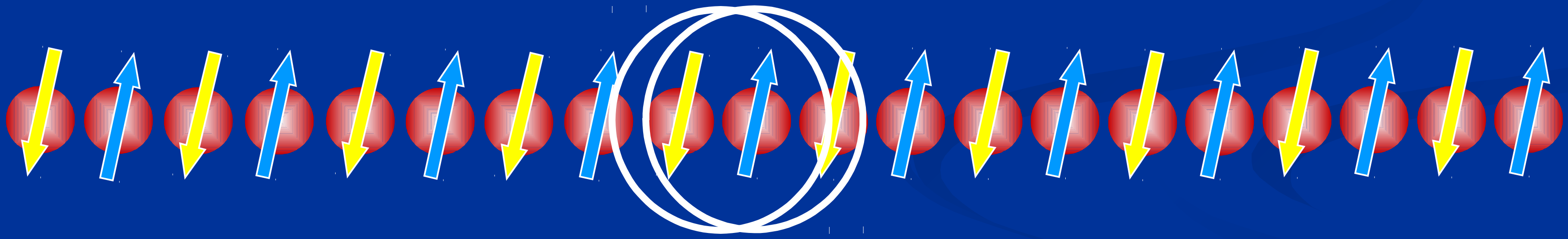
$k = k_1 + k_2$



Deconstruction of the electron: spin-charge separation

Spin

Charge



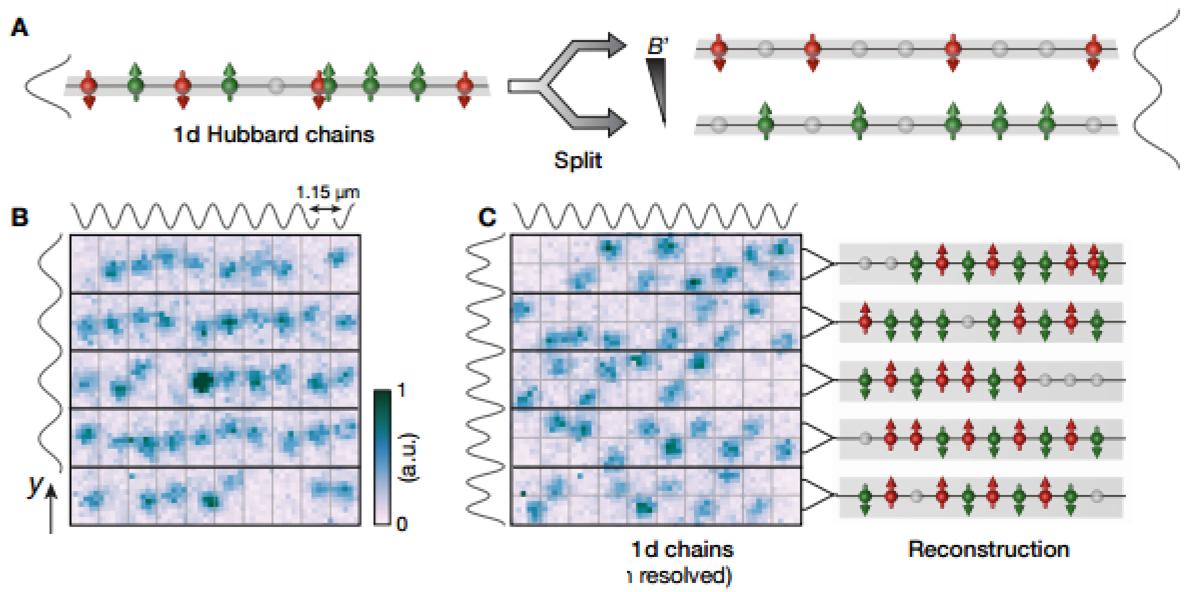
Spinon

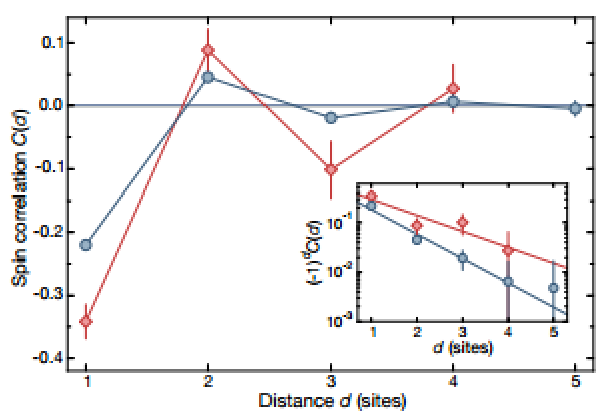
Holon

Spin and Charge Resolved Quantum Gas Microscopy of Antiferromagnetic Order in Hubbard Chains

Martin Boll[1*], Timon A. Hilker[1*], Guillaume Salomon[1*], Ahmed Omran[1], Immanuel Bloch[1,2], and Christian Gross[1†]

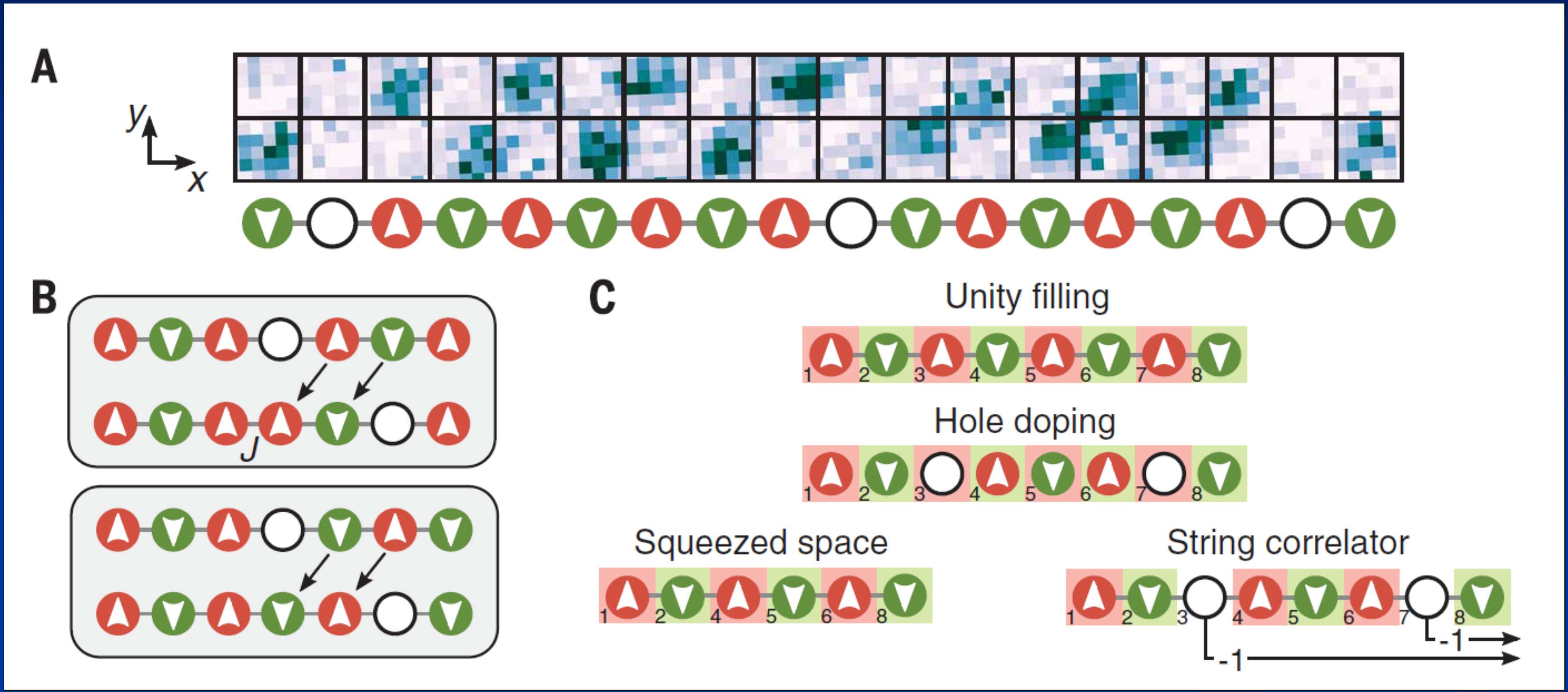
arXiv:1605.05661v2





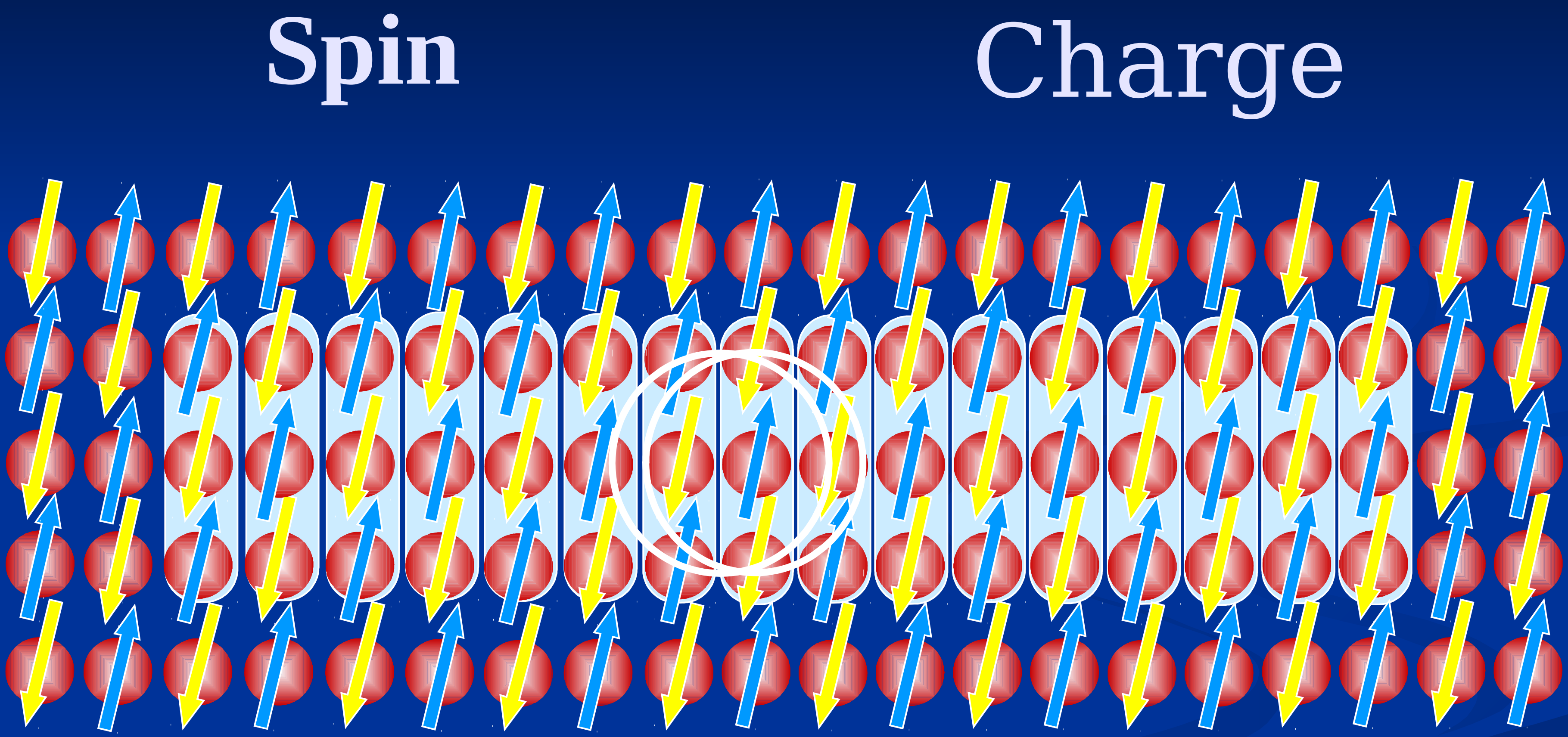
The inset shows the decay of the rectified spin correlations $(-1)^d C(d)$ in a logarithmic plot together with an exponential fit $C(d) \propto \exp(-d/\xi)$, which reveals an average correlation length of $\xi = 0.9(1)$ sites and $\xi = 1.4(4)$ sites for the lowest entropy tube. All error bars represent one s.e.m.

Doped Hubbard model



Hilker *et al.*, *Science* **357**, 484–487 (2017)

Spin-Charge Separation higher D ?



Energy increases with spin-charge separation

Confinement of spin-charge:

Quantitative solutions for TLL

Magnetic insulators

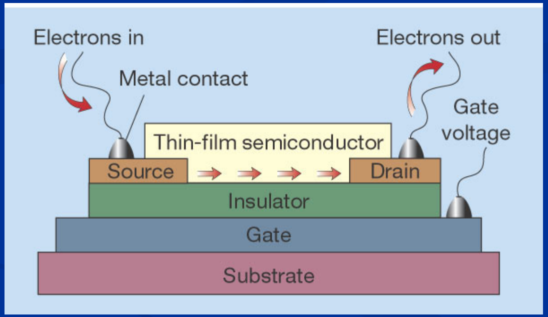
- Interesting problem in itself (spin liquid,…)
- Many materials; dimensions, interactions,….
- Microscopic interactions short range and thus well controled
- Can be used as quantum simulators

S. Ward et al, J. of Physics 25, 014004 (2013)

Hard core bosons on a lattice

$$H = -\frac{J_{xy}}{2}\sum_{ij}[b_i^\dagger b_j + \mathrm{hc}] + J_z \sum_{ij}(n_z - \frac{1}{2})(n_j - \frac{1}{2})$$

- Magnetic field : chemical potential (gate voltage) for the bosons
- In 3D !
- Go from 0 bosons/site to 1 boson/site



Nature **428**, 269 (2004)

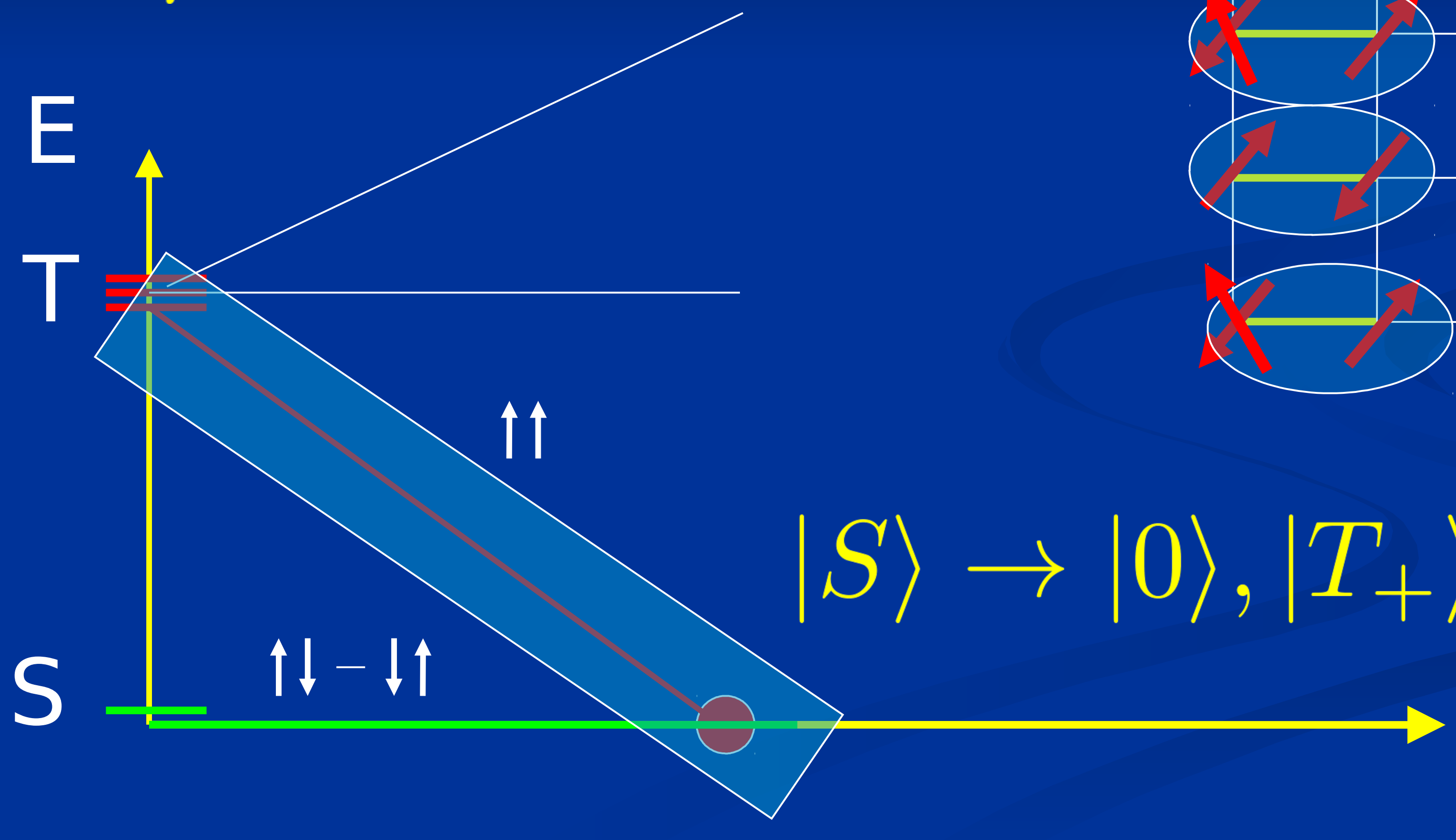
Probes

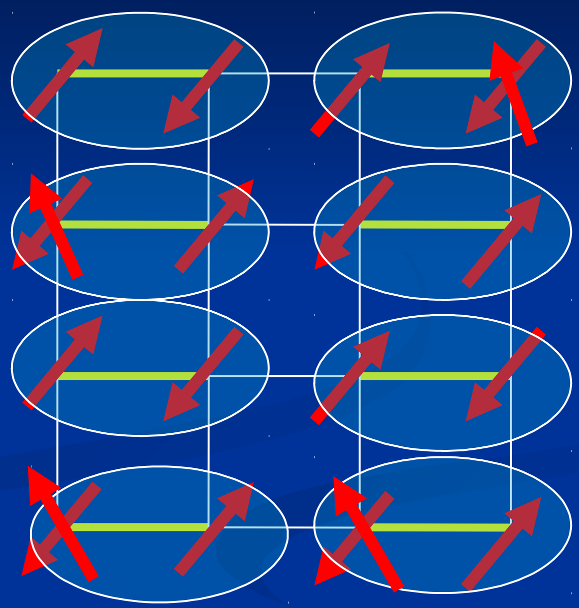
- Magnetization – number of bosons
- Neutrons/NMR : dynamical correlations

Spin Dimers: QS for bosons

TG and A. M. Tsvelik PRB 59 11398 (1999)

$$J_r \gg J$$

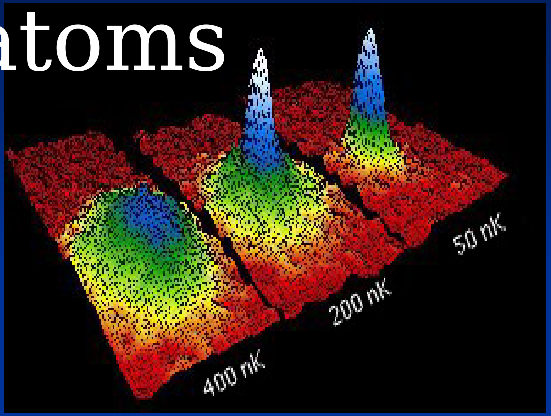




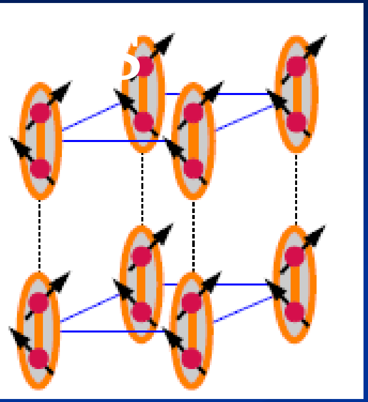
$$|S\rangle \rightarrow |0\rangle, |T_+\rangle \rightarrow |1\rangle$$

BEC and BEC

Cold atoms



Dimers/Spi



Bose gas	Antiferromagnet
Particles	Spin excitations ($\Delta S^z = \pm 1$)
Boson number N	Spin component S^z
Charge conservation U(1)	Rotational invariance O(2)
Condensate wavefunction $\langle \psi(\mathbf{r}) \rangle$	Transverse magnetic order $\langle s_i^x + i s_i^y \rangle$
Chemical potential μ	Magnetic field B
Superfluid density ρ_s	Transverse spin stiffness
Mott insulating state	Integer magnetization plateau

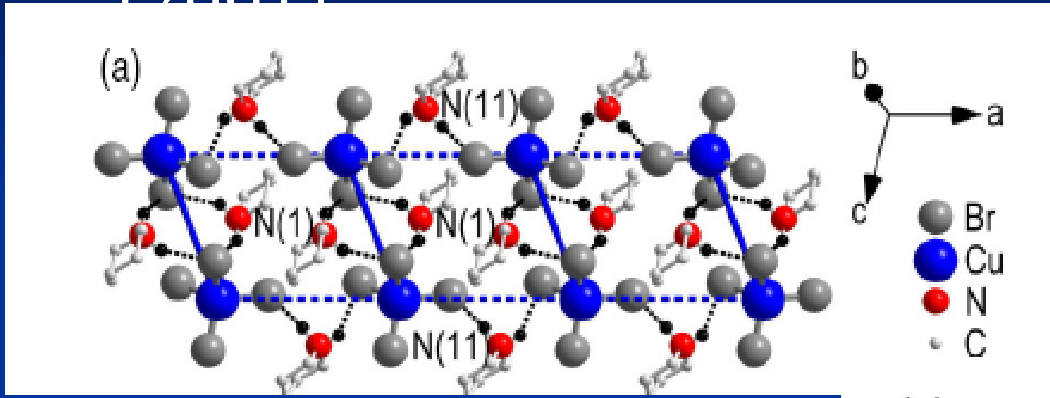
TG, Ch. Rüegg, O. Tchernyshyov, Nat.

The best of both worlds

- Field theory:
  - Asymptotically true: $(r \gg a; t \gg t_0; T \ll J)$
  - Amplitudes and TLL parameters unknown
- Numerics:
  - Efficient at short time, short distance
  - Takes into account the full microscopic model
- Best of both worlds: combine numerics (DMRG) or BA and field theory : essentially exact !

Organic ladder

B. C. Watson et al., PRL 86 5168 (2001)

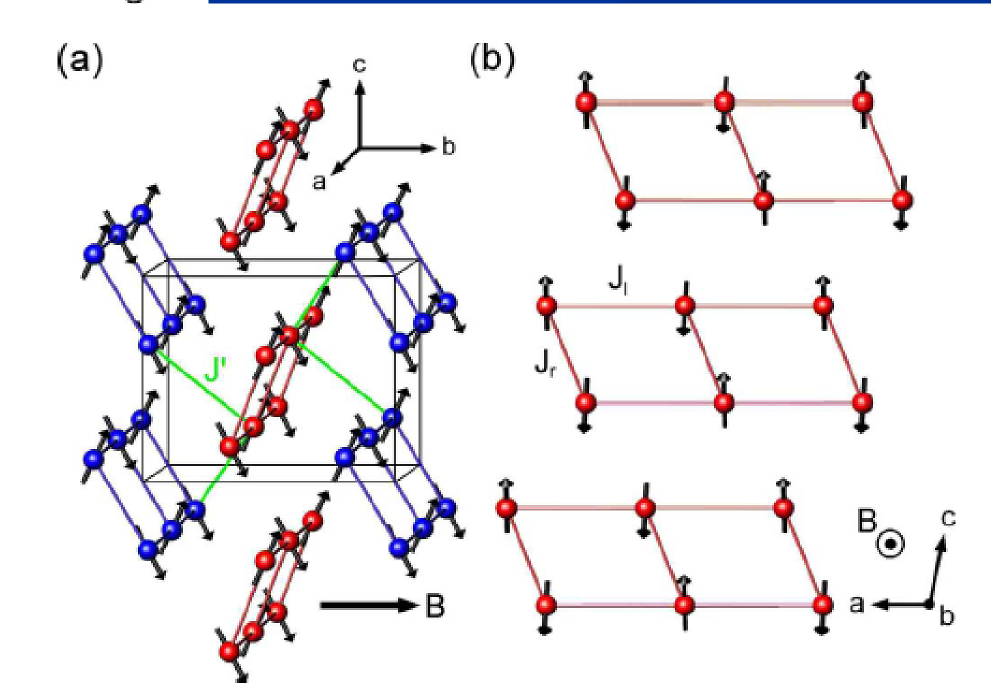


M. Klanjsek et al.,

PRL 101 137207 (2008)

B. Thielemann et al.,

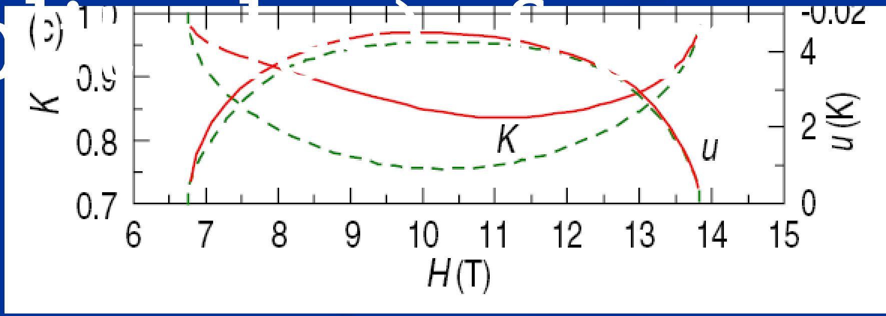
PRB 79, 020408(R) (2009)



Theory

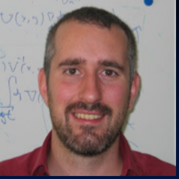
M. Klanjsek et al., PRL 101 137207 (08);
B. Thielemann et al. PRB 79 020408® (09)

- Compute numerically (DMRG) the non-universal parameters (exponents, amp
H

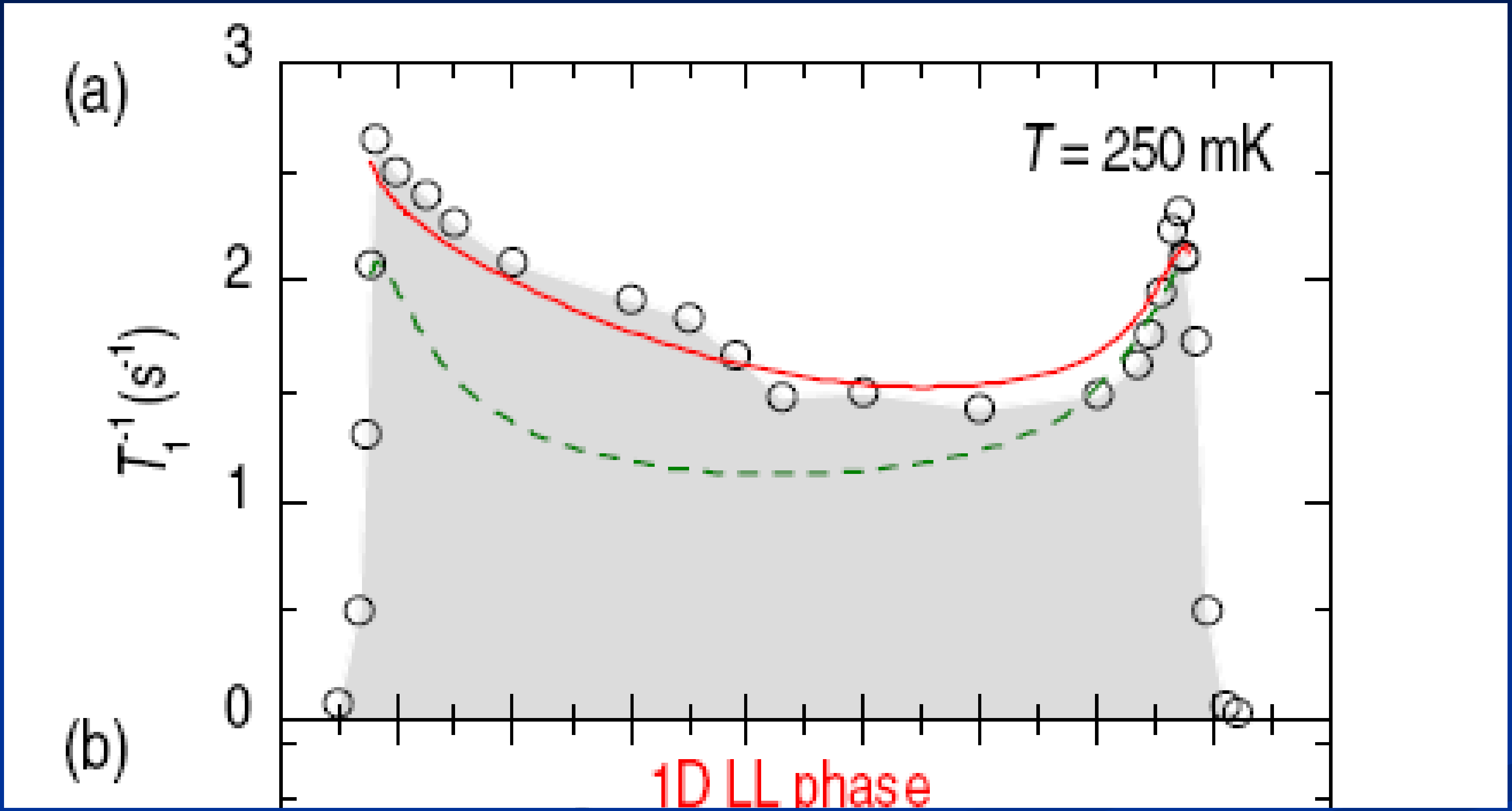


- Inject in TLL theory

$$T_1^{-1} = \frac{\hbar\gamma^2 A_\perp^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right)\left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$




antitative test of T

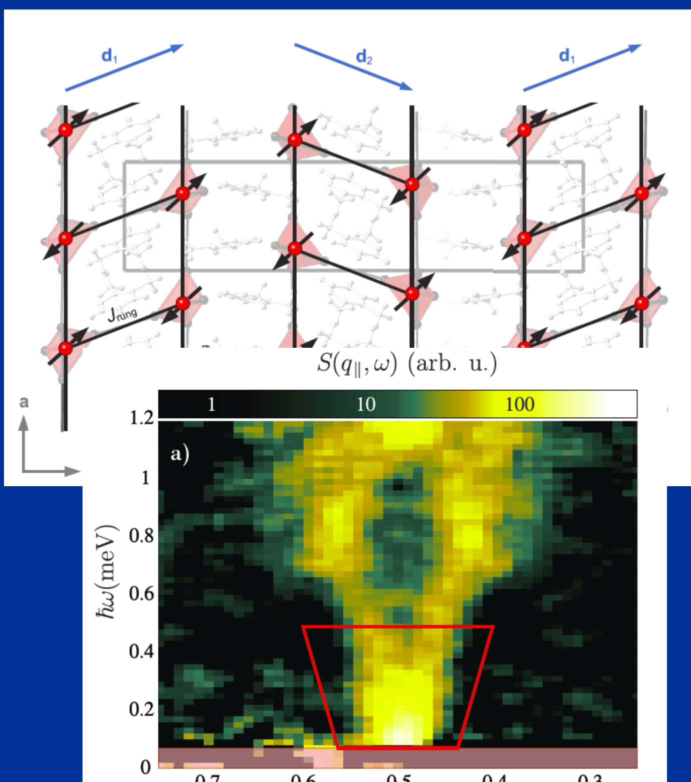



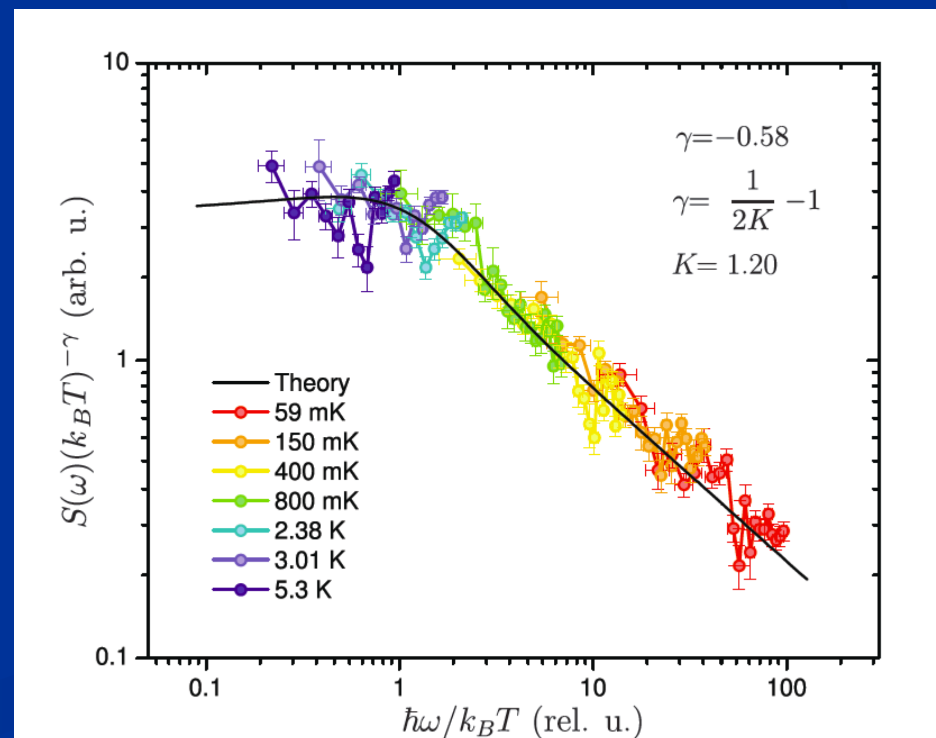
M. Klanjsek et al., PRL 101 137207 (2008)

TLL scaling

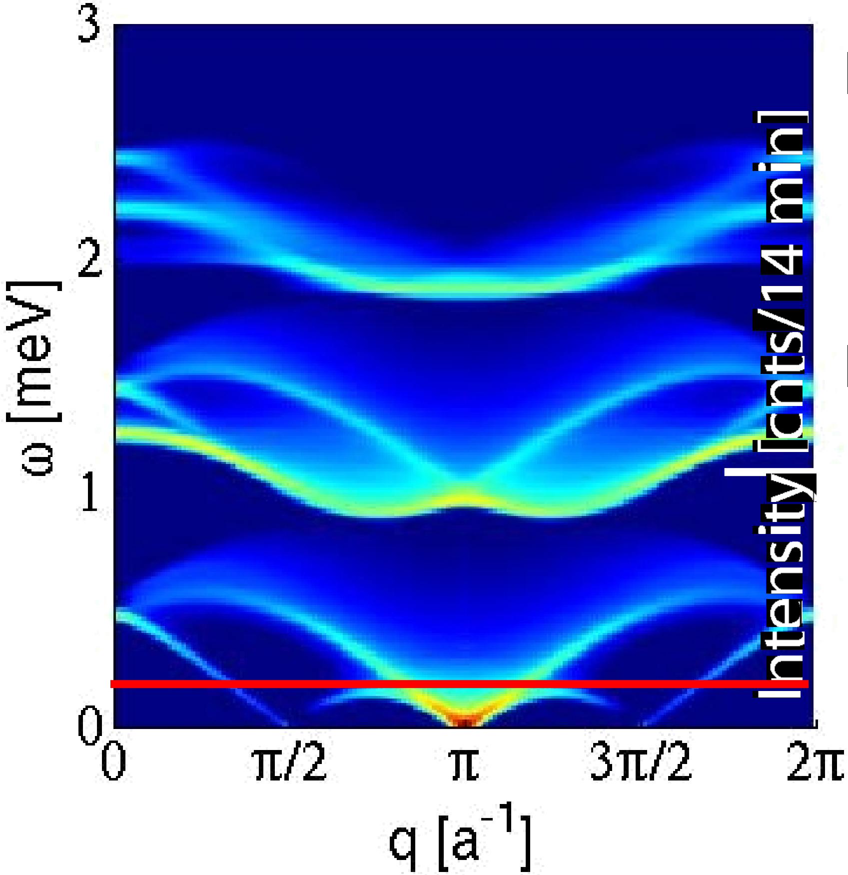
D. Schmidiger et al. PRL 108 167201 (12):

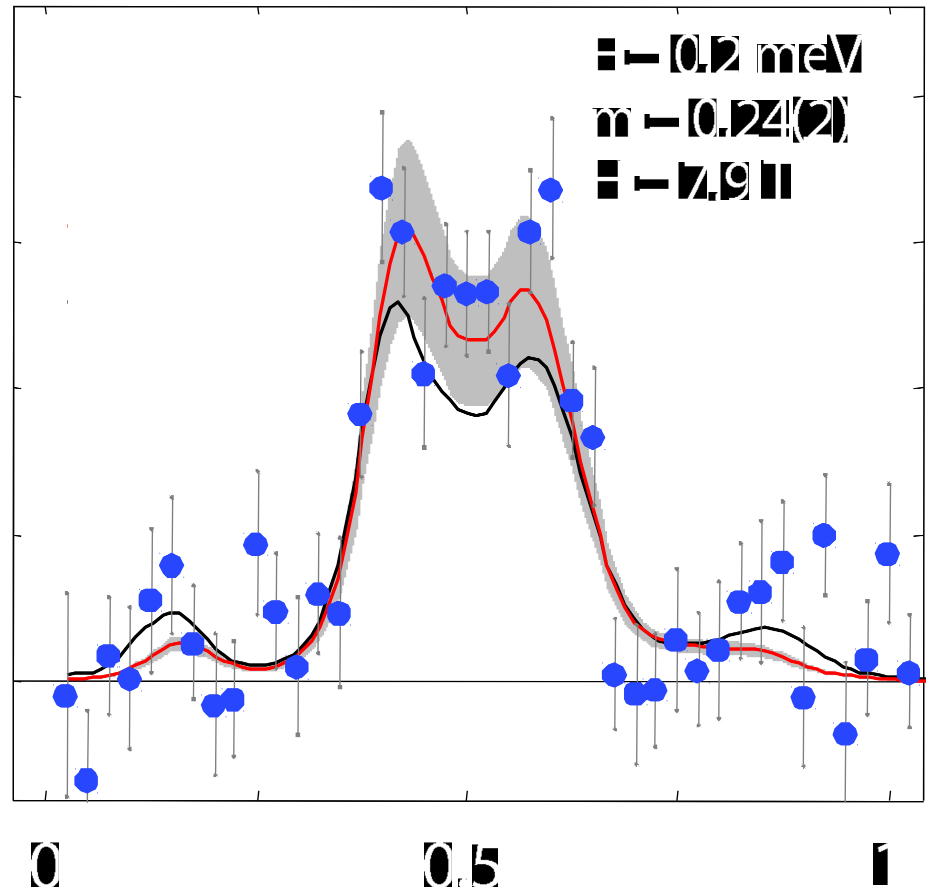
K. Yu et al. arxiv/1406.6876 (14)



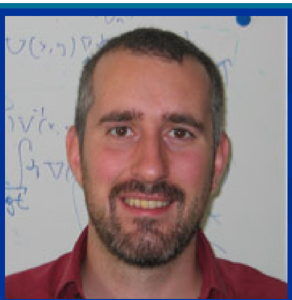


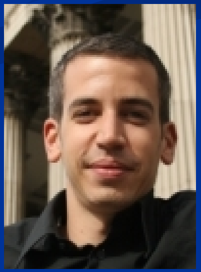
Direct calculation of correlations





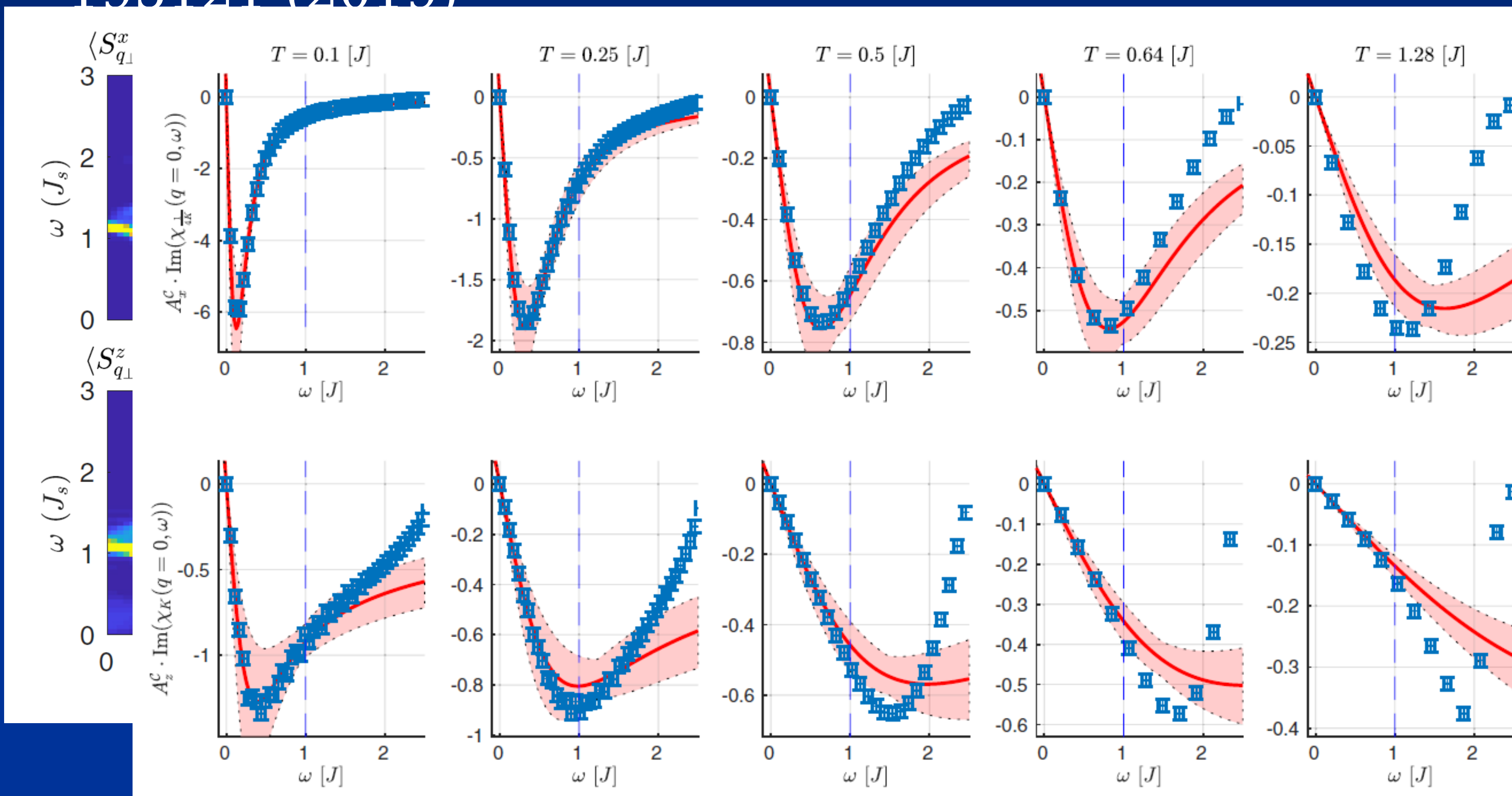
P. Bouillot et al.
PRB 83, 054407 (2011)





Neutron spectra, Finite

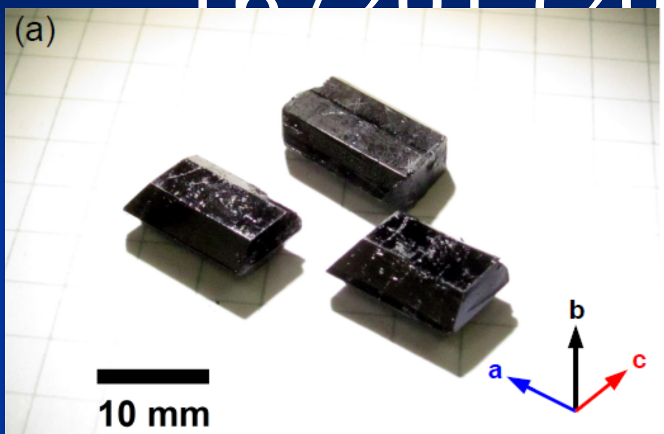
N. Kestin and T. Giamarchi, PRB 99, 195121 (2019)

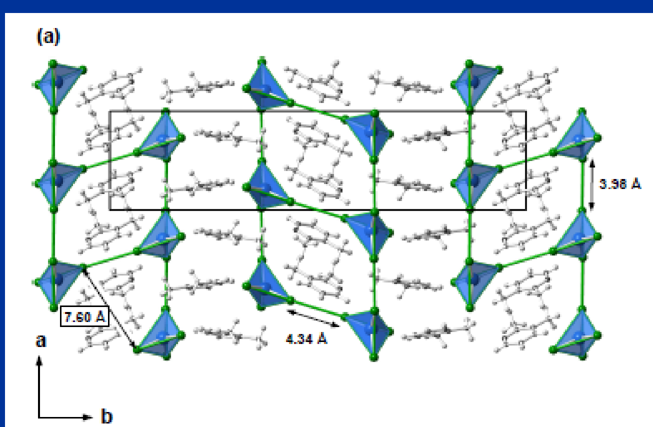


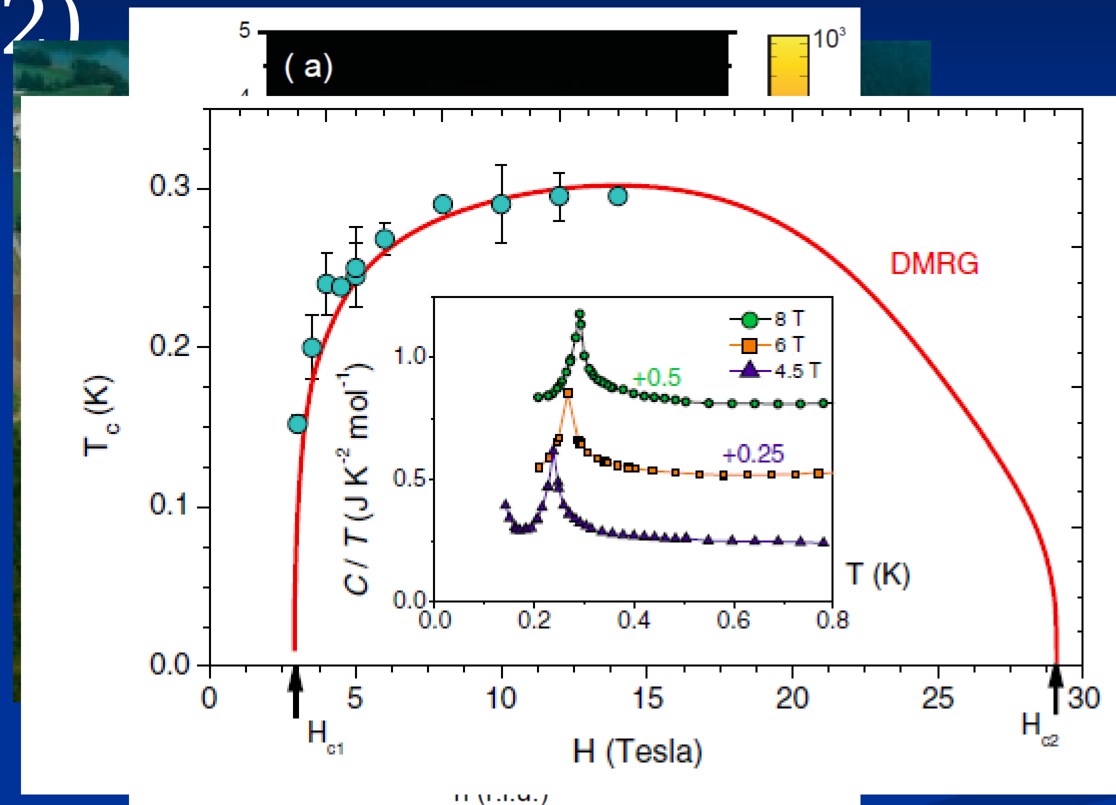
Power of such exact solutions

Hamiltonian reconstruction

D. Schmidiger et al. PRL 108 167201 (2012)



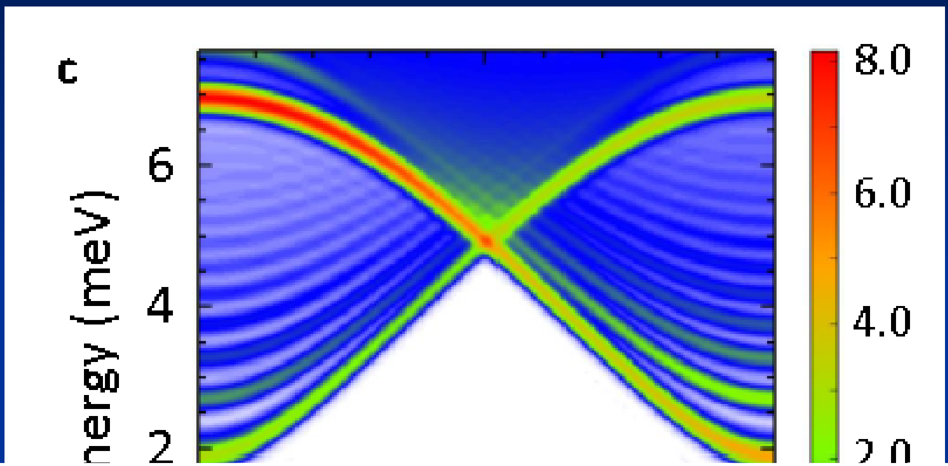


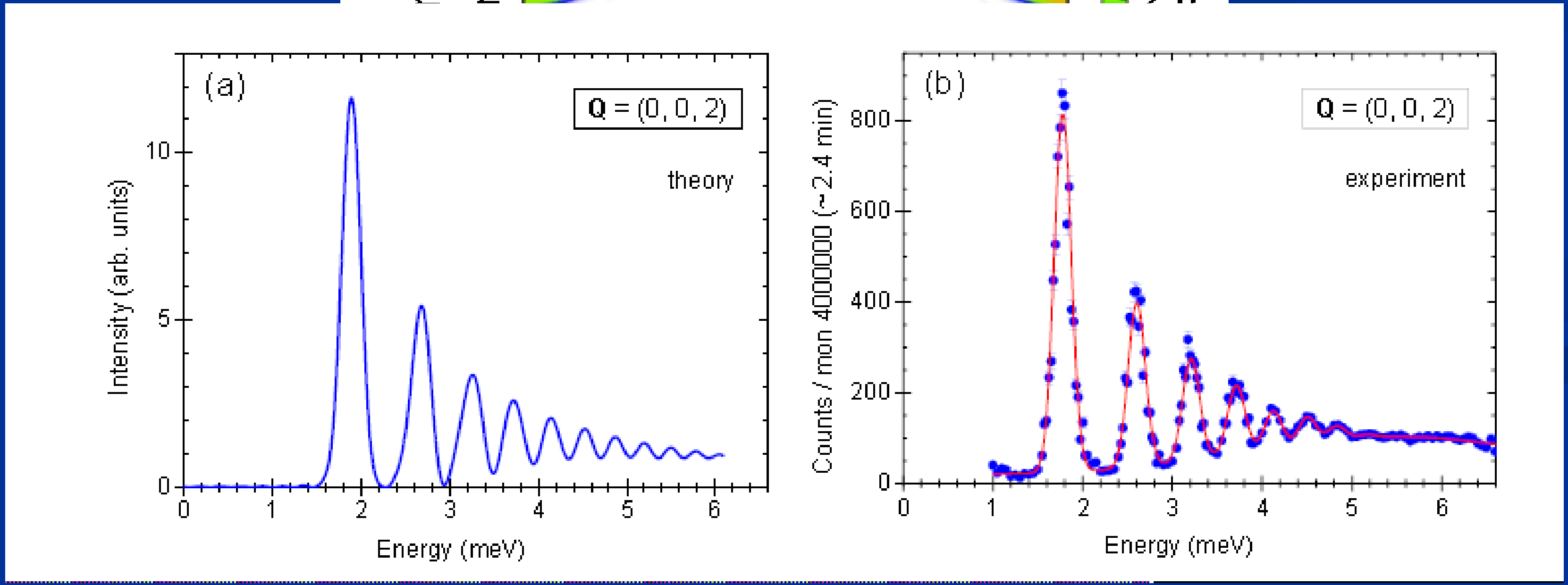


$$\mathcal{H} = J_{\mathrm{leg}} \sum_{l,j} S_{l,j} \cdot S_{l+1,j} + J_{\mathrm{rung}} \sum_{l} S_{l,1} \cdot S_{l,2} - g\mu_B H \sum_{l,j} S^z_{l,j}.$$

Confinement of spinons BaCoVO

Q. Faure, S. Takayoshi, et al. Nat. Phys 14, 716 (2018)





Take home message

- Good theoretical methods to deal with the case of a ``simple'' equilibrium 1d systems (analytics and numerics)
- Stepping stone to go beyond: many exciting questions and problems (out of equilibrium, disorder, many chains, etc.)
- Controled experimental realizations in condensed

Quantitative agreement with the predictions of the Luttinger liquid

End of the story …… ??????

NO! Luttinger liquid plays the same role than Fermi liquid did

Beyond Luttinger liquids

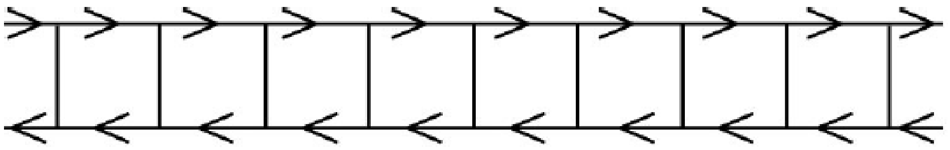
- 1D additional perturbation:
  Lattice (Mott transition), disorder (Bose glass) etc.

Multicomponents, mixtures, ….

Artificial gauge fields

Meissner effect in bosonic ladders

E. Orignac, TG, PRB 64 144515 (2001)


(a)

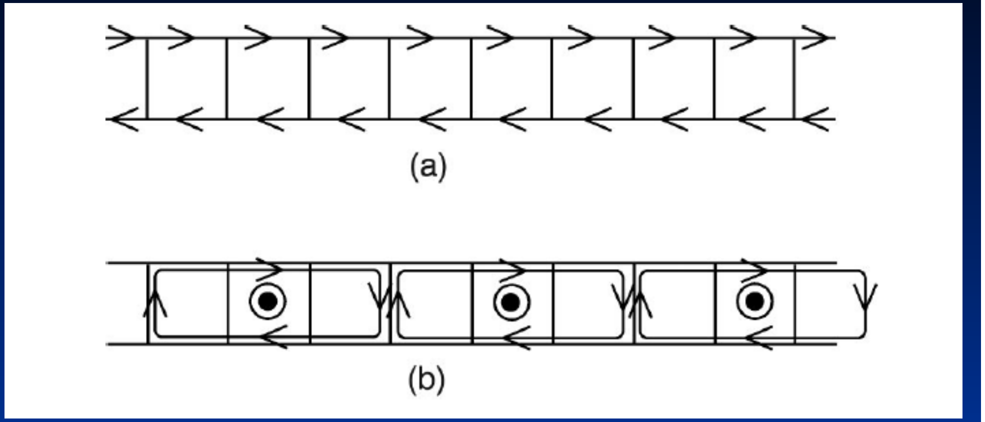
$$H = -t_{\parallel} \sum_{i,p=1,2} \left(b^{\dagger}_{i+1,p} e^{ie^* a A_{\parallel,p}(i)} b_{i,p} + b^{\dagger}_{i,p} e^{-ie^* a A_{\parallel,p}(i)} b_{i+1,p} \right)$$

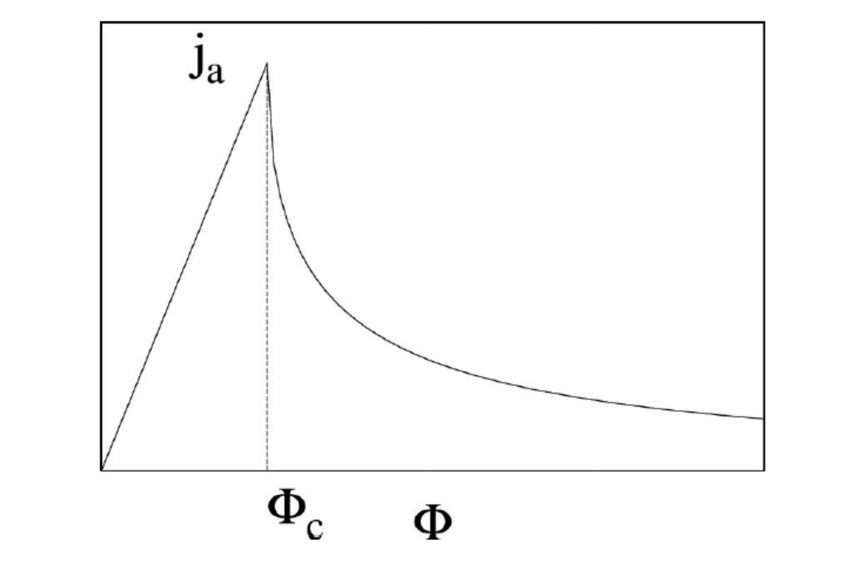
$$- t_{\perp} \sum_{i} \left(b^{\dagger}_{i,2} e^{ie^* A_{\perp}(i)} b_{i,1} + b^{\dagger}_{i,1} e^{-ie^* A_{\perp}(i)} b_{i,2} \right)$$

$$+ U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \qquad (1)$$

$$H = H^0_s + H^0_a - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos\sqrt{8}\,\phi_a ,$$



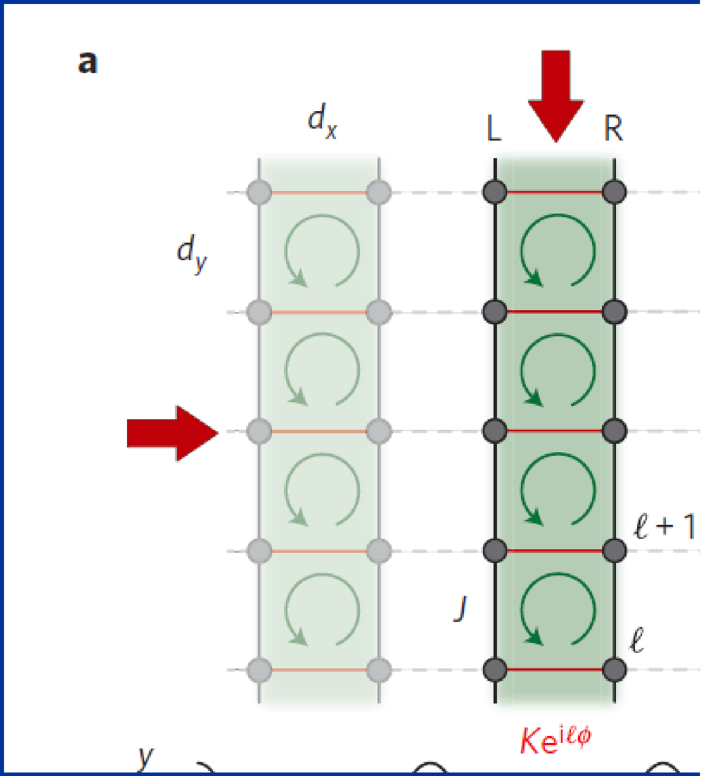


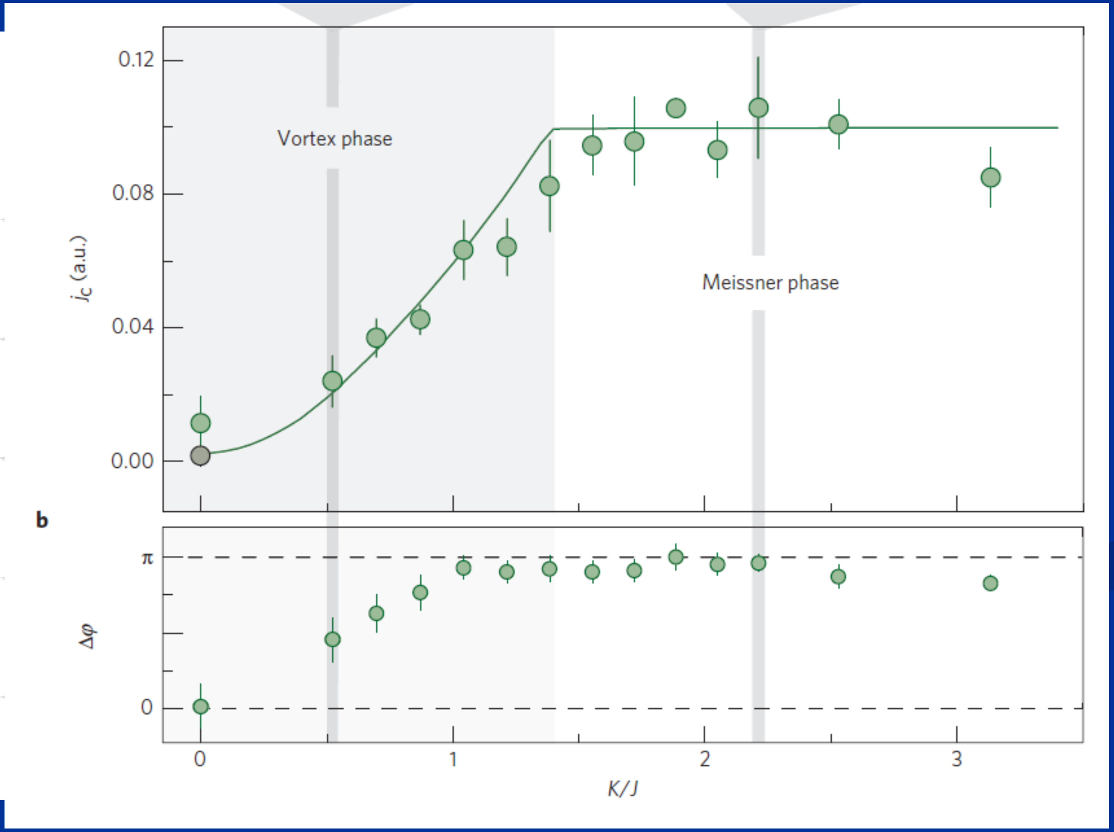
Orbital currents (``Meissner'' effect)

Field ``Hc1'': appearance of vortices

Artificial gauge field (cold atoms)

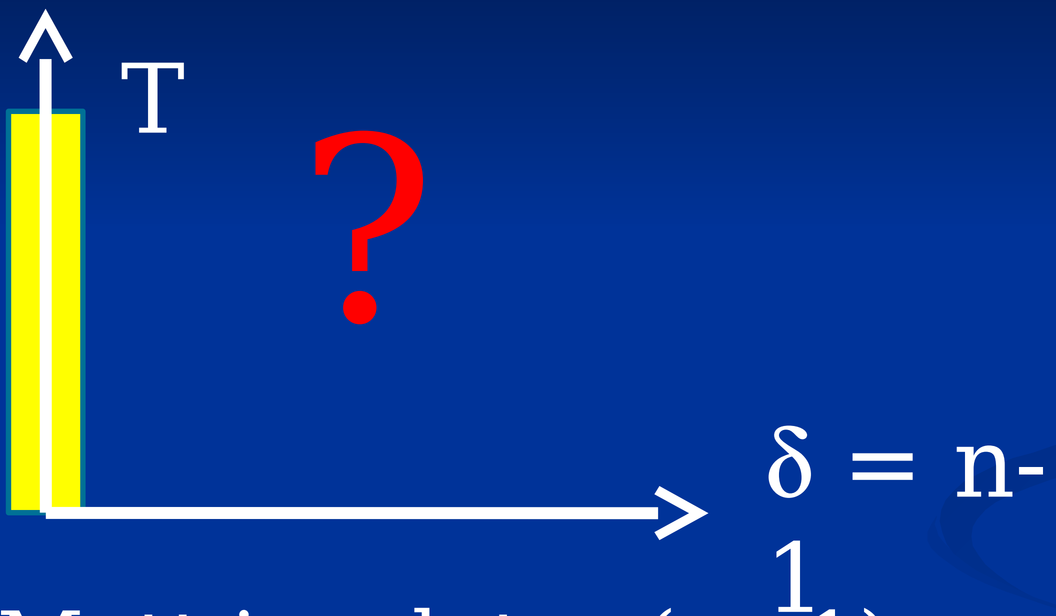
M Atala et al. Nat Phys, 10 588 (2014)

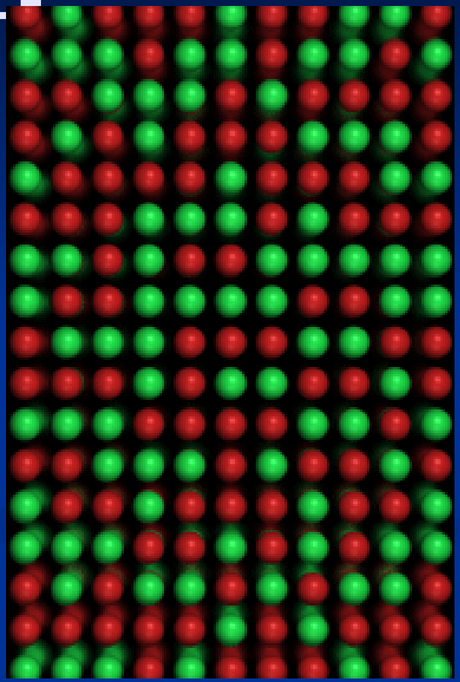




Effect of lattices: Mott transition

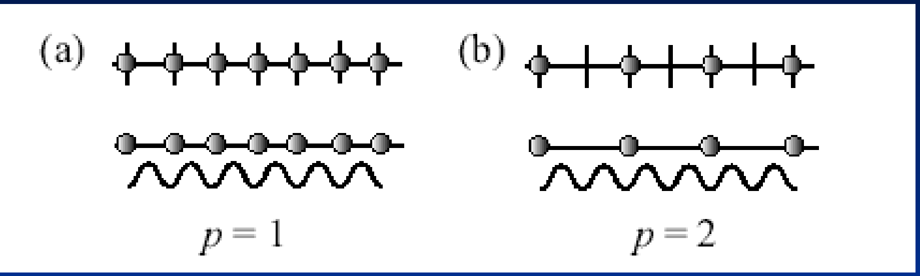
Mott transition





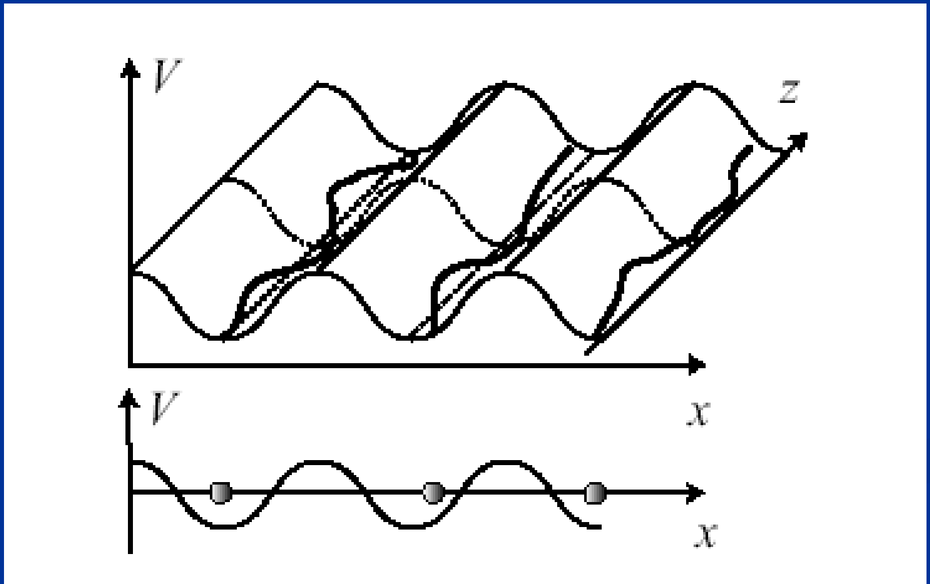
- Mott insulator (n=1)
- T < T_N : antiferromagnetic phase

Periodic lattice



- Incommensurate: $Q \neq 2\pi\rho_0$
- Commensutate: $Q = 2\pi\rho_0$

Competition

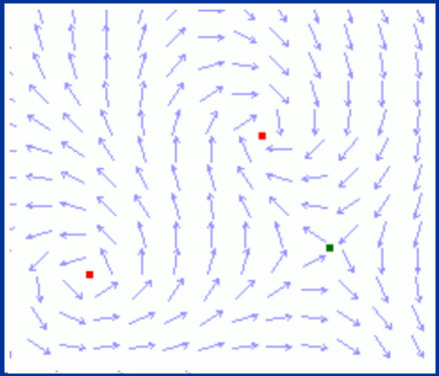


Beresinskii-Kosterlitz-Thouless transition at K=2

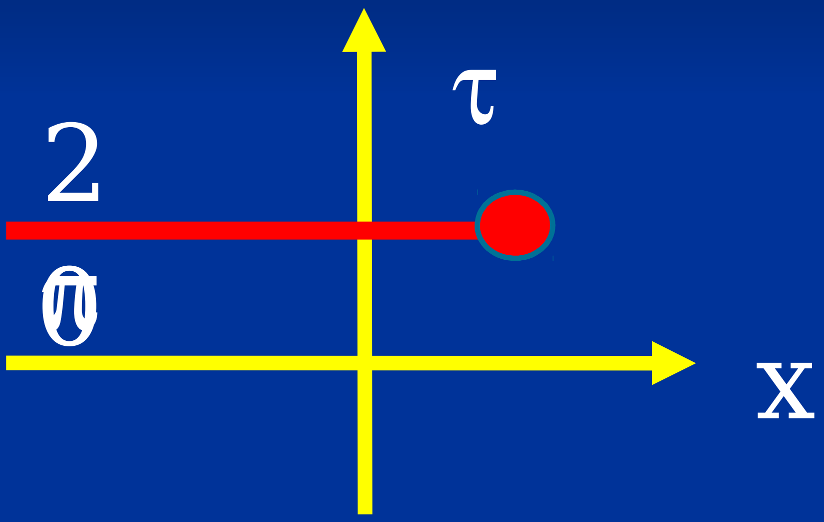
String order

BKT transition

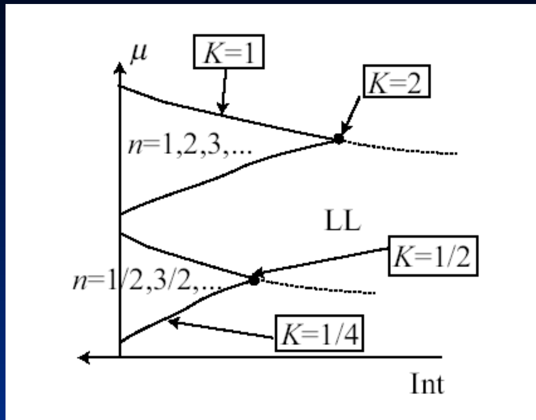
- BKT: remarkable transision going outside the paradigm of Landau’s phase transitions
- A transition without an order parameter
- Topological Vortex excitations

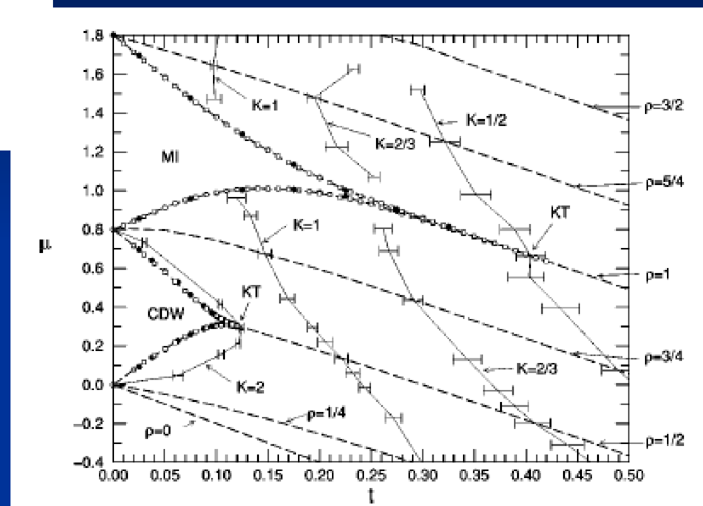


Vortex operator



- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity





Mott insulator: ϕ is locked

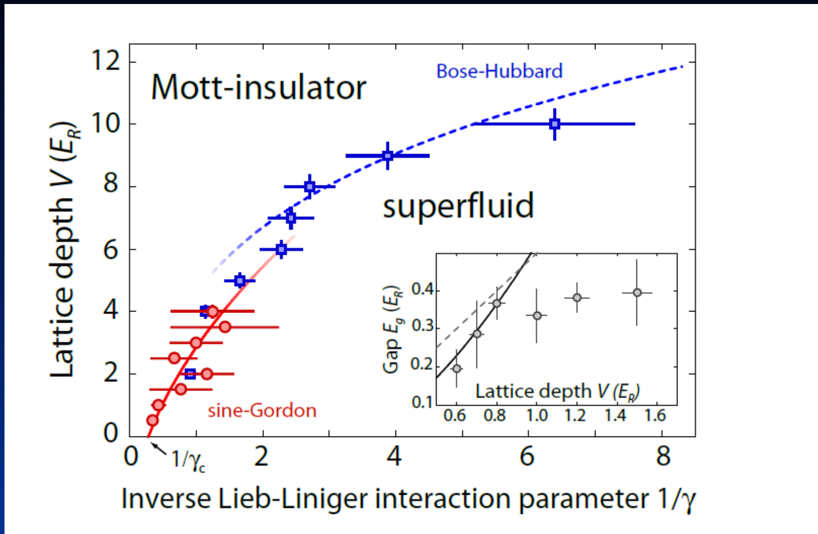
Density is fixed

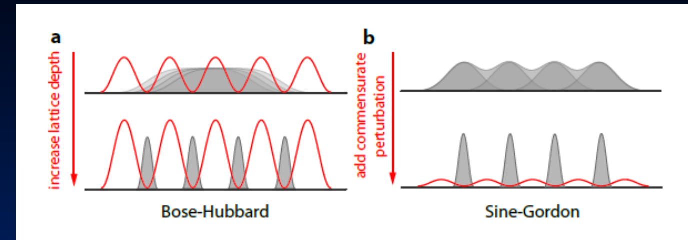
T. Kuhner et al. PRB 61 12474 (2000)

Gap in the excitation spectrum

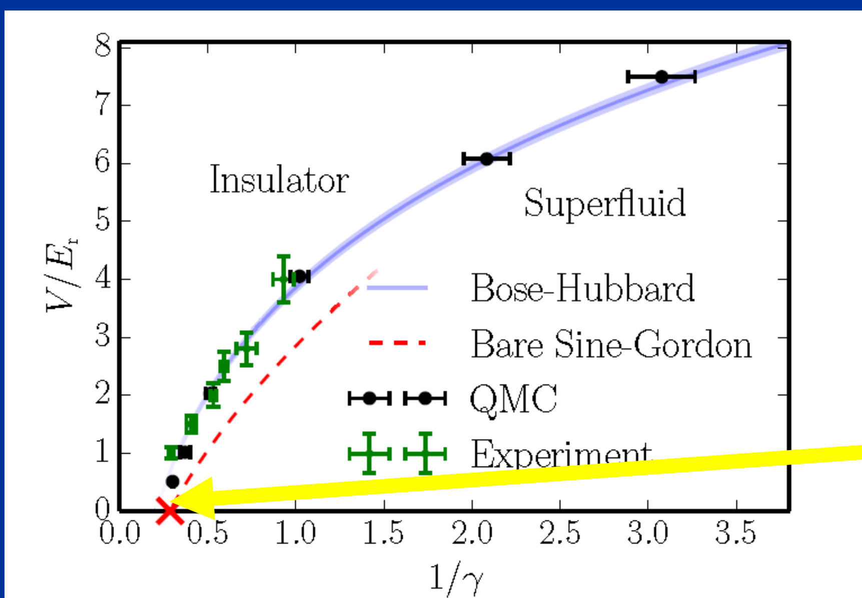
TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures); Oxford (2004);

M. Cazalilla et al., Rev. Mod.





E. Haller et al. Nature 466 597 (2010)



Renormalized Sine-Gordon

Shows:

K* = 2

G. Boeris et al. PRA 93 93,

Non local (topological) order

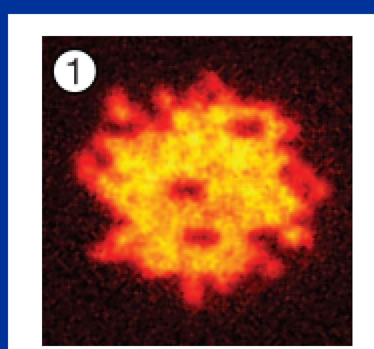
$$\mathcal{O}_{\mathrm{P}}^{2} = \lim_{l\to\infty} \mathcal{O}_{\mathrm{P}}^{2}(l) = \lim_{l\to\infty} \left\langle \prod_{k \leq j \leq k+l} e^{i\pi\delta\hat{n}_j} \right\rangle$$

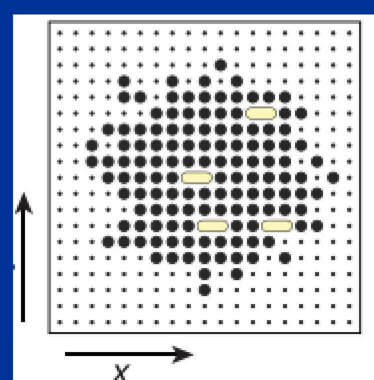
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

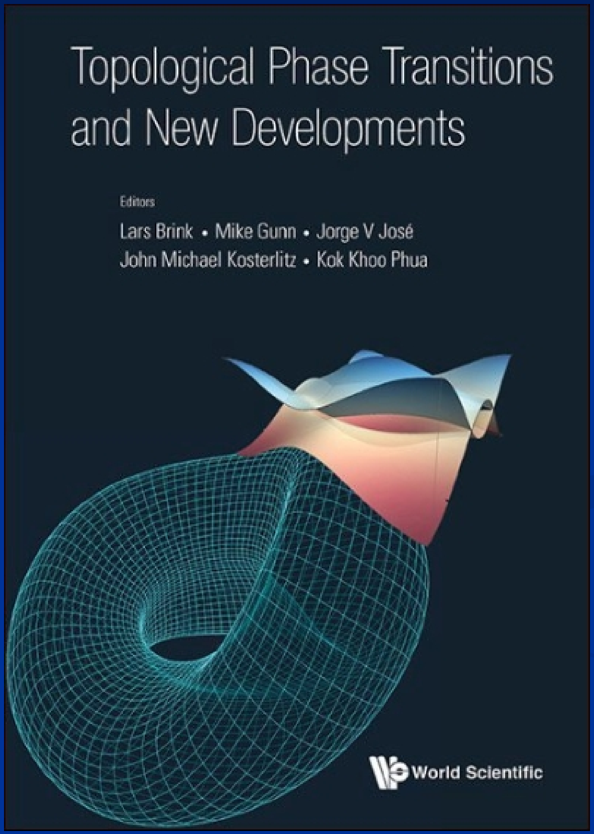
M. Endres,[1*] M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

Science (2011)





Topological excitations is the norm in 1D



Topological Phase Transitions and New Developments, pp. 147-164 (2018)

Clean and dirty bosons in 1D lattices

Disorder (equilibrium)

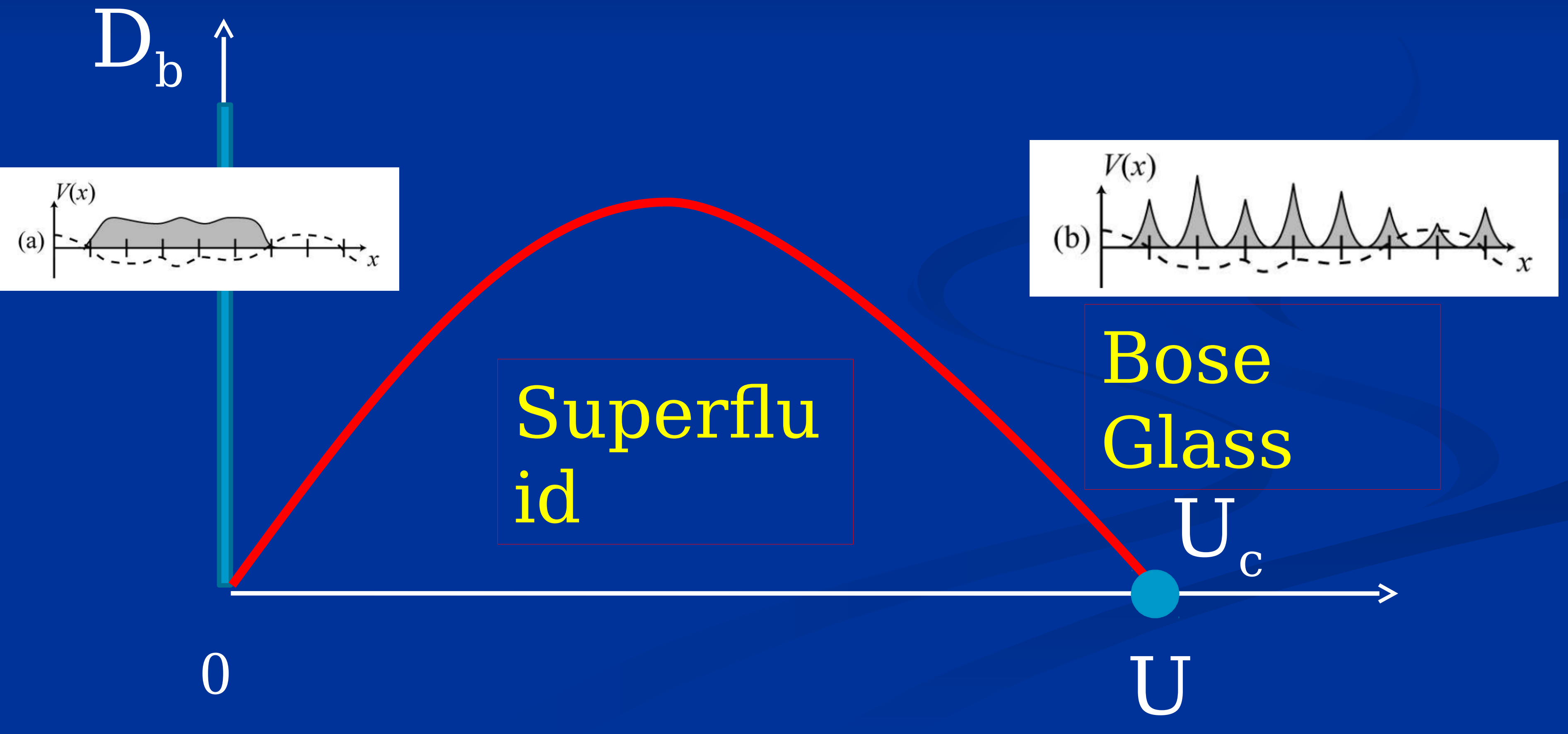
Example: localization of 1D interacting bosons

TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

Competition Disorder vs Interactions

Existence of a Many-Body localized phase: Bose glass

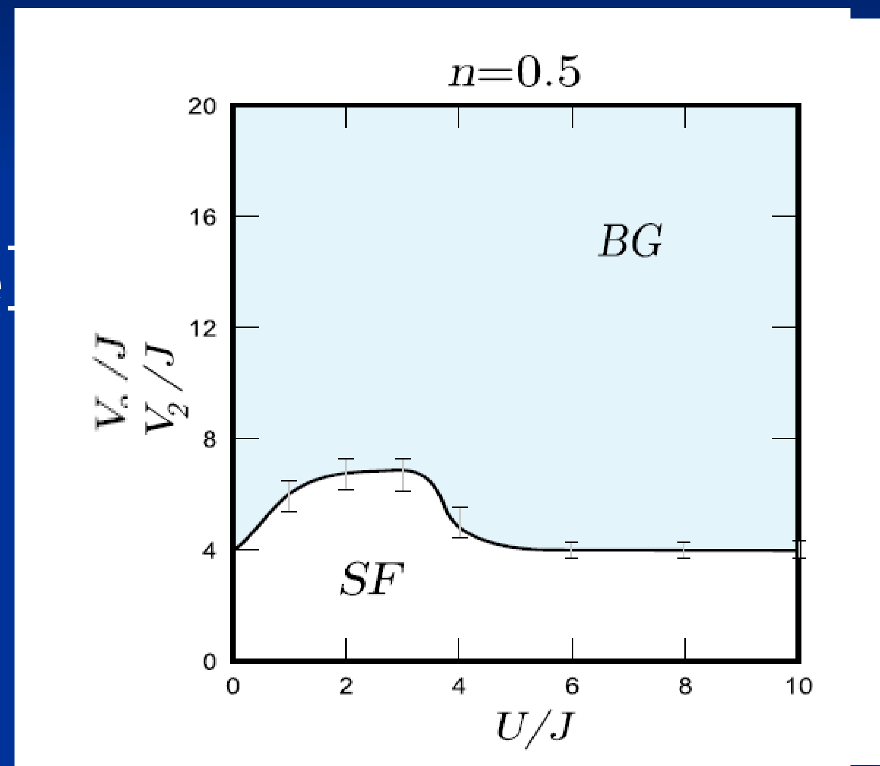
Bose glass phase



Other potentials: Biperiodics

- U= 0
  Aubry-André model
- Localization transition

Effect of interactions?
Same as ``true'' disorder ?



J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

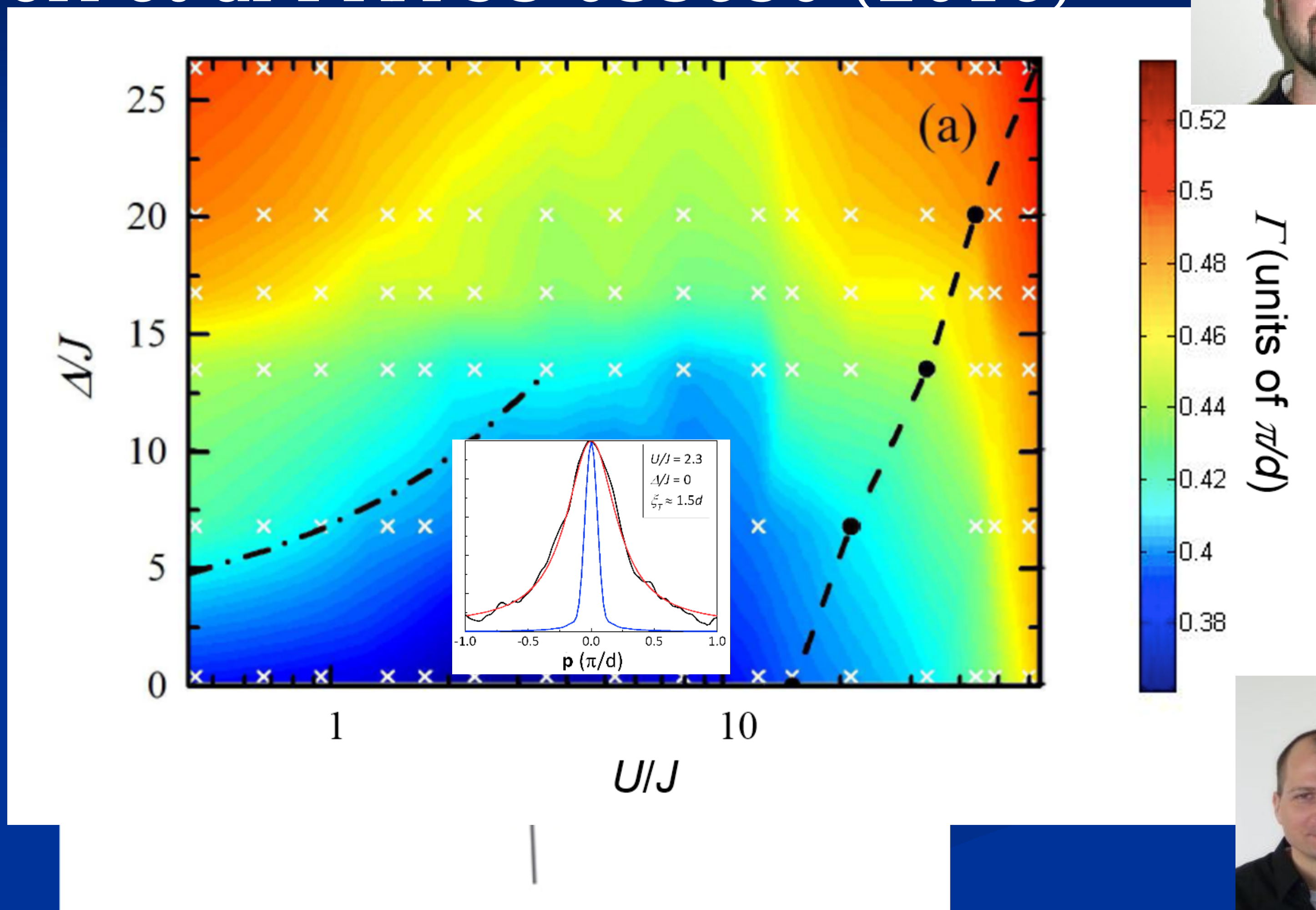
G. Roux et al. PRA 78 023628 (2008);
T. Roscilde, Phys. Rev. A 77,

Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (201
L. Gori et al PRA 93 033650 (2016)



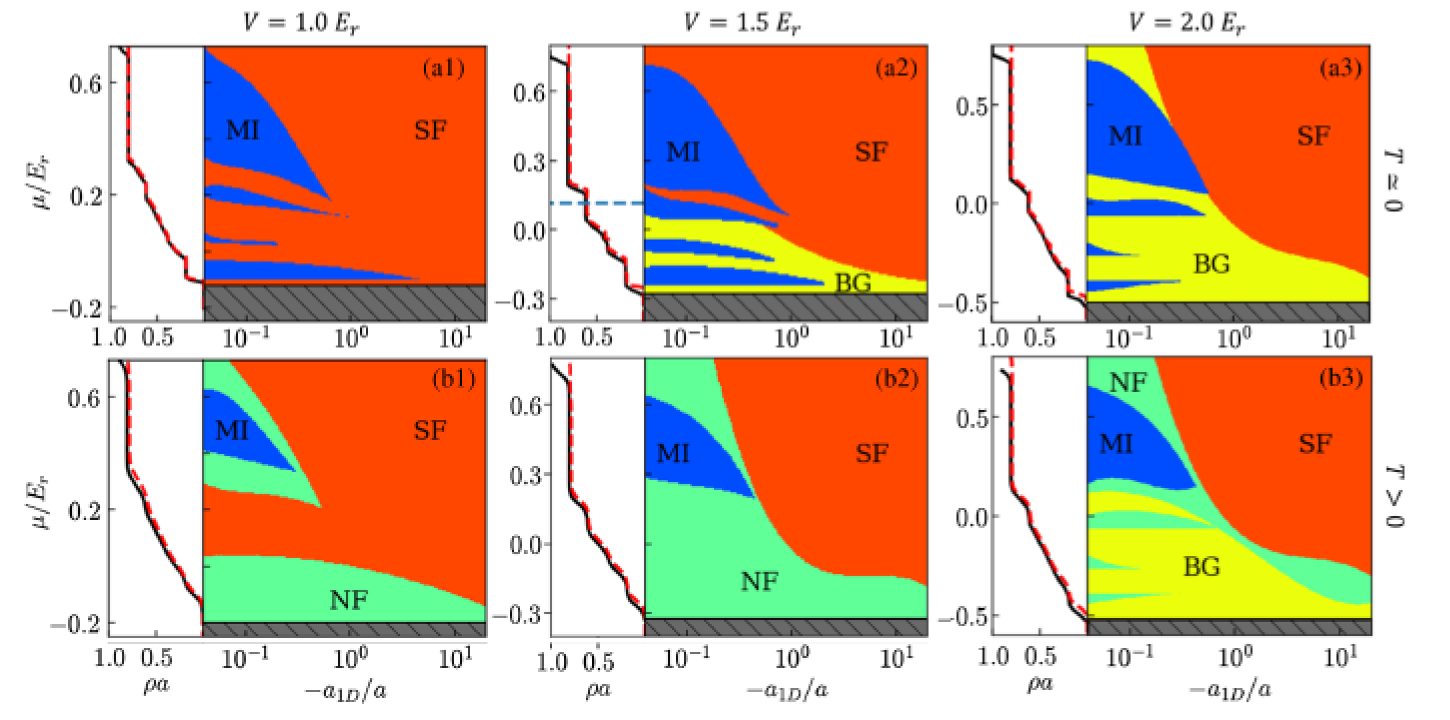




PHYSICAL REVIEW LETTERS **125**, 060401 (2020)

Lieb-Liniger Bosons in a Shallow Quasiperiodic Potential: Bose Glass Phase and Fractal Mott Lobes

Hepeng Yao,[1] Thierry Giamarchi,[2] and Laurent Sanchez-Palencia[1]

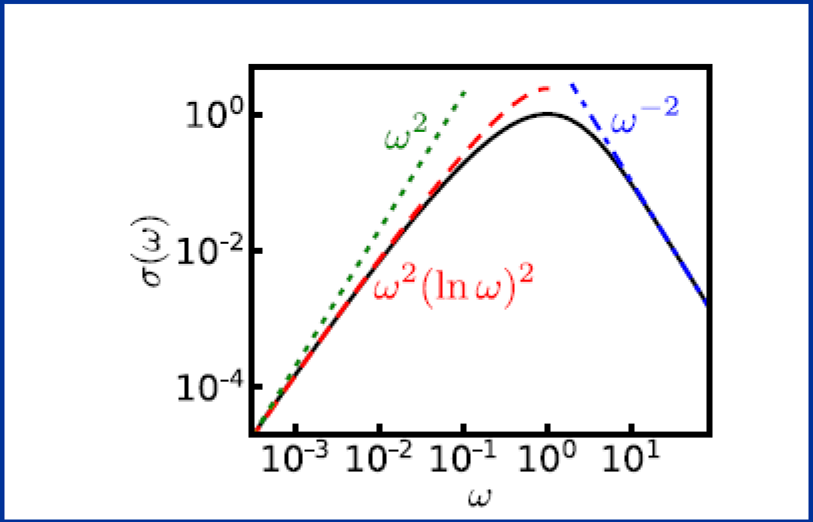


TLL and the great beyond

Disorder and interactions

No interactions: Beresinskii solution

V. L. Berezinskii, JETP 38, 620 (1974)

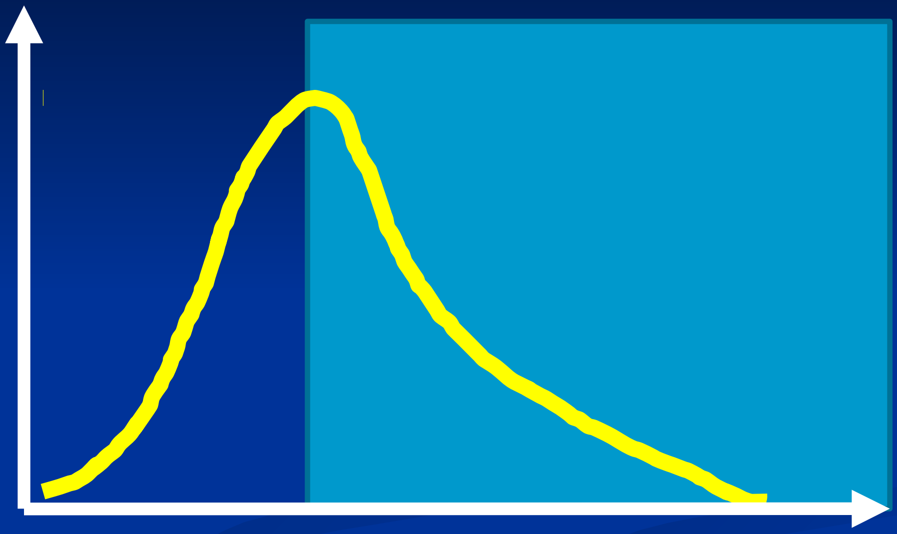


Log^2:
Bert Halperin

Peak:

a.c. transport

$$\xi_{\mathrm{loc}} \sim \alpha \left(\frac{1}{K^2 \tilde{D}_{\mathrm{b}}}\right)^{\frac{1}{3-2K}}$$



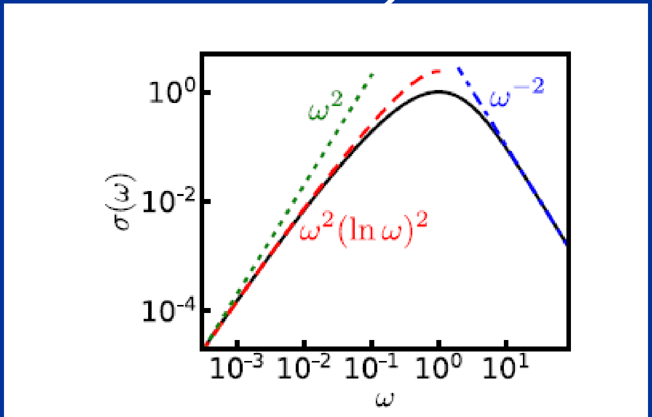
Dynamical conductivity of disordered quantum chains

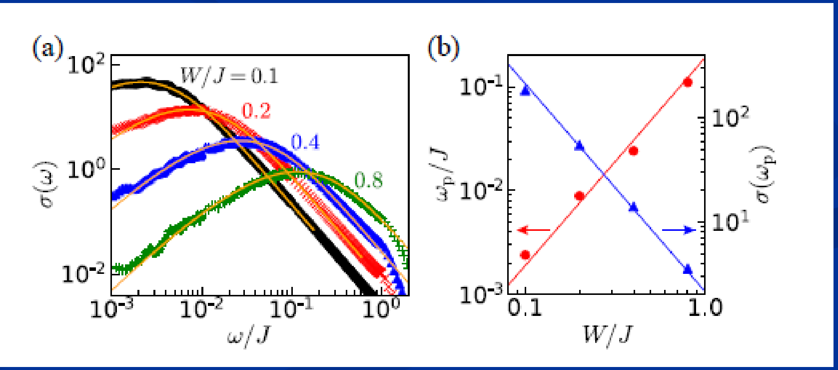
Shintaro Takayoshi[1] and Thierry Giamarchi[2]

arXiv:2206.00023

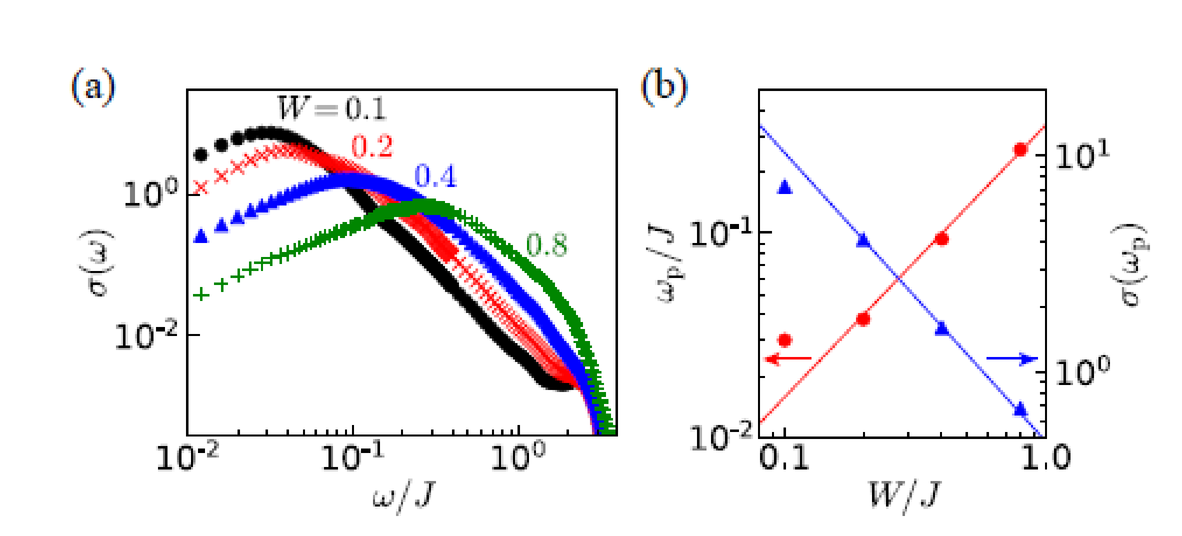
$$\hat{\mathcal{H}} = J\sum_{l=1}^{N-1}\left[\frac{1}{2}(\hat{a}_l^\dagger \hat{a}_{l+1} + \mathrm{H.c.}) + \Delta\left(\hat{n}_l - \frac{1}{2}\right)\left(\hat{n}_{l+1} - \frac{1}{2}\right)\right] - \sum_{l=1}^{N} h_l\left(\hat{n}_l - \frac{1}{2}\right), \quad (1)$$

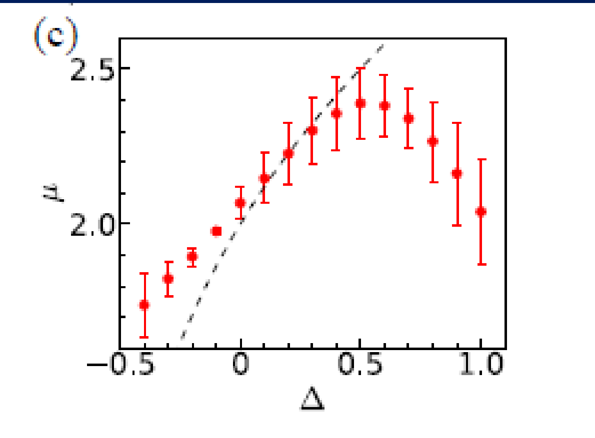
Non-interacting (Berezinskii solution)

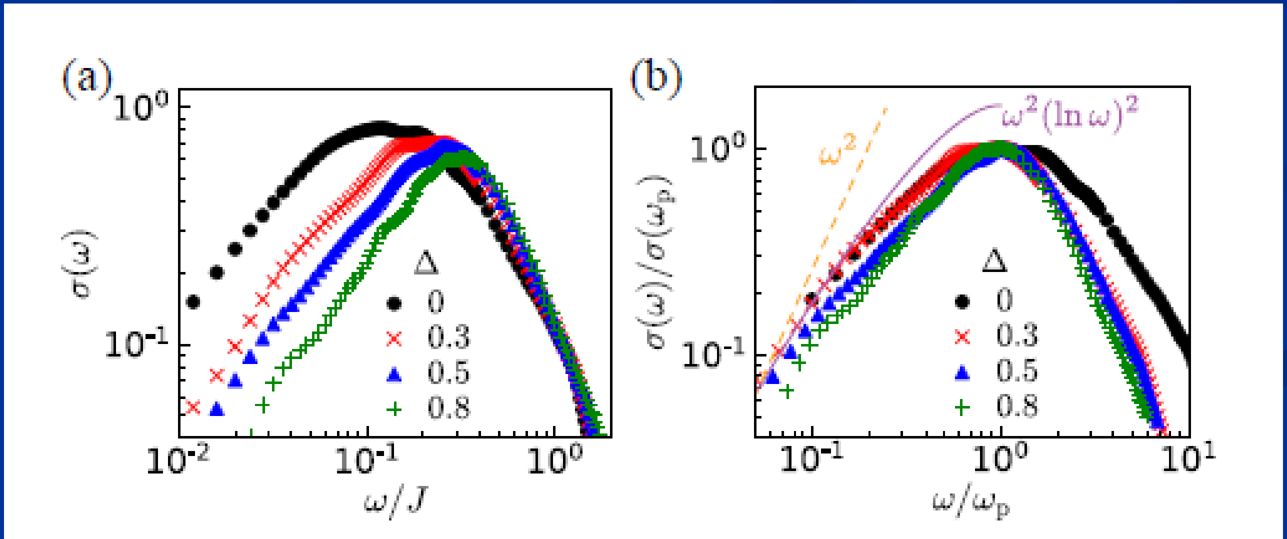




With interactions (DMRG)







d.c. transport

VRH with interactions

T. Nattermann, TG, P. Le doussal, PRL 91, 056603 (2003)

$$\frac{S}{\hbar} = \int_0^L dx \int_0^{\beta\hbar u} dy \frac{1}{2\pi K}[(\partial_y \phi)^2 + (\partial_x \phi)^2],$$

$$S_{\text{dis}}/\hbar = -\frac{1}{2}\int \frac{dxdyA(x)}{2\pi K \alpha^2} e^{i[\phi(x,y)-\zeta(x)]} + \text{H.c.},$$

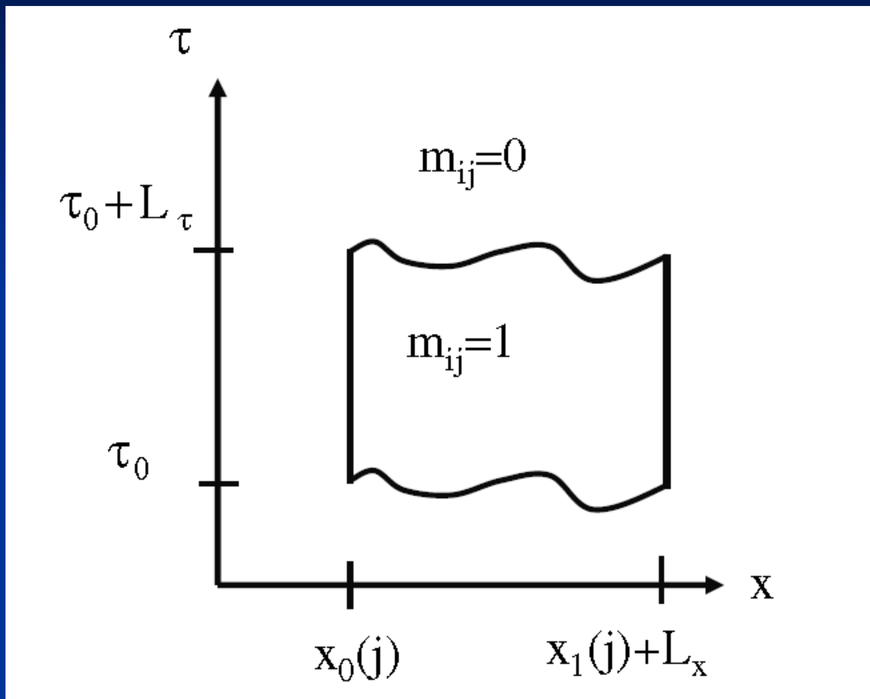
$$S_E/\hbar = \int dxdy\tilde{E}\phi(x,y),$$

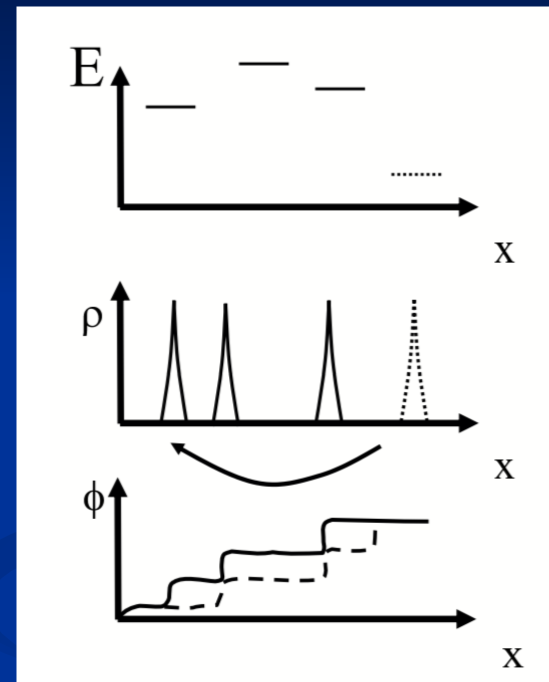
$$\frac{H}{u^*\hbar} = \frac{1}{2\pi K^* \alpha}\sum_{i=1}^{N}[(\phi_{i+1} - \phi_i)^2 - A^* \cos(\phi_i - \zeta_i)],$$

$$\frac{H}{u^*\hbar} = \frac{2\pi}{K^* \alpha}\sum_{i=1}^{N}(n_{i+1} - n_i - f_i)^2 - \frac{N}{2\pi K^* \alpha},$$

$$n_i^0 = m_0 + \sum_{j<i}[f_i],$$

VRH: solitonic excitations





$$\sigma(T) \propto e^{-(S^*/\hbar)} = \exp\left[-\frac{4\pi}{K^*}\sqrt{2\beta\Delta}\right].$$

$$\sigma \propto e^{-(d+1)\left(\frac{\beta}{N_0 d^d \xi_{\rm loc}^d}\right)^{\frac{1}{d+1}}}$$

d.c. transport at finite T

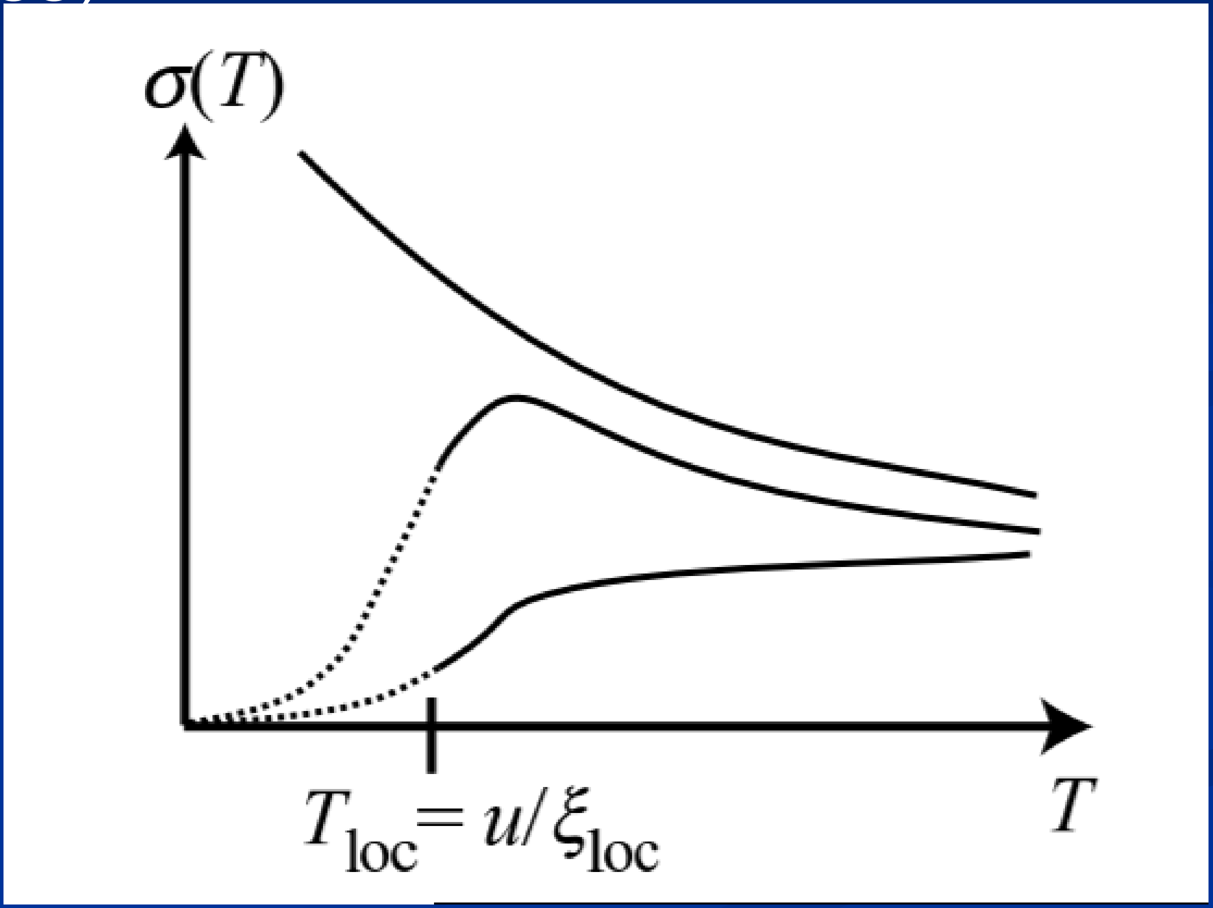
- In 1D all states are exponentially localized
- Finite T: sweeps energy E with probability f(E)
- No interactions $\sigma(T)=0$ for all T !

- Needs coupling to a thermal bath (phonons, etc.)
- Or with interactions can the system be its own thermal bath ?

With a bath

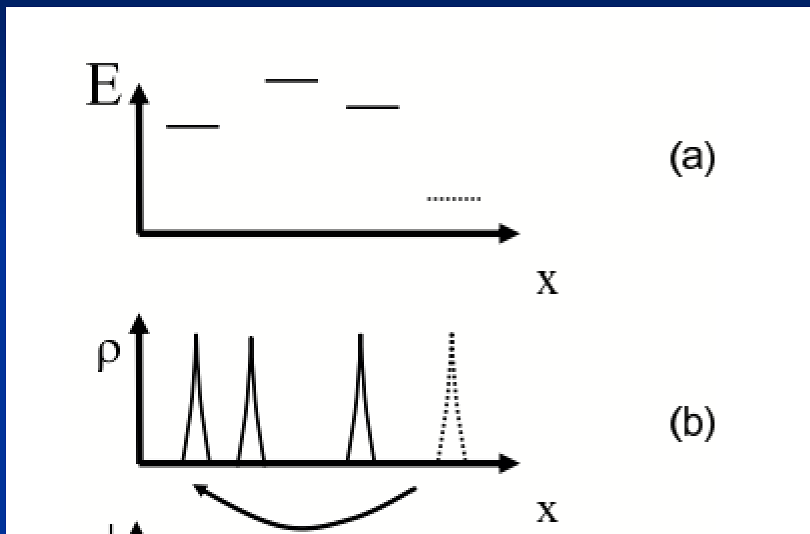
d.c. transport (high T)

TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)



Transport with a Thermostat

- Mott variable range hopping



$$e^{-\beta E} e^{-L/\xi_{\text{loc}}}$$

$$N_0 L^d E$$

$$\sigma \propto e^{-\frac{\beta}{N_0 L^d} - L/\xi_{\text{loc}}}$$

$$\sigma \propto e^{-(d+1)\left(\frac{\beta}{N_0 d^d \xi_{\text{loc}}^d}\right)^{\frac{1}{d+1}}}$$

- 1D with interactions

T. Nattermann, TG, P. Le doussal, PRL 91, 056603 (2003)
arXiv:cond-mat/0403487

$$\sigma(T) \propto e^{-(S^*/\hbar)} = \exp\left[-\frac{4\pi}{K^*}\sqrt{2\beta\Delta}\right].$$

Without a bath

- With interactions can other particles be a ``bath’’ for one particle ?

- Can the system reach the thermodynamic equilibrium ? And explore ergodically the phase space ?

Many body localization

Basko, Aleiner, Altshuler; Gornyi, Mirlin Polyakov; Huse, ...
Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)

No thermostat $\sigma(T) = 0$ if $T<T^*$ even with interactions

The system is not ergodic even at finite temperatures/energies

Equilibrium (time scales ?)

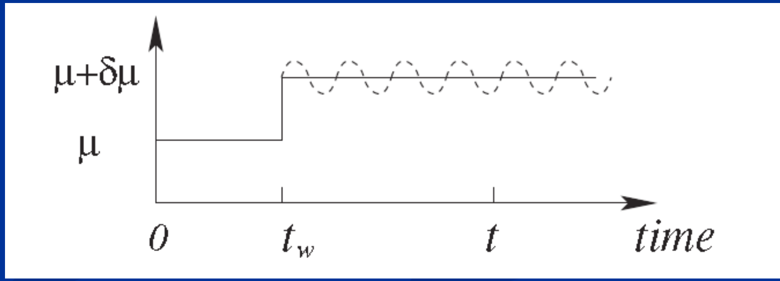
disordered interacting

PRL **96,** 217203 (2006) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2006

Dynamic Compressibility and Aging in Wigner Crystals and Quantum Glasses

Leticia F. Cugliandolo,[1,2] Thierry Giamarchi,[3] and Pierre Le Doussal[2]

$$H = \int_x \left\{ \frac{\hbar^2 \Pi^2(x)}{2m\rho_0} + \frac{c}{2}[\nabla u(x)]^2 \right\} - \int_x U(x)\rho_0 \cos\{Q[x - u(x)]\},$$



Non ergodic system (glass)

Observables depend on time

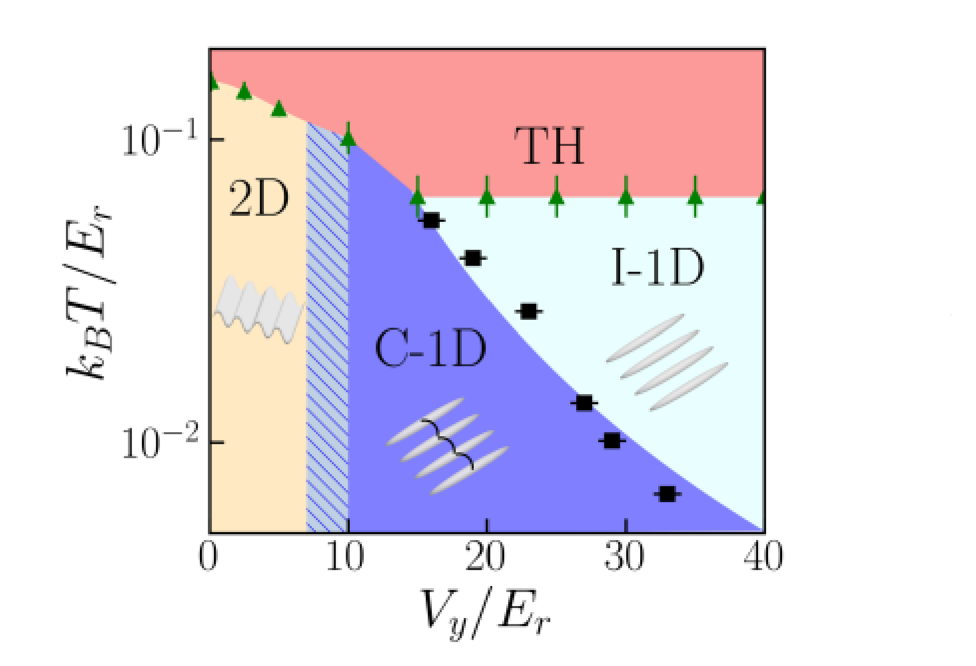
Link with MBL ???

Take home message

- Remarkable interplay between localization and interactions
- Consequences and challenges both in equilibrium (LIP) and out of equilibrium (MBL).
- Experimental possibilities to explore these phenomena
- Need better tools !

Dimensional crossover

Bosons



Strongly-interacting bosons at 2D-1D dimensional crossover

Hepeng Yao, Lorenzo Pizzino, Thierry Giamarchi

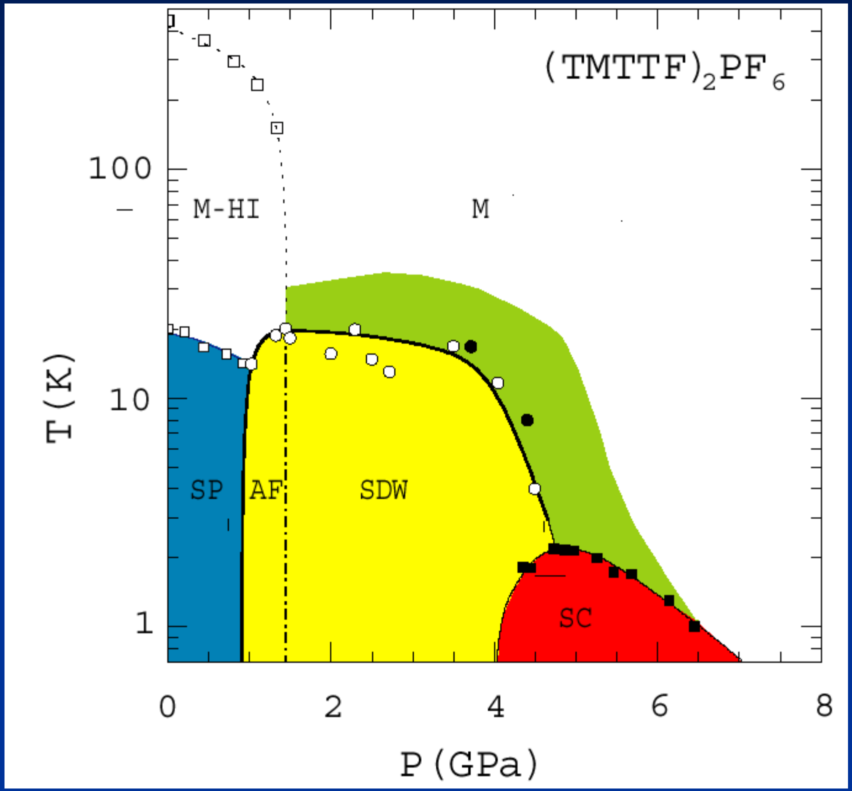
SciPost Phys. 15, 050 (2023) · published 7 August 2023

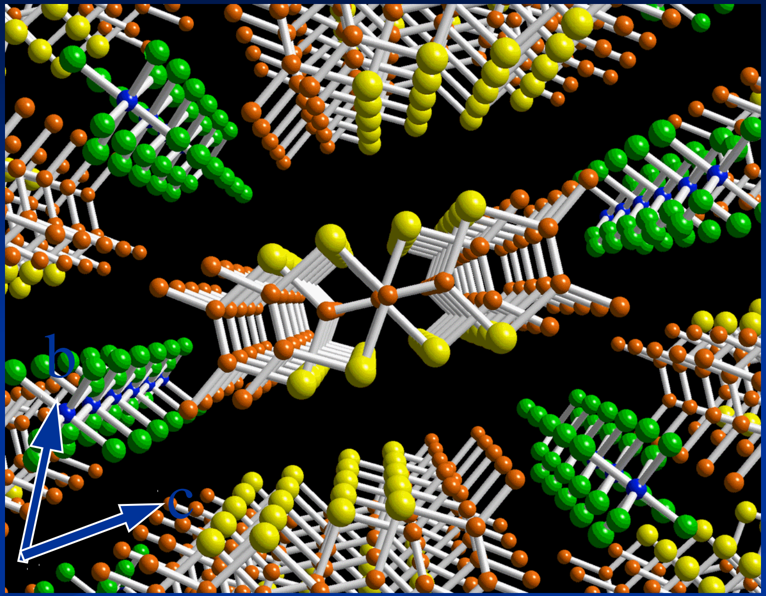
Cooling bosons by dimensional reduction

Yanliang Guo,[1,*] Hepeng Yao,[2,*] Sudipta Dhar,[1] Lorenzo Pizzino,[2] Milena Horvath,[1] Thierry Giamarchi,[2] Manuele Landini,[1] and Hanns-Christoph Nägerl[1,†]

Arxiv/2308.04144

Deconfinement

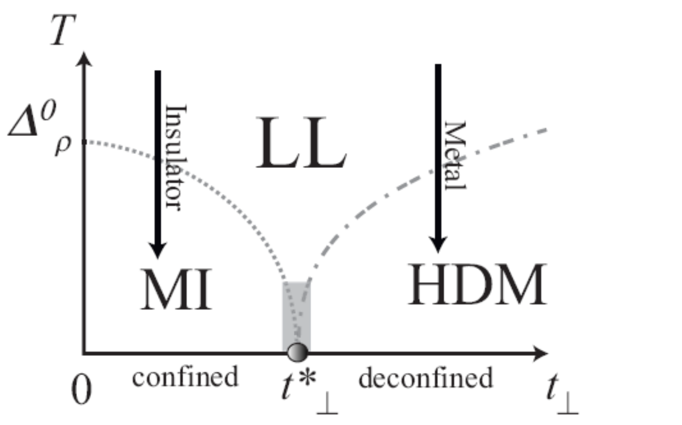


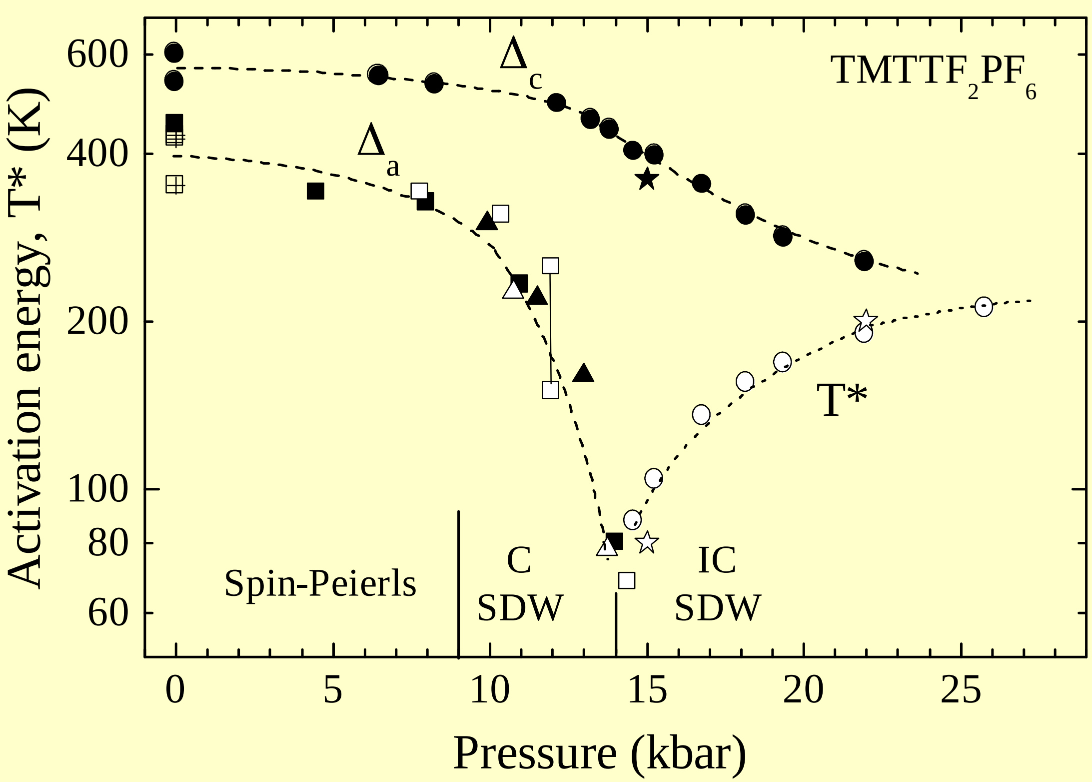


TG Chemical Review 104 5037 (2004)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Deconfinement

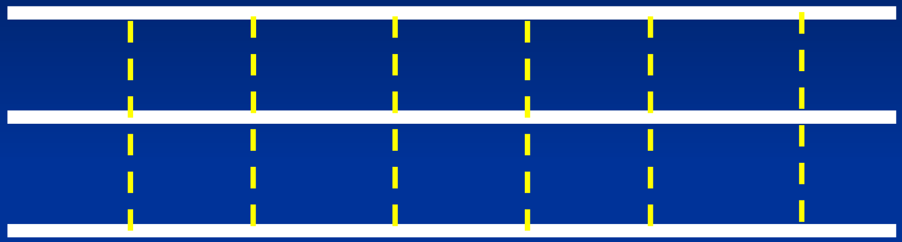




TG Chemical Review 104 5037 (2004)

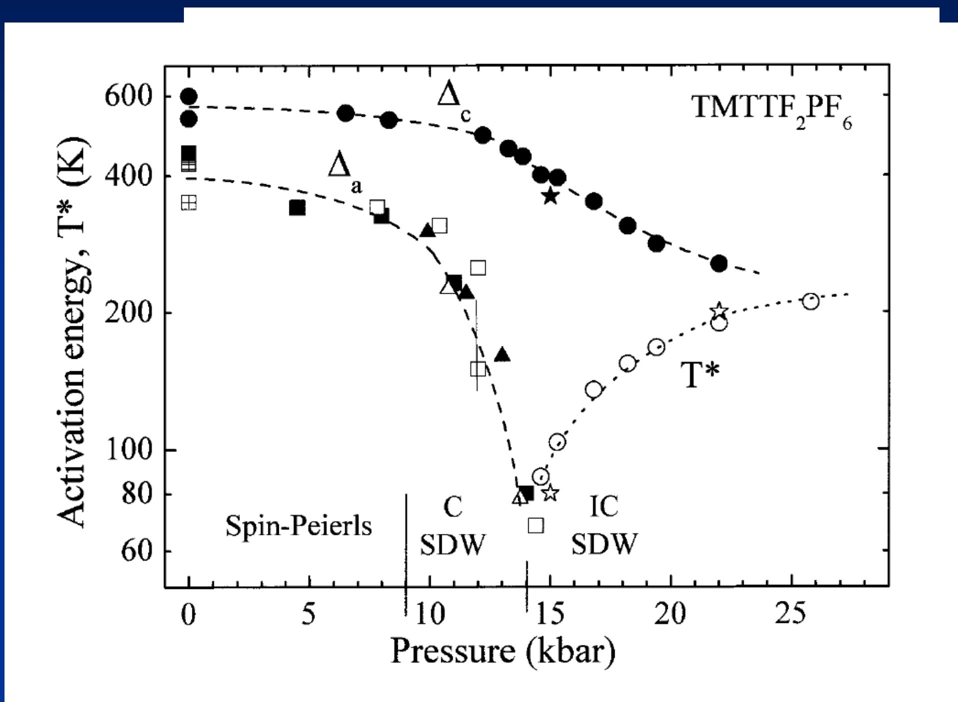
P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, PRB 81

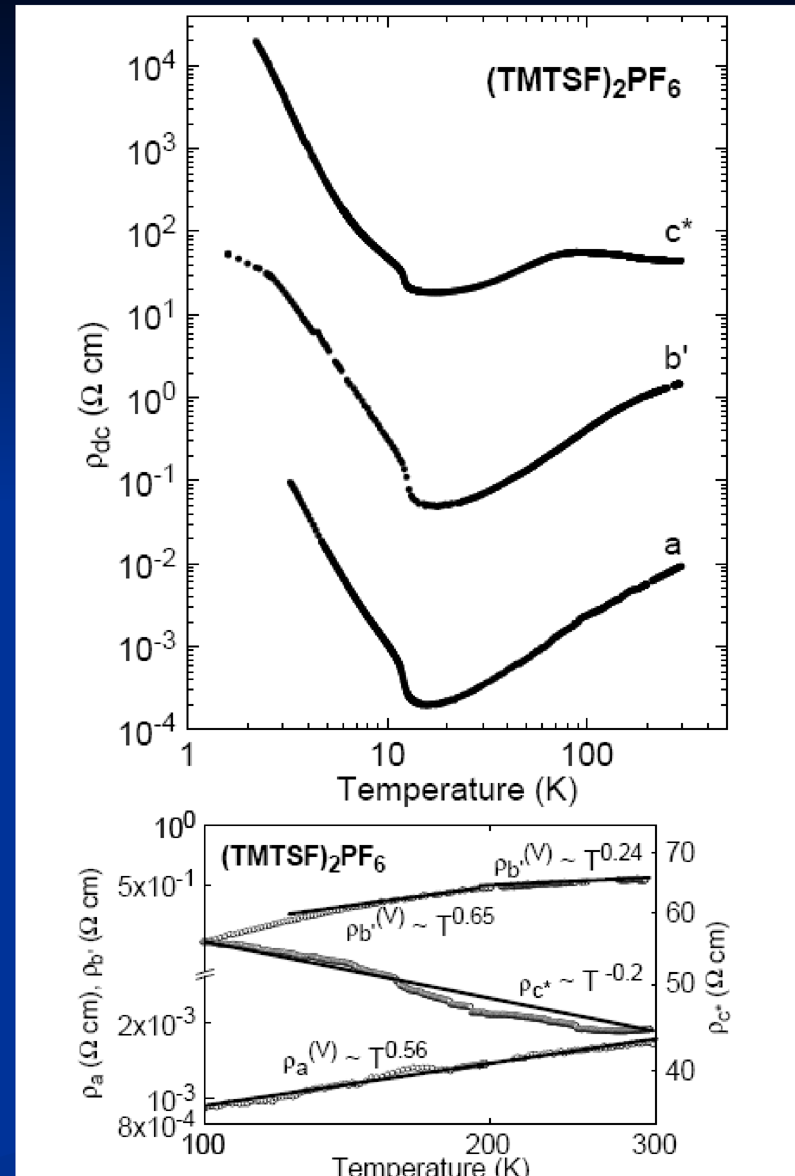
Transverse transport



Perpendicular hopping t’

T > t’: tunneling, not usual transport





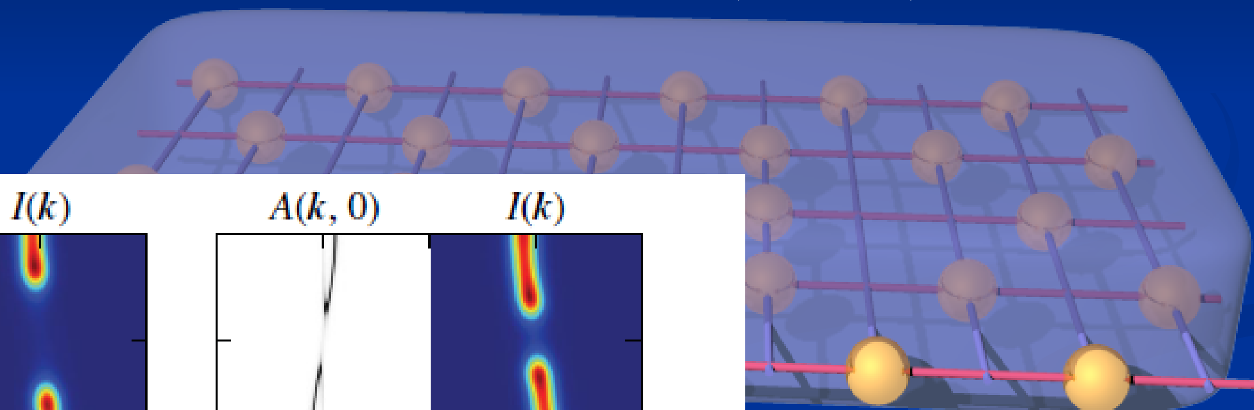
V. Vescoli et al. Science 281 191 (1998), Euro Phys J B 13 503 (2000)

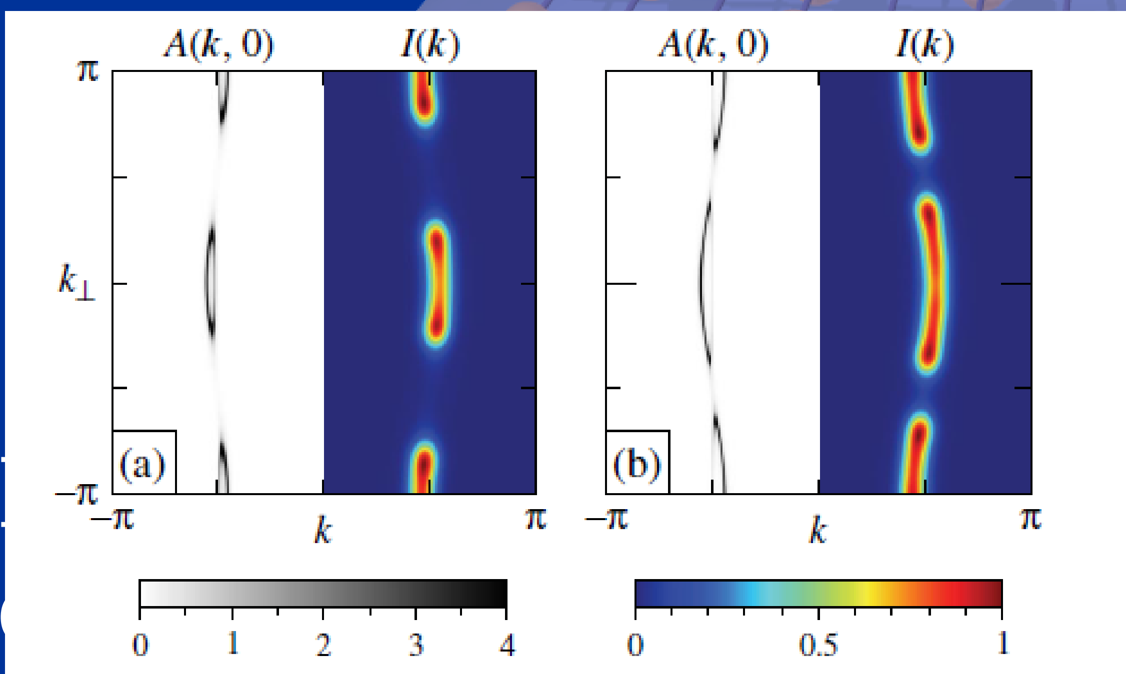
P. Auban-Senzier et al. J. Phys IV, 114 (2004)

M. Dressel *et al.*, Phys. Rev. B **71**, 075104 (2005).

Chain (Cluster) - DMFT

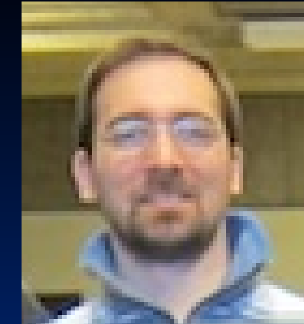
S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)





stently

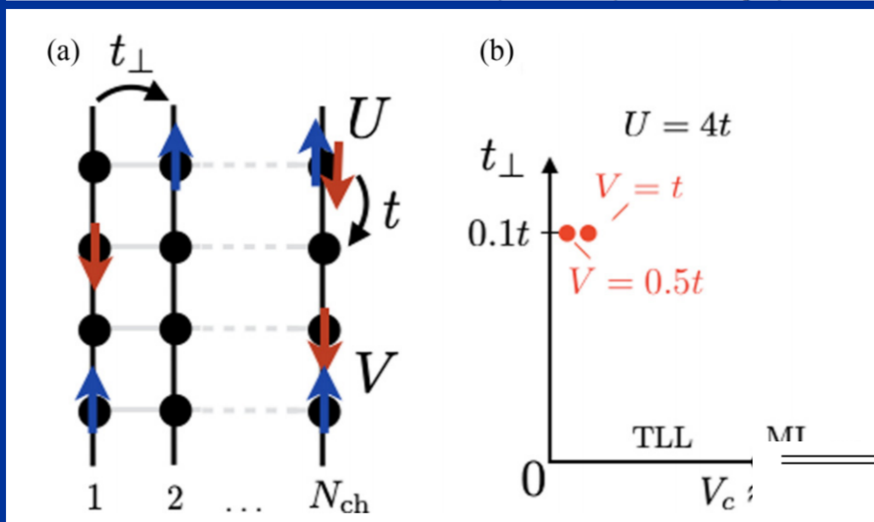
DMRG



PHYSICAL REVIEW B **100**, 075138 (2019)

Understanding repulsively mediated superconductivity of correlated electrons via massively parallel density matrix renormalization group

A. Kantian,[1] M. Dolfi,[2,3,*] M. Troyer,[2] and T. Giamarchi[4]



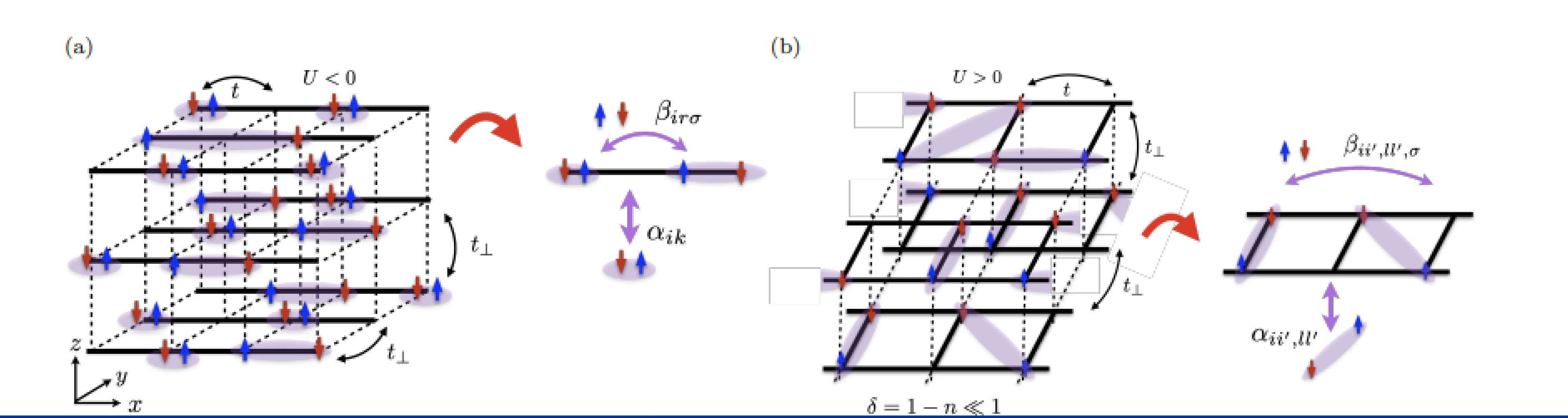
	$\overline{\Delta}_s[N_{\mathrm{ch}}]$ $[t \times 10^{-3}]$			
	Straight extrapolation		Extr. + estimates	
N_{ch}	$V/t = 0.5$	$V/t = 1$	$V/t = 0.5$	$V/t = 1$
2	5.59	7.59	5.36	7.31
4	9.68	10.2	10.77(10)[a]	10.268(11)
6	9.37	11.4	4.3(1.4)[b]	11.20(97)
8	9.45		7.6(1.3)[c]	

DMRG + Mean Field

Solving 2D and 3D lattice models of correlated fermions - combining matrix product states with mean field theory

Gunnar Bollmark,[1] Thomas Köhler,[1] Lorenzo Pizzino,[2] Yiqi Yang,[3] Johannes S. Hofmann,[4] Hao Shi,[5] Shiwei Zhang,[6] Thierry Giamarchi,[2] and Adrian Kantian[1,7]

Phys. Rev. X **13**, 011039 – Published 15 March 2023

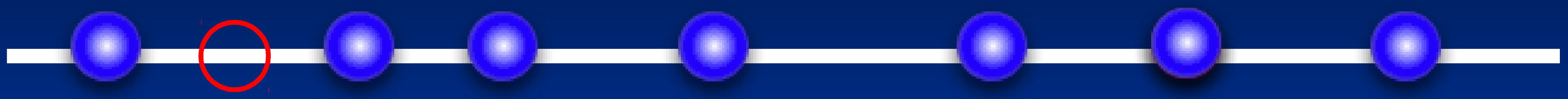


Intermediate systems (system + bath)

Problem

- One impurity coupled to a 1D quantum bath
- Equilibrium physics: ``polaron’’
- Not so ``simple’’ if out of equilibrium: Anderson orthogonality catastrophy

Global vs local quench



- Local quench
- X-ray edge problem
- M finite: destroyed by recoil, but not in 1D !

How to treat ?

- Bosonization

$$H = \frac{\hbar}{2\pi} \int dx [\frac{uK}{\hbar^2} (\pi \Pi(x))^2 + \frac{u}{K} (\nabla \phi(x))^2]$$

- u,K : depend on interactions, density

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi} \nabla \phi(x)\right] \sum_p e^{i2p(\pi \rho_0 x - \phi(x))}$$

Mobile impurity

M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007):

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} [g\delta(x_i - x_j) + U(x_i - x_j)]$$

$$\frac{m^*}{m} = \frac{3\gamma}{2\pi^2}$$

$$\gamma = mg/\hbar^2\rho_0$$

Single spin down pa

$$G_\perp(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

$$\sum_q \epsilon(q) c_q^\dagger c_q + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b_{-k}^\dagger) \rho_\downarrow(-k)$$

General result

$$G_\perp(x,t) \simeq t^{-\alpha}\left[\beta \ln\left(\frac{t}{t_F}\right) + \frac{it\hbar}{2m_*}\right]^{-1/2} \times \exp\left\{\frac{im_* x^2}{2t\hbar - 4i\beta m_* \ln(t/t_F)}\right\}.$$

Comparison with trapped regime:

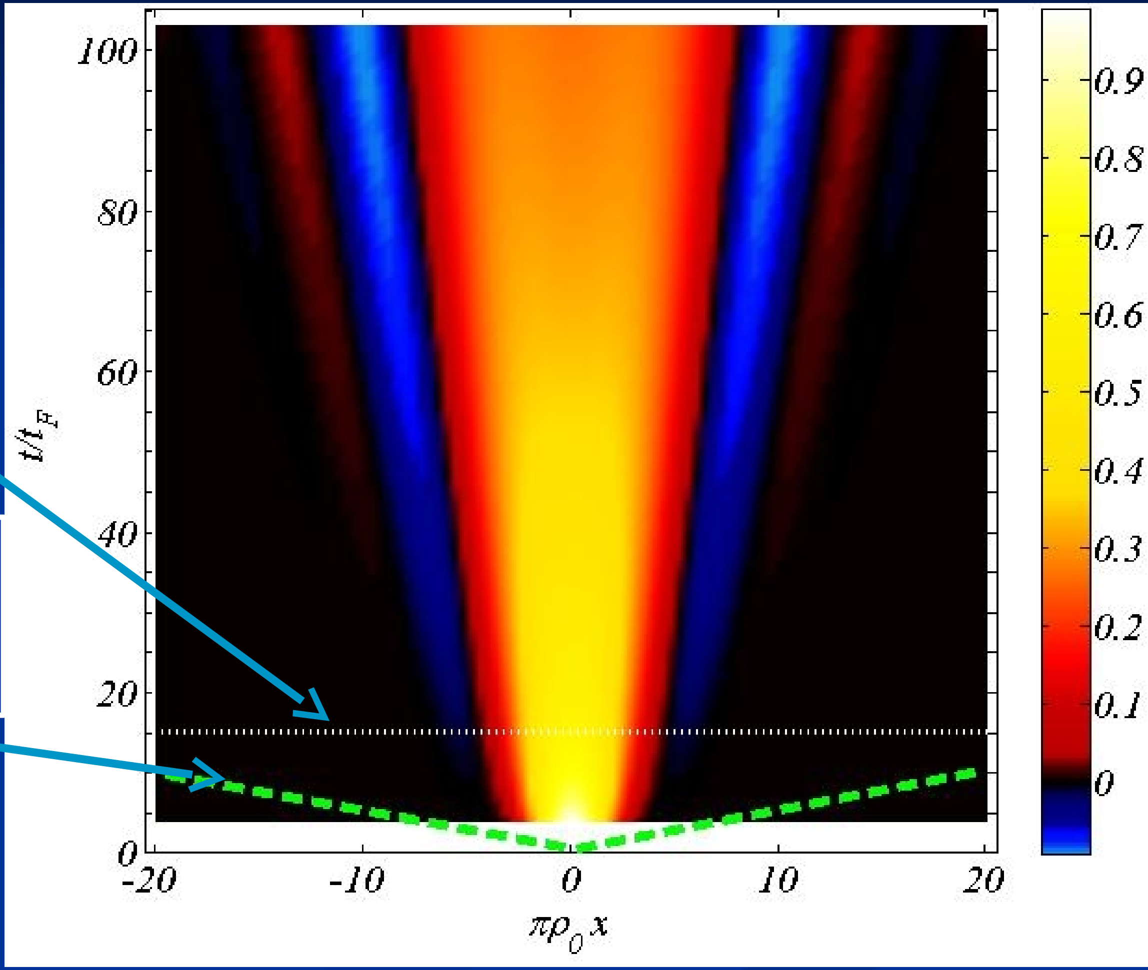
$$\alpha = 0, \qquad \beta = \frac{K}{2(\pi\rho_0)^2}$$

Exponent depends only on K, new universality class : Ferromagnetic

Propagation of the impurity

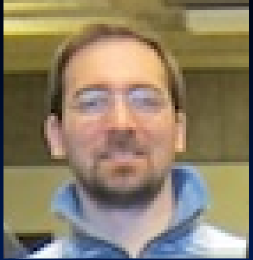
Trapped/ope n regimes

Light cone of spinless bosons



$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

$$G_\perp \simeq e^{-(x^2/2\ell^2)} t^{-\alpha} G_\perp^H, \qquad \ell(t) = \frac{2K^{-(1/2)}}{\pi\rho_0}\frac{t/t_F}{\sqrt{\ln t/t_F}}\frac{m}{m_*}.$$



Two regimes: Infrared dominated vs polaronic

A. Kantian, U. Schollwoeck, TG, PRL 113 070601 (2014)

$$A(p,\omega) \propto \frac{\theta(\omega - \epsilon_p)}{(\omega - \epsilon_p)^{\Delta(p)}}$$

$$\Delta(p) \approx \Delta(0) + \beta p^2$$

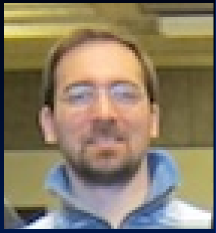
- Very efficient method: linked cluster

$$G_{\mathrm{LCE}}(p,t) = -ie^{-i\varepsilon_p t} e^{F_2(p,t)},$$

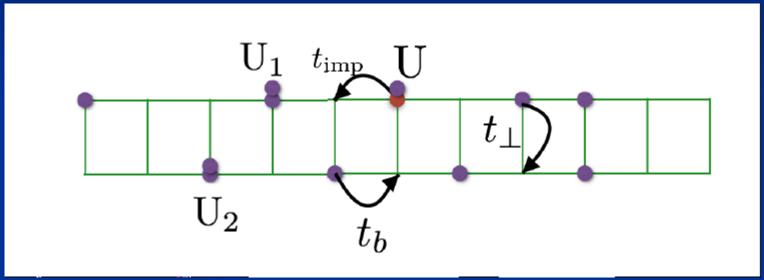
$$F_2(p,t) = \int du \frac{1 + itu - e^{itu}}{u^2} R(u),$$

$$R(u) = \int dq V(q)^2 \delta(u + \varepsilon_p - \varepsilon_{p+q} - v|q|),$$

Towards 2D: ladders



N.A. Kamar, A. Kantian, TG, PRA 100 023614 (2019)



$$H_s = \frac{1}{2\pi}\int dx\left[u_s K_s(\partial_x\theta_s)^2 + \frac{u_s}{K_s}(\partial_x\phi_s)^2\right],$$
$$H_a = \frac{1}{2\pi}\int dx\left[u_a K_a(\partial_x\theta_a)^2 + \frac{u_a}{K_a}(\partial_x\phi_a)^2\right]$$
$$- 2\rho_0 t_\perp \int dx\, \cos[\sqrt{2}\theta_a(x)].$$

- Mixture of massive and massless modes

A. $U \ll \Delta_a$

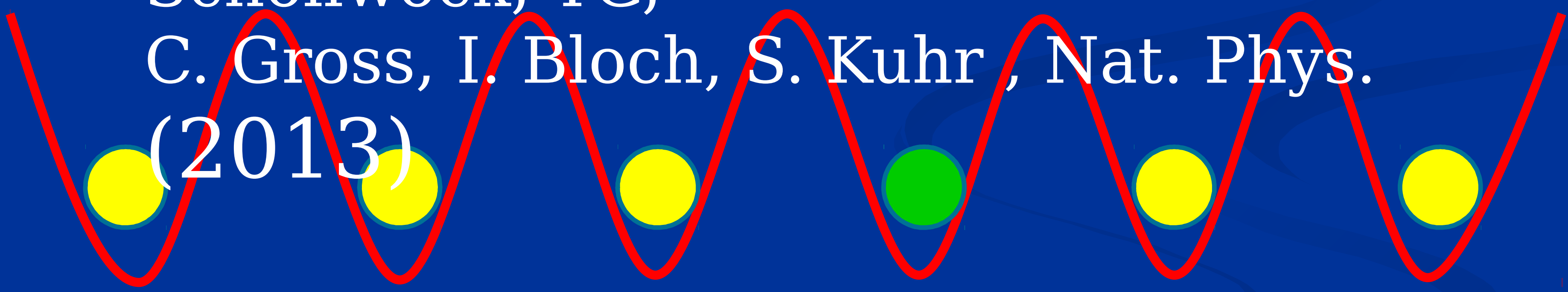
B. $U \gg \Delta_a$

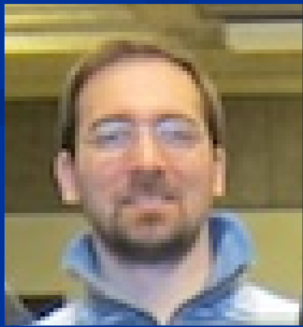
$$|G(p,t)| = e^{-\frac{K_s U^2}{4\pi^2 u_s^2}\left(1+\frac{12 t_{\rm imp}^2 p^2}{u_s^2}\right)\ln(|t|)}.$$

$$= |t|^{-K_s/4\left(\frac{U_\phi}{\pi u_s}\right)^2}.$$

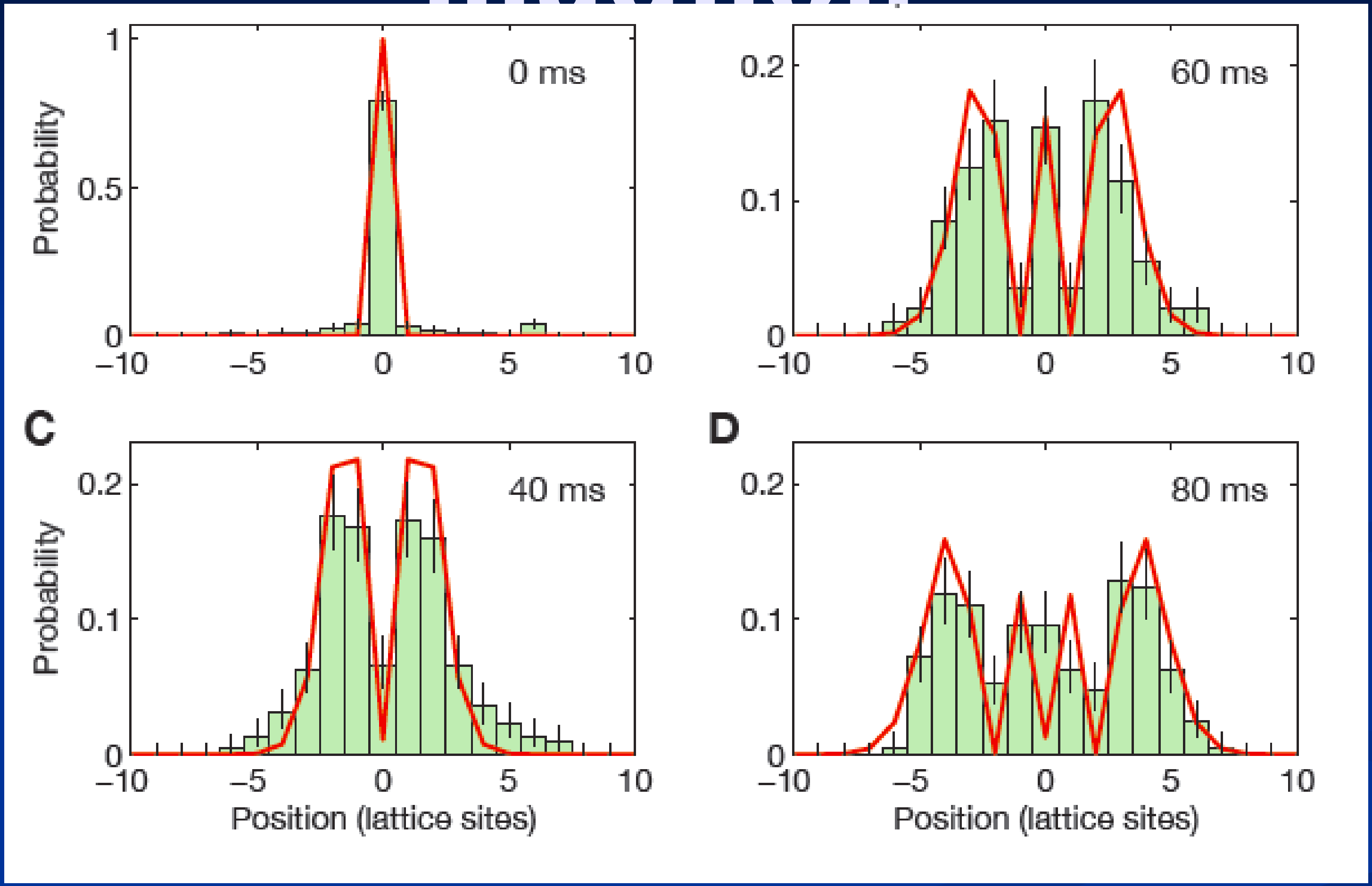
Mobile impurity

T. Fukuhara, A. Kantian, M. Endres, M. Cheneau,
P. Schauss, S. Hild, D. Bellem, U. Schollwock, TG,
C. Gross, I. Bloch, S. Kuhr , Nat. Phys. (2013)





Coherent propagation of a magnon



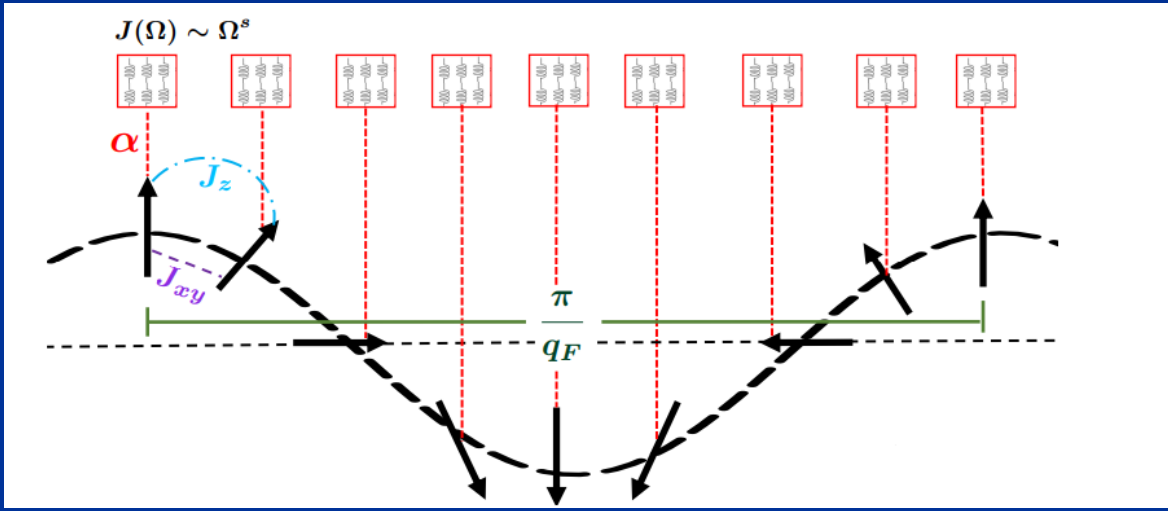
Heisenberg model :

Other baths ?

arXiv:2307.07989 [pdf, other] cond-mat.dis-nn

Localization induced by spatially uncorrelated subohmic baths in one dimension

Authors: Saptarshi Majumdar, Laura Foini, Thierry Giamarchi, Alberto Rosso



$$\frac{dn_A(t)}{dt} = -\gamma n_A(t).$$

arXiv:2305.13090 [pdf, other] cond-mat.quant-gas

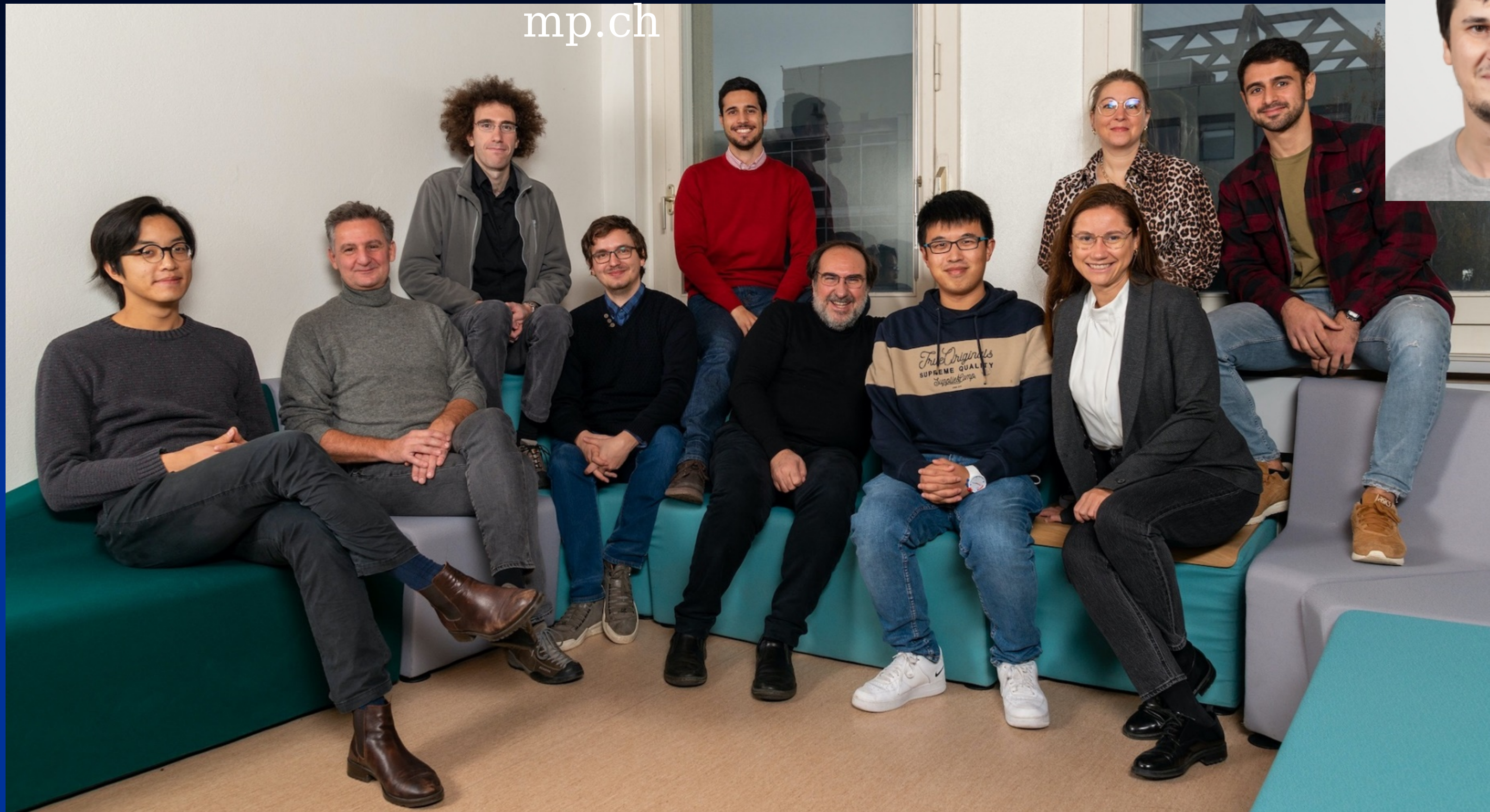
Modeling Particle Loss in Open Systems using Keldysh Path Integral and Second Order Cumulant Expansion

Authors: Chen-How Huang, Thierry Giamarchi, Miguel A. Cazalilla

Conclusions

- Tour of one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful open problems: out of equilibrium, disorder, coupled chains, impurities, etc…….
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Remarkable and complementary realization in condensed matter and

http://giamarchi.dqmp.ch



(T. Jin, C. Berthod, G. Morpurgo, C. Halati, J. Ferreira, TG, H. Yao, N. Caballero, F. Hartmeier, L. Pizzino, L. Gotta)

Spin systems: A. Tsvelik, P. Bouillot, C. Kollath, …

Exp Groups: C. Berthier, B. Grenier, C. Ruegg, V. Simonet, A. Zheludev, …

Cold atoms: M. Cazalilla, A. Ho, M. Zvonarev, V. Cheianov, U. Schollwoeck, P. Torma, …

Exp Groups: I. Bloch, J.P. Brantut, T. Esslinger, R. Hulet, M.