Workshop Numerical Schemes for SDEs and SPDEs, Lille June 1-3, 2016

Program

- 1st June 8h30-9h-30 : Welcome
9h30-11h : D. Talay (mini course)
11h-11h30 : Coffee Break
11h30-12h30 : P. Etoré
12h30-14h : Lunch
14h15-15h15 : Y. Ouknine
15h15-16h15 : K. Zygalakis
16h15-16h45 : Coffee Break
16h45-17h45 : A. Lejay
19h30 : Social Diner : Restaurant "La Fossetta" in the City Center, 15 rue des fossés
- 2nd June 8h30-10h : D. Ounaissi (Thesis defense)
10h-10h30 : Coffee Break
10h30-11h30 : D. Talay (mini course)
11h30-12h30 : M. Giles
12h30-13h45 : Lunch
14h-15h : E. Gobet
15h-16h : G. Pichot
16h-16h30 : Coffee Break
16h-17h : E. Tanré

- 3th June
8h-30-9h30 : A. Jentzen
9h30-10h30 : S. Mazzonetto
10h30-11h : Coffee Break
11h-12h L. Lenôtre

Pierre Etoré

Title. Time inhomogeneous Stochastic Differential Equations involving the local time of the unknown process, and associated parabolic operators

Abstract. In this work we first aim at extending the results of J.F. Le Gall (1984) on Stochastic Differential Equations with Local Time (SDETL), to the case where all the coefficients appearing in the SDETL depend on time. Once this is done and we have get the existence of a unique strong solution X to the SDETL in the time-inhomogeneous case, we investigate the question of the parabolic operator associated to the process X. First we prove a Feynman-Kac formula linking X and the solution $u(t,x)$ of some parabolic transmission problem. In fact we have to prove the existence of this solution $u(t,x)$, because the result is not directly provided by the seminal paper by O.A. Ladyzhenskaya et al. (1966), studying this kind of problems in a purely divergence form. Then we aim at identifying the generator of the Feller process consisting in the space-time process associated to X. Extensions and simulation issues are finally discussed.

Mike Giles

Title. Adaptive timestepping for SDEs with non-globally Lipschitz drift

Abstract. This talk looks at the use of the standard Euler-Maruyama discretisation with adaptive timesteps for SDEs with a drift which is not globally Lipschitz. It is proven that under certain conditions all moments of the numerical solution remain bounded for any finite time interval, and the strong order of convergence is the same as usual, when viewed as accuracy versus expected cost per path. Furthermore, for SDEs which satisfy a certain contraction property, similar results hold over the infinite time interval. The strong convergence results are relevant to the use of this discretisation for Multilevel Monte Carlo simulation. (Joint research with Wei Fang)

Emmanuel Gobet

Titre. Convergence rate of strong approximations of compound random maps

Abstract. We consider a random map $x \mapsto F(\omega, x)$ and a random variable $\Theta(\omega)$, and we denote by $F^N(x)$ and $\Theta^N(\omega)$ their approximations : We establish a strong convergence result, in L_p-norms, of the compound approximation $F^N(\omega, \Theta^N(\omega))$ to the compound variable $F(\omega, \Theta(\omega))$, in terms of the approximations of F and Θ. This result allows for instance to study the approximation of composition of two SDEs through their initial conditions, and give numerical solutions to some SPDEs arising from stochastic utilities in mathematical finance. Other applications will be given.

Arnulf Jentzen

Title. Nonlinear stochastic ordinary and partial differential equations : regularity properties and numerical approximations

Abstract. In this talk we present a few! recent results on regularity properties and numerical approximations for stochastic ordinary and partial differential equations with non-globally Lipschitz continuous nonlinearities. In particular, we establish strong convergence of an explicit and easily-implementable full-discrete numerical approximation scheme for stochastic Kuramoto-Shivashinsky equations, for stochastic Burgers equations, for stochastic Ginzburg-Landau equations, and for stochastic Navier-Stokes equations. The talk is based on joint works with Martin Hairer, Martin Hutzenthaler, Thomas Mueller-Gronbach, Diyora Salimova, and Larisa Yaroslavtseva.

More details on this topic can also be found at
https ://www.math.ethz.ch/sam/research/projects.html ?details=33.

Antoine Lejay

Title. Modelling and simulating diffusions processes with discontinuous coefficients.

Abstract. In this talk, we present some issues regarding the modelling simulation of diffusion processes with discontinuous coefficients or moving in media with permeable or semi-permeable barrier. In particular, we explain their relationship with the Skew Brownian motion, the importance of the

local time. We also explain the strong relationship with partial differential equations with discontinuous solutions or fluxes.

Lionel Lenôtre

Title. Simulation of Diffusion Processes on the Real Line.

Abstract. Diffusion processes on the real line are involved in various applications such as geophysics, brain imaging, finance, biology, etc... While many numerical methods exist to simulate diffusion processes when they can be represented as a classical stochastic differential equations, it is a challenge to simulate them when there is not. Such a situation is encountered in geophysics when one wants to simulate the transport of pollutants in highly heterogeneous media.

In this presentation, we provide a solution to a part of the problem presented above through an algorithm for simulating Feller processes on the real line possessing an infinitesimal generator that is strictly of the second order. Without entering into details, this algorithm make use of the resolvent kernels of these Feller processes. In order to illustrate the deployment of this algorithm, we treat the case of a Feller process with piecewise constant coefficients.

Sara Mazzonetto

Title. Exact simulation of Brownian diffusions with drift admitting jumps.

Abstract. In this talk we propose a theoretical retrospective rejection sampling scheme for Brownian diffusions whose drift admits a finite number of jumps. To do this, we provide an explicit representation of the transition density of Brownian motions with drift perturbed by semipermeable barriers (i.e. skew Brownian motions), in the case of two jumps. We also propose a generalised rejection sampling method for sampling from such densities.

Geraldine Pichot

Title. Comparison of algorithms for the simulation of diffusion processes in one-dimensional discontinuous media.

Abstract. In this presentation, we describe benchmark test cases designed to quantify the bias of schemes simulating diffusion in one dimensional discontinuous media. We apply these test cases to four schemes with constant time steps. It gives rules to choose between these schemes according to the physical quantities of interest.

Youssef Ouknine

Title. Reflected BSDEs when the obstacle is not right-continuous and limit theorems.

Abstract. In the first part of my talk, we study reflected backward stochastic differential equations (RBSDEs) with lower obstacle which is assumed to be right upper-semicontinuous but not necessarily right-continuous. We prove existence and uniqueness of the solutions to such RBSDEs in appropriate Banach spaces. The result is established by using some tools from the general theory of processes such as Mertens Doob Meyer decomposition of optional strong (but not necessarily right continuous) supermartingales, some tools from optimal stopping theory, as well as an appropriate generalization of Itô's formula due to Gal'chouk and Lenglart. In the second part of my talk, some limit theorem are proved (monotonic limit theorem, penalization etc....) Dynkin Game are introduced to solve reflected BSDE with two barrieres . this caraterize the so called Mokobodski hypothesis .

Denis Talay

Title. Mathematical analysis of singular computational models.

Abstract. Stochastic computational models are used to simulate complex physical or biological phenomena and to approximate (deterministic) macroscopic physical quantities by means of probabilistic numerical methods. By nature, most often they involve singularities and therefore their analysis needs the development of specific techniques.

We will give examples of recent developments and open questions with a particular emphasis on stochastic particle systems with singular interactions and the related novel issues in stochastic analysis and PDE analysis.

Etienne Tanré

Title. A particle system for singular non-linear McKean-Vlasov stochastic differential equations.

Abstract. We build and approximate the solutions to a nonlinear McKean-Vlasov equation driven by a singular self-excitatory interaction of the mean-field type.

Such an equation is intended to describe an infinite population of neurons which interact with one another : The state of the neurone is characterizes by the value of its membrane potential, assumed to solve a McKean-Vlasov equation. A neuron 'spikes' at the hitting time of a fixed threshold by its membrane potential. Each time a proportion of neurons 'spike', the whole network instantaneously receives an excitatory kick. The instantaneous nature of the excitation makes the system singular and prevents the application of standard results from the literature.

Making use of the Skorohod M1 topology, we prove that, for the right notion of a 'physical' solution, the nonlinear equation can be approximated either by a finite particle system or by a delayed equation. As a by-product, we obtain the existence of 'synchronized' solutions, for which a macroscopic proportion of neurons may spike at the same time. [Common work with F. Delarue (Univ. Nice), J. Inglis (Inria) and S. Rubenthaler (Univ. Nice)]

Konstontinos Zygalakis

Title. Long Time Approximation of Ergodic SDEs and Multi-Level Monte Carlo.

Abstract. Understanding the long time behaviour of solutions to ergodic stochastic differential equations is an important question with relevance in many field of applied mathematics and statistics. Hence, designing appropriate numerical algorithms that are able to capture such behaviour correctly is extremely important. One aspect of correct design is the accuracy with which one approximates quantities of interest. In the general case, any traditional numerical analysis approach based on simply discretizing the underlying SDE, introduces bias in such estimation, which however can be removed if an accept-reject step is added to the algorithm, as done in the computational statistics approach. In addition, the computational complexity (when measured in the number of random variables used by the algorithm) of the numerical analysis approach can be an order higher than the computational statistics approach. In this talk, we develop a framework that allows the use of the multi-level Monte Carlo (MLMC) methodology to calculate quantities of interest. In that context, we study the (over-damped) Langevin equations with strongly convex potential and show that, when appropriate contracting couplings for the numerical integrators are available, one can obtain a time-uniform estimates of the MLMC variance in stark contrast to the majority of the results in the MLMC literature. As a consequence, one can approximate expectations with respect to the invariant measure in an unbia-

sed way without the need of a Metropolis- Hastings step, while having the same computational complexity as in the computational statistics approach.